# Financial Frictions, FX Reserves, and Exchange Rate Management in Emerging Economies

Sauhard Srivastava*

Department of Economics, University of Minnesota

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**Abstract**

Emerging economies, even those with considerable external debt, hold substantial amounts in foreign exchange (FX) reserves. This paper identifies a distinct channel through which financial market frictions explain why net external debtor economies might prefer maintaining reserves to reducing their external debt. In a small open economy with free capital mobility and financial frictions, the model shows that the central bank optimally maintains FX reserves instead of reducing the economy's external debt. This is because reserve operations influence the exchange rate; specifically, reserve accumulation depreciates the exchange rate, diluting the real value of existing debt payments and minimizing resource losses. Furthermore, the model shows that the optimal reserve accumulation policy under commitment is time-inconsistent as the central bank faces incentives to mitigate the external debt burden. A time-consistent equilibrium features even greater reserve accumulation. Finally, a quantitative analysis of the model demonstrates that in the presence of volatile capital flows, the economy optimally maintains a portfolio of external debt and foreign reserves with FX interventions stabilizing the exchange rate and smoothing consumption.

**Keywords:** Foreign exchange reserves, financial frictions, FX interventions, exchange rate depreciation, debt dilution, time-inconsistency.

**JEL Classifications:** E21, E44, E58, F31, F32, F34, G15.

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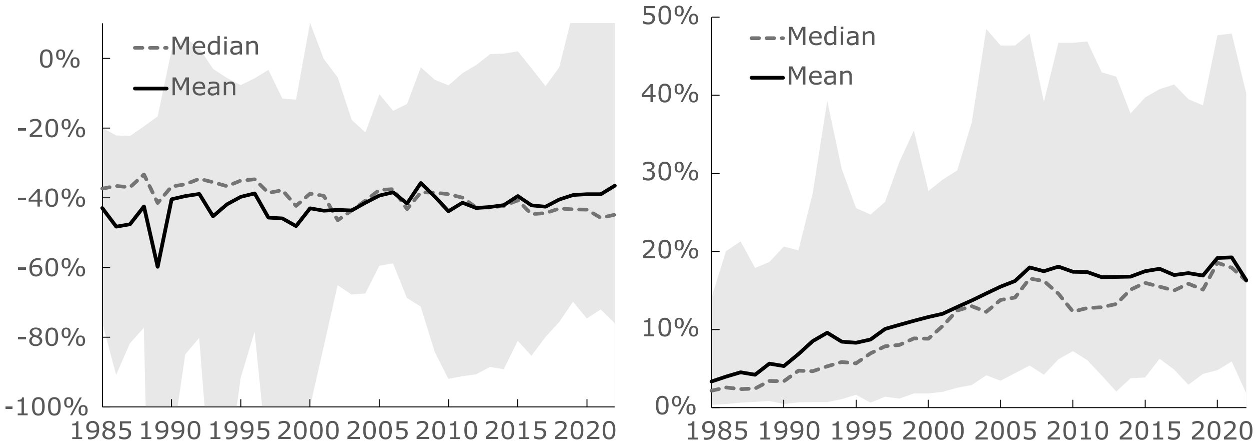
# 1 Introduction

It is well-documented that central banks in emerging economies accumulated substantial amounts of foreign exchange (FX) reserves between the mid-1980s and mid-2000s. They continue to maintain exceptionally high reserves, far exceeding those held by most advanced economies. Notably, however, even net external debtor economies are holding considerable amounts of reserves (Figure 1). This raises the question: why are net debtors holding such large amounts of reserves? Typically, foreign exchange reserves are low-interest-bearing assets, while external debt carries an interest premium. Thus, it seems counterintuitive not to use these reserves to reduce the country's debt. This paper identifies a distinct channel through which international financial market frictions could rationalize why debtors might prefer holding reserves to reducing their debt.

The proposed theory is motivated by examining how this reserves trend relates to broader changes in international financial markets and central banks' exchange rate policies. Two empirical facts are particularly relevant. First, this reserve accumulation has coincided with an increase in international capital mobility and the imposition of exchange rate controls in emerging economies. Second, countries that impose exchange rate controls hold significantly higher reserves than countries with free-floating currencies. These findings are discussed extensively by Ilzetzki, Reinhart and Rogoff (2019) and summarized in Appendix A.

These facts suggest an underlying relationship between reserve accumulation and exchange rate controls in a world of increased capital mobility. The existing literature has proposed a variety of explanations, the most common being a mercantilist motive, i.e., countries build reserves to undervalue their exchange rate and maintain export competitiveness. While this motive might explain reserve accumulation by persistent surplus economies, it is less applicable to debtor economies. Moreover, it fails to account for the simultaneous trends of reserve accumulation and expansion of global financial markets.

To this end, this paper develops a theory linking reserve accumulation to exchange rate controls, while emphasizing the role of international financial markets. Existing economic theory has not only addressed this minimally, but has also traditionally argued against the effectiveness of exchange rate controls in the presence of free capital mobility.(see, e.g., Backus and Kehoe (1989)). A Ricardian equivalence logic suggests that reserve accumulation by public agents would be offset by increased external borrowing by private agents, so that only the net foreign asset position of the economy matters, implying that exchange rates cannot be influenced by reserve operations. If




(a) IIP to GDP (excluding reserves) (b) Reserves to GDP

Notes: A sample of 20 largest (by 2022 GDP) persistent deficit emerging economies i.e., those which are net external debtors (excluding reserves), is considered. For a list of countries in the sample, see Appendix A. The left panel shows the mean and median international investment position (IIP) of this sample, and the right panel shows the official foreign exchange reserves. The shaded region represents the maximum and minimum. In 2022, the median IIP was -44.86% and median reserves were 16.27% in this sample. Source: Computed using the External Wealth of Nations database (Milesi-Ferretti 2022).

Figure 1: Net-external debtors holding reserves

this were the case, then an economy that holds both debt and reserves would benefit from using all its reserves to pay off its debt.

However, recent literature has emphasized the significance of frictions in international financial markets. In the presence of such frictions, this equivalence result breaks, and FX interventions can indeed be effective. These frictions create a wedge between domestic and international markets, making the composition of external assets and liabilities - not just their net position - relevant for exchange rate dynamics. Building on this framework, this paper establishes that in a world with free capital mobility but with financial frictions, it may not be optimal for a debtor economy to use its reserves to pay off its debt.

The paper presents a model of a pure exchange small open economy (SOE) consisting of households, international financial intermediaries, and a central bank. While there is free capital mobility, the economy faces financial market frictions in the spirit of Gabaix and Maggiori (2015). The financial markets are segmented so that the SOE does not have direct access to international markets, but can only access domestic markets where agents trade bonds denominated in domestic currency. The economy's connection to the international financial markets is through the intermediaries who access both domestic and international markets and trade bonds denominated in both local and international currency. Crucially, there is a limited repayment commitment on the part of financial intermediaries: they can divert funds after taking asset positions and therefore face a credit constraint. This friction endogenously results in the failure of uncovered interest parity (UIP). Rather, the extent of UIP deviation matters, and becomes a key determinant of the amount of assets supplied by the intermediaries in the domestic bonds market. Finally, the central bank has direct access to saving (but not borrowing) in foreign assets, i.e., reserves.

First, the paper describes the equilibrium dynamics of the exchange rate over the infinite horizon. As established by Gabaix and Maggiori (2015), the friction makes the asset price role of the exchange rate more prominent. Specifically, the currency appreciation rate, rather than simply being determined by the UIP condition, becomes a key equilibrium variable that clears the domestic bonds market. The exchange rate adjusts in response to exogenous changes in international financial flows- appreciating during booms and depreciating during disruptions.

Secondly, as established in the literature, the friction makes FX interventions effective by preventing households from borrowing one-for-one in response to reserve accumulation by the central bank. Absent the friction, only the net foreign asset position matters. However, with the friction, the distinct choices of household borrowing and central bank reserve accumulation generate different outcomes for the exchange rate and consumption.

Given this, and considering an economy that begins with both external debt and foreign reserves, this paper establishes that a benevolent central bank that has the option to run down its reserves to zero and reduce the country's external debt may not find it optimal to do so. This is because such an intervention induces an exchange rate appreciation. On one hand, this appreciation allows the household to borrow lesser, but on the other hand, since debt is denominated in domestic currency, this appreciation also increases the real value of existing debt payments thereby leading to a larger resource loss. Said differently, central bank's reserve accumulation, while making the household borrow more, also induces an exchange rate depreciation that dilutes the real value of existing debt payments. The central bank internalizes these general equilibrium effects of reserve operations, and at the optimum, equates the marginal benefits of diluting the real value of previous debt payments to the marginal costs associated with increased household borrowing.

Third, given that the intermediaries' supply of assets is an increasing function of the appreciation rate, this paper shows that the central bank's optimal reserve policy under commitment; one where it announces future exchange rates (or future reserve holdings), is time-inconsistent. This time-inconsistency problem arises from the fact that the central bank's future exchange rate policy affects current private borrowing/lending decisions. The central bank has an incentive to announce a higher future exchange rate (and thus a lower appreciation rate) to reduce intermediaries' current lending and thereby mitigate households' future debt burden. However, once the future period arrives, now taking the previously borrowed amount by households as given, the central bank finds it sub-optimal to implement the announced plan, opting instead for a lower-than-announced exchange rate to allow for higher household consumption.

Consequently, if a central bank lacks commitment and deviates to a lower-than-promised exchange rate in the future (resulting in a higher ex-post appreciation rate), the intermediaries anticipate this, and as a result, lend more in the present. The central bank then responds by accumulating even more reserves to prevent an excessive exchange rate appreciation. Thus, this paper shows, that a time-consistent equilibrium features even greater reserve accumulation than under commitment.

Finally, the paper presents a quantitatively solved version of the time-consistent equilibrium in the presence of exogenous shocks to financial flows. It is shown that, given this capital flow volatility, the central bank follows a leaning-against-the-wind reserve accumulation policy where it builds reserves in times of financial booms and runs down reserves in times of disruptions. While reserve stockpiling in financially stable times induces an exchange rate depreciation, reduces household consumption, and increases household borrowing; in times of disruptions, these reserves allow the central bank to decrease the extent of capital outflow led exchange rate depreciation, thereby providing higher consumption to the household. In essence, this procyclical reserve accumulation policy serves as an insurance against volatile financial flows, allowing for a relatively stable exchange rate and a smoother stream of consumption. As a result, managed floating emerges as the optimal policy regime in this model, with the economy maintaining a portfolio with both external debt and foreign reserves in the long-run equilibrium.

The rest of this paper is organized as follows. Section 2 summarizes the related literature. Section 3 presents the model and the main analytical results of this paper. Section 4 contains a quantitative analysis of the model. Section 5 concludes.

## 2 Related Literature

This paper is primarily related to the literature studying motives for foreign exchange reserve accumulation. This has been the subject of extensive empirical and theoretical literature going back to the 1980s. A variety of motives for reserve accumulation have been proposed, most commonly, mercantilist motives (Aizenman and Lee 2007), growth and capital accumulation motives (Korinek and Servén 2016; Benigno, Fornaro and Wolf 2022), precautionary and self-insurance motives (Gourinchas and Obstfeld 2012; Durdu, Mendoza and Terrones 2009), and safe-asset motives (Caballero, Farhi and Gourinchas 2017). More recently, studies have linked international reserves to macroprudential policy (Arce, Bengui and Bianchi 2022; Davis, Devereux and Yu 2023; Jeanne and Sandri 2023), lender of last resort policies (Bocola and Lorenzoni 2020), and sovereign default risk hedging (Bianchi and Sosa-Padilla 2023; Barbosa-Alves, Bianchi and Sosa-Padilla 2024).

This paper is also related to the literature on reserves and exchange rate controls that goes back to Krugman (1979). A benchmark result on the ineffectiveness of FX interventions was established by Backus and Kehoe (1989). More recently, the literature has highlighted the role of frictions in financial markets that breaks the benchmark ineffectiveness result. Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021) discuss the role of financial frictions in general equilibrium models of exchange rate determination and have emphasized the significance of the financial sector in influencing the exchange rates. Amador, Bianchi, Bocola and Perri (2020) highlight the role of FX interventions at the zero lower bound in an environment with limits to international arbitrage. Fanelli and Straub (2021) discuss optimal exchange rate policies in an environment with distributional consequences. Bianchi and Lorenzoni (2022) discuss and compare capital controls and exchange rate controls in the presence of frictions and aggregate demand externalities. Itskhoki and Mukhin (2023) characterize the linkages between exchange rates and monetary policies in the presence of nominal rigidities. Basu, Boz, Gopinath, Roch and Unsal (2020) discuss the interactions between exchange rate, capital control, macroprudential, and monetary policy instruments in an environment with financial, trade, and housing frictions and characterize optimal policies.

# 3 Model

This section presents a dynamic general equilibrium model of a pure exchange small open economy (SOE) with free capital mobility but facing a financial intermediation friction. The model consists of three types of agents- a representative consumer, international financial intermediaries, and a central bank. Time is discrete and denoted by $t$. The agents are infinitely lived.

The following subsections first describe the environment and the agents' optimization problems. The competitive equilibrium conditions are then derived to formulate the optimal policy problem. Finally, the optimal policy and main analytical results are presented.

### 3.1 Representative Consumer

The representative consumer is infinitely lived and consumes two goods- an internationally tradeable consumption good: $c_t$, and a domestic non-tradeable good: $m_t$. Each period, the consumer receives a fixed endowment of the two goods: $y$ and $m^s$, respectively. The consumer also receives a transfer $T_t$ from the central bank.

The non-tradeable good, $m$, is the domestic unit of account and its price is normalized to 1 every period [1]. Let $p_t$ denote the relative price of the consumption good, $c_t$, in units of the domestic numeraire. The tradeable consumption good is assumed to be the world numeraire, its world price, $p_t^* = 1$. The law of one price holds in the tradeable good market: $p_t = e_t p_t^*$; where $e_t$ is the exchange rate expressed in units of the domestic numeraire.

In addition to consuming the two goods, the consumer also participates in a competitive domestic bonds market where it can save or borrow using one-period risk-free bonds denominated in units of the domestic numeraire: $\tilde{b}_{t+1}$. $\tilde{b}_{t+1} > 0$ implies saving whereas $\tilde{b}_{t+1} < 0$ implies borrowing from the domestic bonds market. The consumer is subject to a no-Ponzi-games constraint. Let $R_{t+1}$ denote the gross risk-free interest rate in this market. Moreover, the bonds markets are segmented: while the consumer can access the domestic bonds market, it does not have direct access to the international bonds market.

Let $\beta$ denote the consumer's discount factor, $\sigma$ be the risk aversion parameter, and $\omega$ be the relative utility preference parameter for the tradeable consumption good. For simplicity, it is assumed that the consumer's preferences are separable and homothetic in the two goods. The consumer's optimization problem is described by (1) and involves maximizing expected lifetime utility subject to the budget constraints (and a no-Ponzi-games constraint).

$$\begin{gathered}\max_{\{c_t, m_t, \tilde{b}_{t+1}\}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \omega \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{m_t^{1-\sigma}}{1-\sigma} \right) \qquad \text{s.t.} \\ e_t c_t + m_t + \tilde{b}_{t+1} = e_t y + R_t \tilde{b}_t + m^s + T_t\end{gathered} \tag{1}$$

The optimality conditions for (1) are given by an Euler equation, (2), an intra-temporal relation between $c_t$ and $m_t$, (3) and a transversality condition, (4).

$$c_t{}^{-\sigma} = \beta R_{t+1} \mathbb{E}_t \left( \frac{e_t}{e_{t+1}} c_{t+1}{}^{-\sigma} \right) \tag{2}$$

[1] $m_t$ is a good with an endowment every period. It has the unit of account role but is not a store of value.

$$m_t^{\sigma} = \frac{1}{\omega} e_t {c_t}^{\sigma} \tag{3}$$

$$\lim_{J\to\infty} \beta^J \mathbb{E}_t \left( c_{t+J}^{-\sigma} \frac{\tilde{b}_{t+J}}{e_{t+J}} \right) = 0 \quad \forall t \tag{4}$$

### 3.2 Financial Intermediaries

The world is populated by a unit mass of identical financial intermediaries who intermediate borrowing and lending transactions in the financial markets. The intermediaries are assumed to be owned outside the SOE. The intermediaries have access to an international bonds market where they trade one-period risk-free bonds denominated in units of the world numeraire, i.e., the tradeable consumption good: $q^*_{t+1}$. Let $R^*$ denote the gross interest rate in the international bonds market. This rate is assumed to be fixed, and crucially, it is assumed that $R^* < \beta^{-1}$.

While intermediaries have access to the international bonds market, the SOE has free capital mobility, so they also have unrestricted access to the domestic bonds market. Let $\tilde{q}_{t+1}$ denote the asset position of the intermediaries in the domestic bonds market denominated in units of the domestic numeraire.

A positive asset position in either market denotes saving whereas a negative position denotes borrowing. The intermediaries take offsetting positions and face a balance sheet constraint, (5).

$$\frac{\tilde{q}_{t+1}}{e_t} + q^*_{t+1} = 0 \tag{5}$$

The intermediaries are risk neutral and seek to maximize their expected return, $\mathbb{E}_t \nu_{t+1}$, from their asset positions:

$$\mathbb{E}_t \nu_{t+1} = \mathbb{E}_t \left[ \frac{e_t}{e_{t+1}} R_{t+1} - R^* \right] \frac{\tilde{q}_{t+1}}{e_t} \tag{6}$$

The key financial market friction in this environment is now described. Following Gabaix and Maggiori (2015), the financial friction takes the form of a limited repayment commitment on the part of the intermediaries. Every period, immediately after taking their asset positions, an intermediary can divert a portion $\Gamma_t \left| \frac{\tilde{q}_{t+1}}{e_t} \right|$, with $\Gamma_t > 0$, of the position it intermediates: $\left| \frac{\tilde{q}_{t+1}}{e_t} \right|$. If the intermediary diverts the funds, the firm gets unwounded and the lenders recover a portion $\left( 1 - \Gamma_t \left| \frac{\tilde{q}_{t+1}}{e_t} \right| \right)$ of their claims $\left| \frac{\tilde{q}_{t+1}}{e_t} \right|$. Since the lenders anticipate the incentives of the intermediary to divert funds, the intermediary is subject to a credit constraint such that expected discounted returns from the intermediation business are weakly higher than the returns earned by diverting the funds. This constraint is described by (7)[2].

$$\frac{1}{R^*}\mathbb{E}_t \nu_{t+1} \geqslant \Gamma_t \left|\frac{\tilde{q}_{t+1}}{e_t}\right| \times \left|\frac{\tilde{q}_{t+1}}{e_t}\right| \tag{7}$$

The intermediary's constrained optimization problem is then given by (8).

$$\max_{\tilde{q}_{t+1}} \frac{1}{R^*}\mathbb{E}_t \left[\frac{e_t}{e_{t+1}} R_{t+1} - R^*\right] \frac{\tilde{q}_{t+1}}{e_t} \qquad \text{s.t. to (7)} \tag{8}$$

Since, the objective in (8) is linear in $\tilde{q}_{t+1}$ and the constraint (7) is convex in $\tilde{q}_{t+1}$, the constraint,(7), always binds at the optimum. The solution to (8) is then given by a supply of funds rule, (9).

$$\frac{\tilde{q}_{t+1}}{e_t} = \frac{1}{\Gamma_t}\left[\frac{R_{t+1}}{R^*}\mathbb{E}_t\left(\frac{e_t}{e_{t+1}}\right) - 1\right] \tag{9}$$

First note that given $\Gamma_t$ and the interest differential, $\frac{R^*}{R_{t+1}}$, the intermediary's saving is a linearly increasing function of the expected appreciation rate, $\mathbb{E}_t\left(\frac{e_t}{e_{t+1}}\right)$, with $\tilde{q}_{t+1} = 0 \Leftrightarrow \mathbb{E}_t\left(\frac{e_t}{e_{t+1}}\right) = \frac{R^*}{R_{t+1}}$. Secondly, the key variable in this equation is $\Gamma_t$ which is implicitly a measure of friction in the financial markets. $\Gamma_t = 0$ is the frictionless model where the solution to the optimization problem gives the uncovered interest parity condition[3]. For any finite value of $\Gamma_t > 0$ uncovered interest parity fails. In other words, $\Gamma_t > 0$ drives a wedge between the interest differential, $\frac{R^*}{R_{t+1}}$ and the expected appreciation rate, $\mathbb{E}_t\left(\frac{e_t}{e_{t+1}}\right)$. The supply of funds by the intermediaries is an increasing function of this wedge: given the interest differential, the higher the expected appreciation rate, higher is the amount that intermediaries are willing to save in the domestic bonds market. Moreover, the supply is also a function of $\Gamma_t$: an increase in frictions is equivalent to financial disruptions where the intermediaries are willing to lend smaller amounts. As illustrated by Gabaix and Maggiori (2015), it turns out that the expected appreciation rate, $\mathbb{E}_t\left(\frac{e_t}{e_{t+1}}\right)$ is not only an essential equilibrium variable but is also the key variable that adjusts in response to exogenous shocks to $\Gamma_t$.

In what follows, the competitive equilibrium framework is presented for both deterministic and stochastic settings. In the deterministic setting, it is assumed that $\Gamma_t = \Gamma > 0, \forall t$, and the stochastic setting assumes that $\Gamma_t > 0$ follows an exogenous Markov switching regime.

[2]For further discussion on such limited commitment constraints and their micro-foundations see Gabaix and Maggiori (2015) and the references therein.

[3]If $\Gamma_t = 0$ intermediaries are simply a veil, the solution to their optimization problem gives the UIP condition: $\frac{R^*}{R_{t+1}} = \mathbb{E}_t\left(\frac{e_t}{e_{t+1}}\right)$

### 3.3 Central bank

Since the SOE is assumed to have free capital mobility, the central bank does not have access to any distortionary capital taxation instrument. It only has access to lump-sump transfers/taxes $T_t$ denominated in units of the domestic numeraire. However, the central bank can bypass the intermediaries and directly access the international bonds market, where it can *save* in one-period risk-free foreign assets denominated in units of the tradeable consumption good, henceforth called reserves: $a_{t+1}$. The central bank faces a period-by-period budget constraint, (10).

$$e_t a_{t+1} + T_t = e_t R^* a_t \tag{10}$$

It should be noted here that while the central bank can save in foreign assets, borrowing directly through the international bonds market is not an option[4]: $a_{t+1} \geqslant 0$.

### 3.4 Competitive Equilibrium

Given $(a_0, \tilde{b}_0)$, central bank policy $\{T_t, a_{t+1}\}$, and an exogenous stochastic process for $\Gamma_t$, a competitive equilibrium is given by stochastic sequences of the exchange rate, $\{e_t\}$, the interest rate on domestic bonds, $\{R_{t+1}\}$, consumption of the two goods, $\{c_t, m_t\}$ and asset positions in the domestic bonds market $\{\tilde{b}_{t+1}, \tilde{q}_{t+1}\}$ so that,

- Given the exchange rates and interest rates, the representative consumer solves its optimization problem (1).
- Given the exchange rates and interest rates, the intermediaries solve their optimization problem, (8).
- The central bank's budget constraint, (10) holds every period.
- The domestic non-tradeable and the domestic bonds markets clear every period:

$$m_t = m^s \tag{11}$$

$$\tilde{b}_{t+1} + \tilde{q}_{t+1} = 0 \tag{12}$$

[4] If the central bank can borrow directly from the international bonds market, the intermediation friction is irrelevant. Moreover, this constraint reflects a real-world reality where central banks have zero to negligible access to borrowed reserves. See the discussion in Davis, Devereux and Yu (2023).

Next, this section proceeds with deriving the competitive equilibrium conditions. Since the endowment for the domestic non-tradeable good, $m^s$, is assumed to be constant, for simplicity and without loss of generality, it is normalized: $m^s = 1$. Then combining (11) and (3), (13) is obtained.

$$e_t = \omega c_t^{-\sigma} \tag{13}$$

Equation (13) establishes an explicit equilibrium relation between the exchange rate, $e_t$, and the consumption of the tradeable good $c_t$. This equation is essentially equivalent to a constant-elasticity-of-demand rule and suggests that an exchange rate depreciation implies that the consumer is willing to consume a smaller amount of the tradeable good.

Secondly, combining (13) with the consumer's Euler equation, (2), pins down the interest rate on domestic bonds:

$$R_{t+1} = \beta^{-1} \quad \forall t \tag{14}$$

In other words, since the endowment for the domestic non-tradeable good is constant, and the domestic bonds are denominated in units of this good, the domestic interest rate on these bonds is also pinned down and constant.

Combining the consumer's budget constraint in (1) with the central bank budget constraint, (10), and market clearing condition, (11), yields a resource constraint for the tradeable consumption good, henceforth called the balance of payments (BoP) condition, (15).

$$e_t c_t + \tilde{b}_{t+1} + e_t a_{t+1} = e_t y + R_t \tilde{b}_t + e_t R^* a_t \tag{15}$$

Let $\frac{\tilde{b}_{t+1}}{e_t} \equiv b_{t+1}$ and $\frac{\tilde{q}_{t+1}}{e_t} \equiv q_{t+1}$ i.e., private agents' savings expressed in units of the tradeable consumption good. Using (12) and (13) the BoP condition can be expressed in units of the tradeable consumption good, (16).

$$c_t - q_{t+1} + a_{t+1} = y + R^* a_t - R\left(\frac{c_t}{c_{t-1}}\right)^{\sigma} q_t \tag{16}$$

The constraint (16) suggests that the 'real' interest rate on external debt- in units of the tradeable good, is given by $R\left(\frac{c_t}{c_{t-1}}\right)^{\sigma}$. where $\left(\frac{c_t}{c_{t-1}}\right)^{\sigma}$ denotes the ex-post appreciation rate.

Secondly, using (13), the intermediaries' supply of assets, (9), can essentially be expressed as a distorted Euler equation, (17).

$$q_{t+1} = \frac{1}{\Gamma_t}\left[\frac{R}{R^*}\mathbb{E}_t\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} - 1\right] \tag{17}$$

As discussed previously, the asset supply by intermediaries is an increasing function of the expected appreciation rate, $\mathbb{E}_t\left(\frac{c_{t+1}}{c_t}\right)^{\sigma}$.

Finally, the transversality condition can be expressed as below:

$$\lim_{J\to\infty} \beta^J \mathbb{E}_t(c_{t+J}^{-\sigma} q_{t+J}) = 0 \quad \forall t \tag{18}$$

Given $(a_0, q_0, c_{-1})$, given a reserve accumulation policy of the central bank $\{a_{t+1}\}$, given an exogenous stochastic process for $\Gamma_t$, and given a no-Ponzi-games condition, the competitive equilibrium is summarized by a system of two stochastic difference equations in $\{c_t, q_{t+1}\}$: (16), (17) and a transversality condition, (18).

The competitive equilibrium conditions explicitly establish the dual role of the exchange rate. First, the exchange rate prices consumption for the household as illustrated by (13). Second, it is also the key asset price that adjusts to clear the domestic bonds market, i.e., the appreciation rate is an equilibrium-determined price that influences the real value of debt payments, as illustrated by (16); as well as governing the amount of external borrowing, as illustrated by (17). The next sub-section further expounds on the asset price role of the equilibrium exchange rate.

### 3.5 Deterministic Dynamics: Laissez-faire

To better illustrate the competitive equilibrium dynamics and the determination of the equilibrium exchange rate, this section describes the solution of a laissez-faire equilibrium in a deterministic setting. In other words, it is assumed, that there is no uncertainty: $\Gamma_t = \Gamma > 0, \forall t$ and that the central bank follows a free-floating exchange rate regime so that it does not accumulate or hold reserves: $a_{t+1} = 0, \forall t$. Using deterministic versions of (16) and (17), a laissez-faire equilibrium is summarized by a system of two difference equations in $\{c_t, q_{t+1}\}$ (19), (20), and a transversality condition, (21).

$$c_t - q_{t+1} = y - R\left(\frac{c_t}{c_{t-1}}\right)^{\sigma} q_t \tag{19}$$

$$q_{t+1} = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} - 1\right] \tag{20}$$

$$\lim_{t\to\infty} \beta^t (c_t^{-\sigma} q_{t+1}) = 0 \tag{21}$$

The steady state of this system is given by $(\bar{c}, \bar{q})$:

$$\bar{c} = y - (R-1)\bar{q}; \qquad \bar{q} = \frac{1}{\Gamma}\left[\frac{R}{R^*} - 1\right]$$

Since it is the case that $R^* < \beta^{-1} = R$ and $\Gamma > 0$, then $\bar{q} > 0$, i.e., the SOE carries a positive debt in the long-run equilibrium. To understand the dynamics of debt and consumption away from the long-run equilibrium, a phase diagram in the $(q, c)$ space is presented. Let $\Delta q = 0$ represent the zero-change locus for q and $\Delta c = 0$ represent the zero-change locus for c. Using (19) and (20) the loci can be expressed as below:

$$\begin{aligned}
\Delta q = 0: &\quad c = y + q\left(1 - R^*(1 + \Gamma q)\right) \\
\Delta c = 0: &\quad q = \frac{1}{\Gamma}\left[\frac{R}{R^*} - 1\right]
\end{aligned}$$

The left panel of Figure 2 presents the phase diagram depicting these zero-change loci. The intersection of these loci signifies the long-run equilibrium $E_0 = (\bar{q}, \bar{c})$. The top-right quadrant has trajectories where consumption and debt explode. Such paths cannot constitute an equilibrium as they violate the no-Ponzi-games constraint. Similarly, the trajectories in the bottom-left quadrant cannot constitute an equilibrium as they involve the household accumulating wealth without consuming anything asymptotically, thereby violating the transversality condition. However, the top-left and the bottom-right quadrants have a unique saddle path that asymptotically converges to the long-run equilibrium. If the household begins with low levels of debt (or high saving) it consumes more in the initial periods by borrowing and builds debt over time. As debt starts building up, higher debt payments cause consumption to decrease and the economy approaches the long-run equilibrium. Conversely, if the household starts with very high levels of debt, it initially lowers consumption to alleviate its debt burden. As debt decreases over time, the household can gradually increase its consumption, eventually approaching the long-run equilibrium.

Next, the adjustment and dynamics of the equilibrium exchange rate in response to exogenous shocks is demonstrated. The right panel of Figure 2 illustrates the short-run and long-run impact of an unanticipated financial shock: a permanent increase in $\Gamma$. This increase is interpreted as a tightening of the credit constraint faced by the intermediaries, constituting a financial disruption. The shock induces a shift in both the loci and the new long-run equilibrium shifts from $E_0$ to $E_1$. From (13), noting the inverse relation between consumption and the exchange rate, it is observed that on impact, a financial disruption induces an exchange rate depreciation and a credit contraction. In the long run, the exchange rate appreciates towards its new long-run equilibrium. Analogously, a decrease in $\Gamma$, interpreted as a loosening of the credit constraint faced by the intermediaries- a financial boom - induces an exchange rate appreciation and a surge in capital

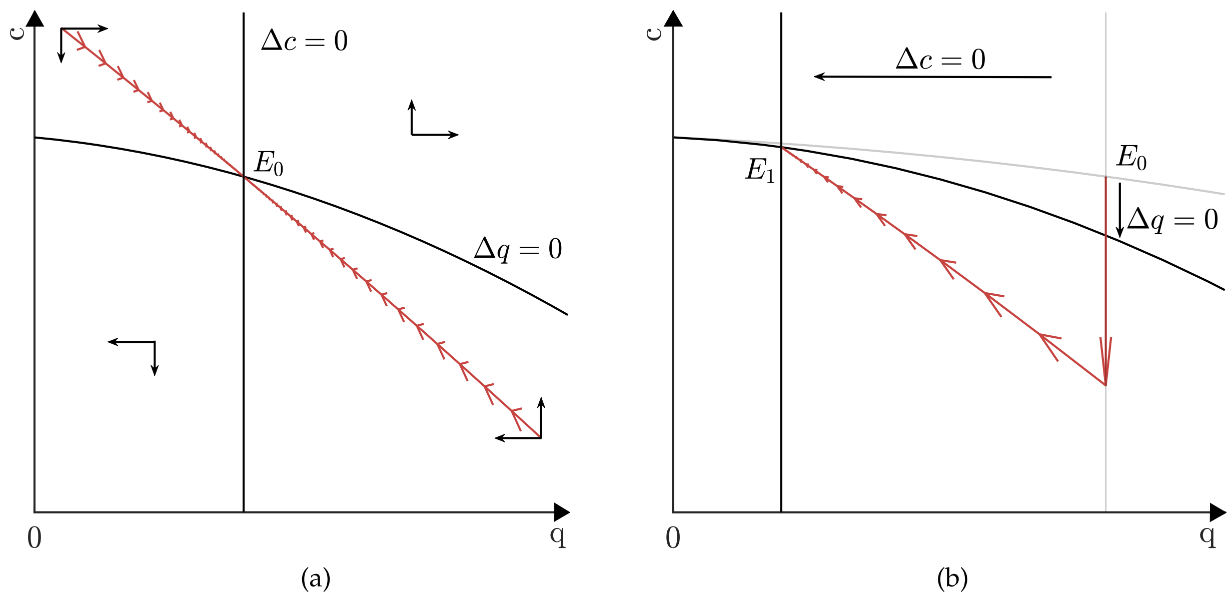



Figure 2: Phase diagram analysis

inflows.

This analysis highlights the asset price role of exchange rates: the exchange rate adjusts to ensure that the financial markets clear. Furthermore, periods of financial booms and busts are accompanied by corresponding exchange rate appreciations or depreciations in response to adjustments in financial flows.

### 3.6 Optimal Reserves: A Ramsey Problem

With an understanding of competitive equilibrium dynamics, the analysis proceeds to set up the policy problem and characterize the optimal reserve accumulation policy for the central bank. The central bank is assumed to be benevolent and a Ramsey equilibrium is described, i.e., the central bank chooses a reserve policy that maximizes the expected lifetime utility of the consumer, subject to the BoP constraint, (16), and the supply of funds, (17). The Ramsey equilibrium is given by the solution to (22).

$$
\begin{gathered}
\max_{\{c_t, q_{t+1}, a_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\omega {c_t}^{1-\sigma}}{1-\sigma} \quad \text{s.t.} \\
c_t - q_{t+1} + a_{t+1} = y + R^* a_t - R\left(\frac{c_t}{c_{t-1}}\right)^{\sigma} q_t \\
q_{t+1} = \frac{1}{\Gamma_t}\left[\frac{R}{R^*}\mathbb{E}_t\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} - 1\right] \\
a_{t+1} \geqslant 0
\end{gathered}
\tag{22}
$$

The solution to this problem addresses the motivating question of this paper: why a country might simultaneously hold both debt and reserves. It will be shown that a Ramsey equilibrium in this environment could indeed feature a portfolio with both debt and reserves held together, with financial frictions playing a key role in determining the optimal portfolio. To this end, it is helpful to analytically analyze a two-period version of this policy problem in a deterministic environment, which is considered next.

### 3.6.1 Two-period problem

In this subsection, a two-period deterministic version of the policy problem (22) is analyzed. Let $t = 1, 2$, with the initial state $(a_1, q_1, c_0)$ given at $t = 1$. The economy is assumed to begin with some positive levels of debt and reserves at $t = 1$: $a_1, q_1 > 0$[5]. After substituting the asset supply equation into the BoP constraints, the policy problem is given by (23).

$$\max_{\{c_1, c_2, a_2\}} \frac{\omega c_1^{1-\sigma}}{1-\sigma} + \beta \frac{\omega c_2^{1-\sigma}}{1-\sigma} \qquad \text{s.t.}$$

$$\begin{gathered} c_1 - \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right] + a_2 = y + R^* a_1 - R\left(\frac{c_1}{c_0}\right)^{\sigma} q_1 \\ c_2 = y + R^* a_2 - R\left(\frac{c_2}{c_1}\right)^{\sigma} \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right] \\ a_2 \geqslant 0 \end{gathered} \tag{23}$$

Furthermore, the two BoP constraints in (23) can be combined by substituting out $a_2$ to obtain an intertemporal resource constraint, (24).

$$c_1 + \frac{c_2}{R^*} + \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right]^2 = y + \frac{y}{R^*} + R^* a_1 - R\left(\frac{c_1}{c_0}\right)^{\sigma} q_1 \tag{24}$$

For the sake of exposition, this constraint can be expressed in a way that explicitly shows the resource costs associated with the initial debt being denominated in domestic currency. The intertemporal resource constraint is then given by (25). The expressions labeled A and B represent two different resource costs. These are discussed below in detail.

$$c_1 + \frac{c_2}{R^*} + \underbrace{\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right]^2}_{B} = y + \frac{y}{R^*} + R^* a_1 - R^* q_1 - \underbrace{R^*\left[\frac{R}{R^*}\left(\frac{c_1}{c_0}\right)^{\sigma} - 1\right] q_1}_{A} \tag{25}$$

[5] $q_1, a_1 > 0$ is the relevant state for answering the motivating question of this paper.

Finally, the non-negativity constraint on $a_2$ can be expressed in terms of $c_1, c_2$ as well, which yields (26).

$$a_2 = \frac{c_2 - y}{R^*} + \frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma}\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right] \geqslant 0 \tag{26}$$

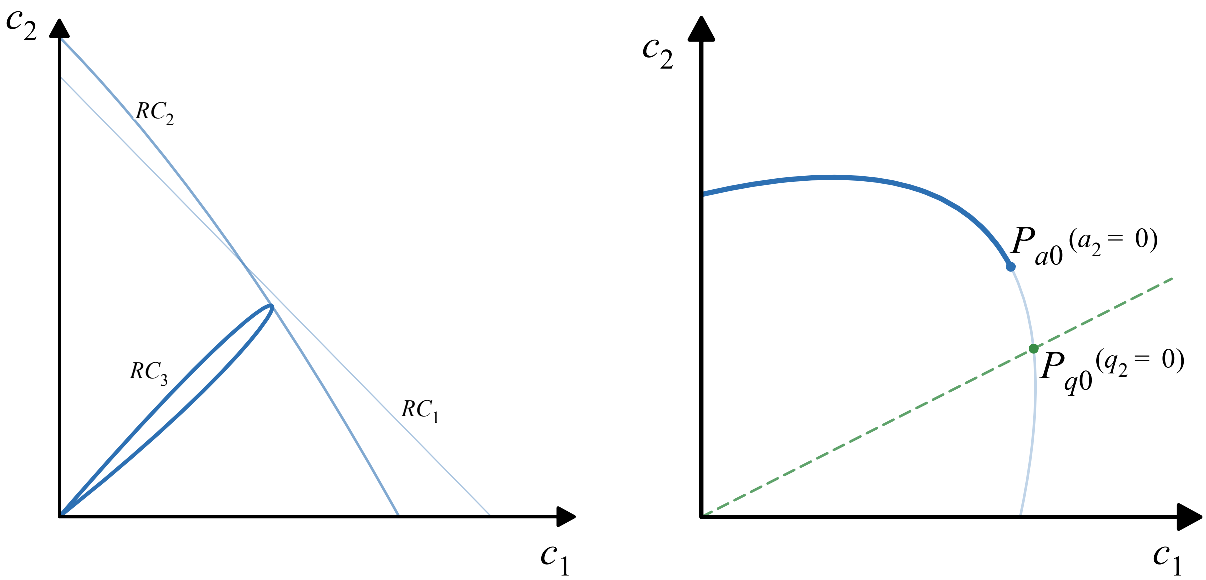
One can observe that the optimal policy problem (23) together with (25) and (26) looks very similar to the standard two-period consumption savings problem. The intertemporal resource constraint, (25), is very similar to the usual intertemporal budget constraint that appears in the canonical consumption-savings problem, except for two peculiar expressions in this equation, the ones labeled A and B.

First, the expression labeled A in (25) is obtained by decomposing the value of previous debt payments, $R\left(\frac{c_1}{c_0}\right)^{\sigma} q_1$, into two parts: the first part being $R^* q_1$, and the second being the expression labeled A. The first part, $R^* q_1$, signifies the debt payments if the debt was denominated in foreign currency. The second part, A, signifies the 'premium' paid over and above the first part due to the debt being denominated in domestic currency. Higher the ex-post appreciation rate, $\left(\frac{c_1}{c_0}\right)^{\sigma}$, higher is the premium paid and vice-versa.

In other words, since the consumer borrows in units of the domestic numeraire, the 'real' debt payments, i.e., payments in units of the tradeable consumption good, are dependent on the appreciation rate of the currency, $\left(\frac{c_1}{c_0}\right)^{\sigma}$, so that the 'real' interest rate is $R\left(\frac{c_1}{c_0}\right)^{\sigma}$. With $c_0$ given at $t = 1$, higher consumption in period-1, $c_1$, also means that the consumer pays a higher interest on previous debt. Intuitively, if the debt is denominated in local currency, an exchange rate appreciation results in higher payments in units of the international currency. Similarly, lower consumption in period 1 results in lower interest payments. In other words, an exchange rate depreciation lowers the burden of domestic currency denominated debt.

Second, the expression labeled B, $\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right]^2$, which is also equal to $\Gamma q_2^2$, represents a quadratic resource cost associated with the use of the financial intermediation technology. The quadratic term signifies that only the magnitude matters, i.e., it does not matter whether the financial intermediation technology is used for borrowing or saving, but what matters is the extent to which it is used. Said differently, this expression, B, represents a resource cost associated with borrowing or saving through the intermediaries- in the form of profits made by them. The magnitude of this cost depends on the interest parity wedge, i.e., how much $\left(\frac{c_2}{c_1}\right)^{\sigma} \lessgtr \frac{R^*}{R}$ with no cost if and only if $q_2 = 0$ i.e., $\left(\frac{c_2}{c_1}\right)^{\sigma} = \frac{R^*}{R}$.

Panel (a) of Figure 3 plots (25) in the $(c_1, c_2)$ plane and illustrates how the expressions A and B alter




(a) (b)

Figure 3: Intertemporal Resource Constraint

the usual downward sloping budget constraint that shows up in the basic consumption-savings problem. First, ignoring both expressions A and B yields the usual downward sloping linear constraint labeled $RC_1$ in panel (a) of Figure 3. Then, after incorporating expression A, but still ignoring expression B, yields $RC_2$. The inward tilt along the $c_1$-axis illustrates the loss in resources due to higher debt payments being made when $c_1$ is higher; whereas the outward tilt along the $c_2$-axis illustrates that by lowering $c_1$ resources are saved by making smaller debt payments which allows higher consumption in period-2. Finally, after incorporating expression B as well, the intertemporal constraint is depicted by $RC_3$ in panel (a) of Figure 3. The peculiar petal shape [6] of this constraint reflects the effect of expression B- the loss in resources, i.e., the costs associated with using the intermediation technology, as $\frac{c_2}{c_1} \to 0$ or $\frac{c_2}{c_1} \to \infty$. It is also noteworthy that $RC_3$ is tangent to $RC_2$ at the point where the loss is 0, i.e., $\left(\frac{c_2}{c_1}\right)^{\sigma} = \frac{R^*}{R}$.

In terms of the model, each point on $RC_3$ represents a competitive equilibrium that would prevail for some choice of $a_2$ by the central bank. Again drawing an analogy with the standard consumption-savings problem, the equivalent of an 'endowment' point here is the point on $RC_3$ where $a_2 = 0$, i.e., the non-negativity constraint (26) binds. Focusing only on the frontier of the petal-shaped $RC_3$, this point is shown in panel (b) of Figure 3 labeled $P_{a0}$. The points to the left of $P_{a0}$, shown by a darker shade, represent the set of feasible competitive equilibria that can be reached by the

[6] The exact shape of $RC_3$ may or may not be a perfect petal as illustrated in Figure 3, the shape and convexity depends on the parameters- $R, R^*, \Gamma, \sigma$.

central bank choosing $a_2 > 0$. In other words, increasing $a_2$ is equivalent to moving left to right along this constraint. The lighter shaded region on the constraint, i.e., the region to the right of $P_{a0}$, represents infeasible competitive equilibria since they require $a_2 < 0$.

Recall that since $q_2$ is endogenous in $\frac{c_2}{c_1}$, as asserted by the asset supply equation, every point on $RC_3$ also corresponds to a unique implied value for $q_2$. The panel (b) of Figure 3 shows the point where the intermediation loss is 0, i.e., $q_2 = 0$, no borrowing or saving with the intermediaries. This point is labeled $P_{q0}$. To the right of $P_{q0}$ it is the case that $\left(\frac{c_2}{c_1}\right)^\sigma < \frac{R^*}{R} \Leftrightarrow q_2 < 0$ i.e., positive saving with the intermediaries. Conversely, to the left of $P_{q0}$, there is positive borrowing through the intermediaries: $\left(\frac{c_2}{c_1}\right)^\sigma > \frac{R^*}{R} \Leftrightarrow q_2 > 0$. Where exactly $P_{a0}$ and $P_{q0}$ lie relative to each other depends on the initial state $(a_1, q_1, c_0)$. In this Figure, $P_{q0}$ lies in the infeasible region. In other words, even if the central bank runs down its reserves to 0, and chooses to be at $P_{a0}$, the competitive equilibrium involves the consumer borrowing a positive amount through the intermediaries, given the initial state.

A movement along this constraint, accomplished by altering $a_2$, illustrates the effectiveness of FX interventions in this environment. As the central bank increases $a_2$ by moving leftwards from the point $P_{a0}$, it is able to alter the competitive equilibrium choices of $c_1, c_2, q_2$ made by the consumer and the intermediaries. Intuitively, as the central bank builds reserves, it induces the consumer to borrow more to offset this additional borrowing. However, as implied by the intermediaries' asset supply equation, the intermediaries are only willing to lend more if and only if they are offered a higher appreciation rate, i.e., the exchange rate must depreciate at $t = 1$. Such an exchange rate depreciation lowers $c_1$ and also decreases the 'real' value of previous debt payments (the expression A in (25)). Moreover, because this depreciation increases the appreciation rate of the currency, it brings in additional borrowing through the intermediaries and therefore also increases the intermediation resource costs (the expression B in (25)). It is noteworthy that due to the exchange rate depreciation, the increase in reserves is not met by a one-for-one increase in borrowing. Only a part of these reserves are financed through additional borrowing, the remaining is financed through a reduction in $c_1$ and dilution of previous debt payments. The key insight is that given the financial frictions, reserve operations have general equilibrium effects- the central bank is able to manipulate the exchange rate and thereby alter consumption, borrowing, and value of debt payments.

To clarify further, it is useful to compare this outcome to the frictionless case, i.e., $\Gamma = 0$, where reserve operations would be ineffective. To see this, observe that in the absence of the friction, the intermediary asset supply is perfectly elastic, i.e., the interest parity condition holds, yielding the usual frictionless Euler equation:

$$\left(\frac{c_2}{c_1}\right)^{\sigma} = \frac{R^*}{R}$$

And the intertemporal constraint is as below (same as (25) but with expression B=0):

$$c_1 + \frac{c_2}{R^*} = y + \frac{y}{R^*} + R^* a_1 - Rc_1^{\sigma}\frac{q_1}{c_0^{\sigma}}$$

The frictionless equilibrium is thus characterized by a unique net foreign asset position $(a_2 - q_2)$. In other words, any change in $a_2$ would be met by a one-for-one offsetting change in $q_2$ such that $(a_2 - q_2)$ does not change and hence no resultant change in $c_1, c_2$. Intuitively, due to a perfectly elastic supply of assets by the intermediaries, a version of Ricardian equivalence holds- in response to the central bank accumulating reserves, the consumer borrows more, one-for-one so that the net foreign asset position does not change. This corresponds to the benchmark ineffectiveness of FX interventions result illustrated by Backus and Kehoe (1989).

Coming back to a world with financial frictions ($\Gamma > 0$), as mentioned previously, an increase in reserves is met by an exchange rate depreciation which lowers $c_1$ and dilutes previous debt payments in addition to an increase in borrowing. While dilution saves resources (the expression A in (25)), the increase in $q_2$ also increases the size of the intermediation costs (the expression B in (25)). The net effect on $c_2$ depends on the relative size of the intermediation costs and the extent of dilution. The central bank therefore faces a tradeoff. Building more and more reserves, on the one hand, saves resources by diluting previous debt payments, whereas on the other hand, the 'premium' associated with the increased borrowing through intermediaries also leads to a resource loss. The optimal reserve choice is thus such that the marginal gain from dilution is equal to the marginal loss resource associated with the intermediation cost.

Demonstrating this choice, panel (a) of Figure 4 shows the optimal solution where the indifference curve is tangent to the intertemporal resource constraint (25) at the point $P^*$. The optimal choice features a positive reserve position $a_2 > 0$ and a positive debt position $q_2 > 0$. This leads back to the question raised in this paper: why does a country hold both external debt and reserves? Why is it not optimal to run down the reserves to pay off the debt? In other words, why is the point $P_{a0}$ not optimal? The answer lies in the fact that while running down the reserves will allow the country to borrow lesser, it also results in an exchange rate appreciation which increases the 'real' payments on previous domestic currency-denominated debt. The resource loss associated

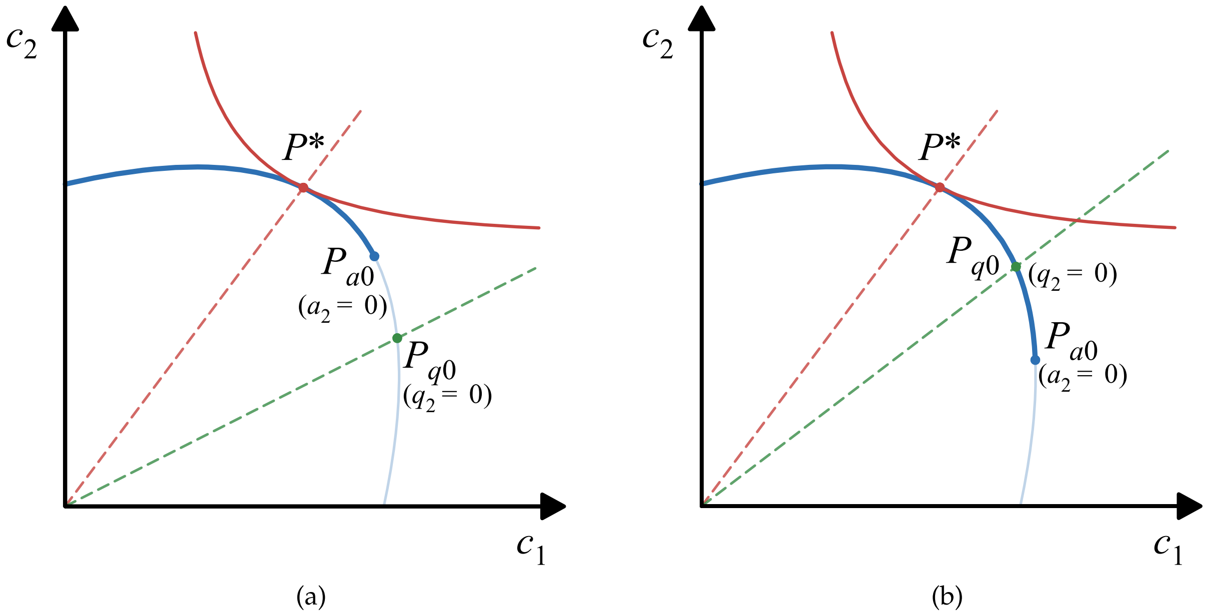



Figure 4: Optimal Portfolio

with this appreciation exceeds the gain generated by lower borrowing. Said differently, by holding reserves, the central bank is equating the marginal benefits of debt dilution to the marginal costs of additional household borrowing. This result is formally stated in Lemma 1. This Lemma establishes the conditions under which $P_{a0}$ is not optimal and $P^*$ lies to the left to $P_{a0}$.

**Lemma 1.** *Suppose that* $c_0 > 0, a_1 > 0, q_1 > 0$ *and suppose there exists* $(\hat{c}_1, \hat{c}_2, \hat{q}_2, \hat{a}_2)$ *that satisfies the constraints of* (23) *such that:*

$\hat{a}_2 = 0,$

$\hat{q}_2 = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma} - 1\right] \geqslant 0$

$\hat{c}_2 = y - R^*(1+\Gamma\hat{q}_2)\hat{q}_2$

$\hat{c}_1 - \hat{q}_2 = y + R^* a_1 - R\left(\frac{\hat{c}_1}{c_0}\right)^{\sigma} q_1$

*If*

$$R^* a_1 + (\sigma-1)R\left(\frac{\hat{c}_1}{c_0}\right)^{\sigma} q_1 > \left[\frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1} - 1\right] y + \Gamma\hat{q}_2^2 + (2\sigma-1)(\hat{q}_2 + \Gamma\hat{q}_2^2)\left(R\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1} + 1\right)$$

*then* $(\hat{c}_1, \hat{c}_2, \hat{q}_2, \hat{a}_2)$ *is NOT a solution to* (23). *Furthermore, the optimal solution involves* $a_2 > 0$ *and* $q_2 > \hat{q}_2$.

*Proof.* See Appendix B.1. □

In panel (a) of Figure 4 $P_{q0}$ lies in the infeasible region, i.e., not using the intermediation technology is not an option even if reserves are set to 0. However, depending on the initial state, it is possible that $P_{q0}$ lies within the set of feasible competitive equilibria. This is illustrated in the panel (b) of Figure 4. The Figure shows that there exists a positive level of reserves that could eliminate the use of the intermediation technology and set $q_2 = 0$. At this point the intermediation leakage is minimized. However, Lemma 2 establishes that as long as previous debt $q_1 > 0$, choosing $q_2 = 0$ can never be optimal. In other words, at $P_{q0}$ the welfare benefits from dilution always exceed the losses from marginally using the intermediation technology. In simpler words, the optimal point $P^*$ always lies to the left of $P_{q0}$. This is illustrated in panel (b) of Figure 4.

**Lemma 2.** *Suppose that* $c_0 > 0, a_1 > 0, q_1 > 0$ *and suppose there exists* $(\hat{c}_1, \hat{c}_2, \hat{q}_2, \hat{a}_2)$ *that satisfies the constraints of* (23) *such that:*

$\hat{q}_2 = 0,$

$\hat{c}_2 = \left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}} \hat{c}_1$

$\hat{a}_2 = \frac{1}{R^*}\left(\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}} \hat{c}_1 - y\right) \geqslant 0$

$\hat{c}_1 + \frac{1}{R^*}\left(\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}} \hat{c}_1 - y\right) = y + R^* a_1 - R\left(\frac{\hat{c}_1}{c_0}\right)^{\sigma} q_1$

*Since* $q_1 > 0$, *then* $(\hat{c}_1, \hat{c}_2, \hat{q}_2, \hat{a}_2)$ *is NOT a solution to* (23). *Furthermore, the optimal solution involves* $q_2 > 0$ *and* $a_2 > \hat{a}_2$.

*Proof.* See Appendix B.2. □

The results of the two-period problem can be generalized to the infinite horizon policy problem. Consider the deterministic version of the policy problem, (22) with $\Gamma_t = \Gamma, \forall t$. Suppose the economy begins in an initial state $(q_0, a_0, c_{-1})$ with $q_0 > 0$ and $a_0 > 0$. Analogous to the two-period problem, the BoP constraints can be combined to obtain an intertemporal resource constraint, (27). As before, the expressions A and B are labeled in (27), where the expression A signifies the resource costs associated with the time-0 debt being denominated in domestic currency and the expression B signifies the resource costs associated with the intermediation technology.

$$\sum_{t=0}^{\infty} \frac{c_t}{R^{*t}} + \underbrace{\frac{1}{\Gamma}\sum_{t=0}^{\infty} \frac{1}{R^{*t}}\left[\frac{R}{R^*}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} - 1\right]^2}_{B} = \sum_{t=0}^{\infty} \frac{y}{R^{*t}} + R^* a_0 - R^* q_0 - \underbrace{R^*\left[\frac{R}{R^*}\left(\frac{c_0}{c_{-1}}\right)^{\sigma} - 1\right] q_0}_{A} \tag{27}$$

For the sake of comparison, the frictionless case is also presented. Analogous to the two-period frictionless case discussed above, the solution over the infinite horizon is entirely characterized by the interest parity condition (frictionless Euler equation):

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \frac{R^*}{R}$$

And an intertemporal resource constraint:

$$\sum_{t=0}^{\infty} \frac{c_t}{R^{*t}} = \sum_{t=0}^{\infty} \frac{y}{R^{*t}} + R^* a_0 - R c_0^{\sigma} \frac{q_0}{c_{-1}^{\sigma}}$$

Since $R^* < R$, consumption approaches 0 and a unique net foreign asset position, $(a_{t+1} - q_{t+1})$, solves the optimal asset accumulation choice[7]. Therefore, absent the friction, a version of Ricardian equivalence holds, i.e., reserve operations would not be effective in altering the optimal consumption path.

In the presence of the friction (expression B in (27) > 0), however, this equivalence does not hold anymore and by choosing to hold reserves, the central bank is able to manipulate the exchange rate, and therefore influence the equilibrium path of consumption, borrowing, and the real value of debt payments. The central bank faces a tradeoff between diluting time-0 debt payments (expression A in (27)), and the intermediation resource costs associated with additional household borrowing in response to the central bank accumulating reserves (expression B in (27)). The optimal sequence of reserves, $\{a_{t+1}\}$, is such that the marginal benefit of diluting time-0 debt is equal to the marginal resource cost of the additional consumer borrowing. Since $R^* < \beta^{-1}$, as one would expect, the central bank eventually finds it optimal to run down the reserves to 0 over time[8]. Reserves decline monotonically to 0 and there exists a $T \geqslant 1$ (with strict inequality for some initial states) such that $a_t > 0$ for all $t < T$. Moreover, once the central bank has run down its reserves, the transition dynamics are identical to the laissez-faire dynamics discussed in section 3.5, i.e., the economy approaches a steady state. This result is formally summarized in Proposition 1.

**Proposition 1.** *Let* $R^* < R$ *and* $\lim_{t\to\infty} \beta^t q_{t+1} = 0$. *Suppose* $q_0 \geqslant 0, a_0 > 0$. *Consider the deterministic version of the policy problem* (22) *with* $\Gamma_t = \Gamma > 0, \forall t$. *The problem has a solution with a steady state limit* $(\bar{q}, \bar{c}, \bar{a})$*:*

- $q_{t+1} > 0, \forall t; \lim_{t\to\infty} q_{t+1} = \bar{q} = \frac{1}{\Gamma}\left[\frac{R}{R^*} - 1\right]$
- $\lim_{t\to\infty} c_t = \bar{c} = y - (R-1)\bar{q}$

[7] These equations are identical to the solution for a standard infinite horizon consumption-savings problem.

[8] This is analogous to the standard infinite-horizon consumption-savings problem with a borrowing constraint and $R^* < \beta^{-1}$, where any initial wealth $a_0 > 0$ is driven down to the borrowing limit over time: $a_0 \geqslant a_1 \geqslant a_2 ... \geqslant -\bar{a}$.

- $\exists T \geqslant 1$ *(with strict inequality for some initial states* $(q_0, a_0)$*) such that* $a_t > 0, \forall t < T$ *and* $a_t = \bar{a} = 0, \forall t \geqslant T$.

Figures 12, 13 in Appendix C, show numerical simulations, illustrating Proposition 1.

The next sub-section, continues to analytically analyze the optimal policy problem and illustrates that the solution is time-inconsistent.

### 3.7 Time inconsistency of the Ramsey optimal reserve policy

This sub-section continues with the analysis of the Ramsey optimal reserve policy that solves the deterministic version of (22). One way to think of the Ramsey planning problem is that at time-0 a Ramsey planner is followed by a sequence of continuation Ramsey planners at times $t = 1, 2, ....$ Consider a transformed version of the policy problem by denoting the net-foreign asset position in period t as $x_t = a_{t+1} - q_{t+1}$. A time-t continuation Ramsey planner takes $(c_t, x_t)$ for $t \geqslant 1$ as state variables passed onto it by the time-(t-1) planner and is obligated to choose the gross asset positions $a_{t+1}$ and $q_{t+1}$ so that they sum up to the 'promised' net position $x_t$. The time-t planner also chooses $(c_{t+1}, x_{t+1})$ and passes them as state variables to the time-(t+1) planner. The time-t planner's objective is to maximize continuation utility subject to three constraints: a promise-keeping constraint, (28), a BoP constraint for period t+1, (29), and the intermediaries' asset supply equation, (30):

$$a_{t+1} - q_{t+1} = x_t \tag{28}$$

$$c_{t+1} + x_{t+1} = y + R^* a_{t+1} - R\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} q_{t+1} \tag{29}$$

$$q_{t+1} = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} - 1\right] \tag{30}$$

However, the time-0 planner faces a different set of constraints. It does not face $(x_0, c_0)$ as state variables. Rather it faces $(c_{-1}, a_0, q_0)$ as state variables and has the ability to choose $(c_0, q_1, a_1)$ without any promise-keeping restriction. The time-0 planner chooses $(c_0, q_1, a_1)$ as well as $(c_1, x_1)$, which are passed as state-variables to the time-1 planner, subject to period-0 and period-1 BoP constraints,(31), (32), and a period-0 asset supply equation, (33):

$$c_0 + a_1 - q_1 = y + R^* a_0 - R\left(\frac{c_0}{c_{-1}}\right)^{\sigma} q_0 \tag{31}$$

$$c_1 + x_1 = y + R^* a_1 - R\left(\frac{c_1}{c_0}\right)^{\sigma} q_1 \tag{32}$$

$$q_1 = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1}{c_0}\right)^{\sigma} - 1\right] \tag{33}$$

The source of time-inconsistency is evident from the fact that while the time-t continuation planner is restricted by accepting $(c_t, x_t)$ as state variables and faces a promise-keeping constraint, (28), the time-0 planner does not face these restrictions. The time-t continuation planner at every $t \geqslant 1$ potentially has an incentive to ignore the promise-keeping constraint and act like a time-0 planner. To better illustrate this problem and understand the direction of a potential deviation from a previous plan, a three-period version of the policy problem is illustrated in the next sub-section.

#### 3.7.1 Three-period policy problem: Commitment vs Deviation

Consider the three-period deterministic version of (22). Let $t = 0, 1, 2$ with the initial state $(a_0, q_0, c_{-1})$ given at $t = 0$. Let $\Gamma_t = \Gamma, \forall t$. As before, it is assumed that the economy begins with some initial debt and reserves: $a_0, q_0 > 0$. After substituting the asset supply equations into the BoP constraints, the time-0 policy problem, i.e., the problem under commitment, is given by (34).

$$\max_{\{c_0, c_1, c_2, a_1, a_2\}} \frac{\omega c_0^{1-\sigma}}{1-\sigma} + \beta\frac{\omega c_1^{1-\sigma}}{1-\sigma} + \beta^2\frac{\omega c_2^{1-\sigma}}{1-\sigma} \quad \text{s.t.}$$

$$\begin{gathered} c_0 - \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1}{c_0}\right)^{\sigma} - 1\right] + a_1 = y + R^* a_0 - R c_0^{\sigma}\frac{q_0}{c_{-1}^{\sigma}} \\ c_1 - \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right] + a_2 = y + R^* a_1 - R\left(\frac{c_1}{c_0}\right)^{\sigma}\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1}{c_0}\right)^{\sigma} - 1\right] \\ c_2 = y + R^* a_2 - R\left(\frac{c_2}{c_1}\right)^{\sigma}\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right] \\ a_1, a_2 \geqslant 0 \end{gathered} \tag{34}$$

Consider now a potential deviation from the commitment plan at $t = 1$, i.e., the planner is allowed to re-optimize and choose $(c_1, c_2, a_2, q_2)$. The problem faced by the time-1 planner is identical to the two-period problem discussed previously, (23). Comparing the $t = 1, 2$ BoP constraints of (34) with those of (23), the source of time-inconsistency is established: the time-1 planner takes $q_1$ as a state variable. In other words, while the period-0 asset supply equation $q_1 = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1}{c_0}\right)^{\sigma} - 1\right]$ is a constraint for the time-0 planner, it is not a constraint for the time-1 planner. The time-0 planner internalizes that the choice of $c_1$ affects the quantity of borrowing received from the intermediaries at $t = 0$ and therefore the debt burden at $t = 1$. On the other hand, since the time-1 planner takes $q_1$ as a given, from his perspective, the choice of $c_1$ does not affect the quantity of debt burden, $q_1$, at $t = 1$.

Let $(c_0^c, c_1^c, c_2^c, a_1^c, a_2^c, q_1^c, q_2^c)$ denote the solution to (34), i.e., the solution under 'commitment', as the planner is required to make this choice at $t = 0$ and then must commit to this solution at $t = 1$. Suppose further that the initial state $(a_0, q_0, c_0)$ is such that $a_1^c, a_2^c > 0$.

Now consider a hypothetical situation, in which, after having made the choices at $t = 0$, the planner is allowed to re-optimize at $t = 1$, by solving (23), taking $(a_1^c, q_1^c, c_0^c)$ as state variables. Let the solution to (23) under this 'deviation' situation be given by $(c_1^D, c_2^D, a_2^D, q_2^D)$. How does $c_1^c$ compare to $c_1^D$? It turns out that if $a_1^c, a_2^c > 0$, then $c_1^D > c_1^c$.

The reason for this is that for a time-0 planner, increasing $c_1$ is more costly at the margin because he internalizes that increasing $c_1$ also increases the quantity of the debt burden, $q_1$. While increasing $c_1$ brings more borrowing through the intermediaries at $t = 0$, it also increases the debt burden at $t = 1$. Said differently, internalizing that the choice of $c_1$ increases the debt burden at $t = 1$, the central bank has an incentive to lower the future private debt burden ex-ante by decreasing $c_1$. [9].

On the other hand, for a 'deviation' planner at $t = 1$, who takes $q_1$ as given, this marginal cost disappears: increasing $c_1$ does not increase the quantity of the debt burden, $q_1$, which was already chosen by the intermediaries and the households at $t = 0$[10]. Given that this additional marginal cost has disappeared and the marginal benefits have not changed, the time-1 deviation planner would like to increase $c_1$ above the 'announced' level by lowering the reserves below the 'announced' level, i.e., $c_1^D > c_1^C$ and $a_1^D < a_1^c$. This result is formally established in Lemma 3.

**Lemma 3.** *Let $(c_0^c, c_1^c, c_2^c, a_1^c, a_2^c, q_1^c, q_2^c)$ be the solution to (34) given $(a_0, q_0, c_{-1})$. Suppose $q_1^c, a_1^c, a_2^c > 0$. Then at time $t = 1$, given $(a_1^c, q_1^c, c_0^c)$, let $(c_1^D, c_2^D, a_2^D, q_2^D)$ be the solution to the two period sub-problem (23).*

*Since $a_1^c, a_2^c, q_1^c > 0$, then $c_1^D > c_1^c$, $a_2^D < a_2^c$, $c_2^D < c_2^c$ and $q_2^D < q_2^c$.*

[9] This intuition is discussed more formally with the help of first-order conditions in Appendix B.3

[10] The choice of $c_1$ at $t = 1$ still affects the real interest rate paid on $q_1$, i.e., $R\left(\frac{c_1}{c_0}\right)^{\sigma}$, but not the quantity itself.

*Proof.* See Appendix B.3 $\square$

To summarize this idea in terms of the model, the central bank's reserve accumulation policy under commitment is time-inconsistent. At $t = 0$, the announced reserve policy $(a_1^c, a_2^c)$ is such that it offers a lower exchange rate appreciation between period 0 and 1, $\left(\frac{c_1^c}{c_0^c}\right)^\sigma$. A lower appreciation rate results in smaller lending by intermediaries at $t = 0$ and hence a smaller debt burden at $t = 1$. At $t = 1$, given the debt burden, $q_1^c$, the central bank would find it optimal to reduce its reserves, $a_2$, below the 'announced' levels. This 'surprise' intervention induces an exchange rate appreciation, thereby allowing the household to consume more: $c_1^D > c_1^c$ and also increases the ex-post appreciation rate[11] $\left(\frac{c_1^D}{c_0^c}\right)^\sigma$.

This line of reasoning applies to the infinite horizon problem as well. The result is formally stated in Proposition 2.

**Proposition 2.** *Let* $\{c_t, a_{t+1}, q_{t+1}\}_{t=0}^\infty$ *be the time-0 Ramsey plan that solves the deterministic version of* (22) *given* $(a_0, q_0, c_{-1})$*. Then at any* $T > 0$*, if* $a_T, a_{T+1}, q_T > 0$*, the continuation of the time-0 plan,* $\{c_t, q_{t+1}, a_{t+1}\}_{t=T}^\infty$*, is NOT a Ramsey plan that solves* (22) *given* $(a_t q_T, c_{T-1})$*.*

Figure 14 in Appendix C shows a numerical simulation illustrating Proposition 2.

Given the time inconsistency of the Ramsey optimal reserve policy, it is imperative to discuss a time-consistent reserve accumulation policy. The next sub-section continues the discussion of the three-period deterministic policy problem, but now assuming that the central bank lacks commitment, the analysis proceeds with describing a time-consistent equilibrium.

### 3.7.2 Three-period policy problem: Equilibrium under Lack of Commitment

The previous section established that the Ramsey optimal reserve accumulation policy is time-inconsistent. In other words, a central bank that lacks commitment would find it optimal to deviate from announced plans in the future. Along the equilibrium path, the market participants would anticipate such deviations and internalize this while making choices in the present. In this

[11] It is useful at this stage to describe an analogy: the Coase conjecture (Coase 1972) about a durable good monopolist who faces two types of consumers- high valuation and low valuation consumers over two time periods. If the high-value consumers are patient enough they can wait until period 2 to buy the good. Considering this, the monopolist would like to 'announce' that he would commit to a high price for the good in both periods. The high-value consumers will find it optimal to buy the good in period 1. In period 2, since the high-value consumers have already bought this durable good, the only remaining consumers in the market are low-value consumers. The monopolist no longer finds it optimal to commit to the previously announced high price in period 2 and would like to deviate to a lower price to serve the low-value consumers.

sub-section, the deterministic three-period policy problem is considered again, but assuming that the central bank lacks commitment, a time-consistent equilibrium is illustrated. This equilibrium is solved using backwards induction.

As before, at $t = 1$, the policy problem is given by (23) taking $(a_1, q_1, c_0)$ as state variables. Let $\mathcal{C}_1(a_1, q_1, c_0)$, $\mathcal{C}_2(a_1, q_1, c_0)$ and $\mathcal{A}_2(a_1, q_1, c_0)$ be the optimal decision rules that solve (23). At $t = 0$, the planner internalizes these decision rules to solve for $(a_1, q_1, c_0)$. At $t = 0$ the policy problem is given by (35).

$$\max_{\{c_0, a_1, q_1\}} \frac{\omega c_0^{1-\sigma}}{1-\sigma} + \beta \frac{\omega \mathcal{C}_1(c_0, a_1, q_1)^{1-\sigma}}{1-\sigma} + \beta^2 \frac{\omega \mathcal{C}_2(c_0, a_1, q_1)^{1-\sigma}}{1-\sigma} \quad \text{s.t.}$$

$$c_0 - q_1 + a_1 = y + R^* a_0 - R\left(\frac{c_0}{c_{-1}}\right)^{\sigma} q_0 \tag{35}$$

$$q_1 = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{\mathcal{C}_1(c_0, a_1, q_1)}{c_0}\right)^{\sigma} - 1\right]$$

The period-0 asset supply equation in (35) suggests that intermediaries internalize that the central bank lacks commitment and take future policy, $\mathcal{C}_1(.)$, as a given when lending at $t = 0$. Similarly, the planner's objective also takes future policies as given. Let $(c_0^l, c_1^l, c_2^l, a_1^l, a_2^l, q_1^l, q_2^l)$ be the time-consistent equilibrium solution to this problem given $(a_0, q_0, c_{-1})$. How does this solution compare to the 'commitment' solution to (34)? Specifically, how does $a_1^c$ compare to $a_1^l$? It will be shown that if $a_1^c, a_2^c, q_1^c > 0$ then $a_1^l > a_1^c$.

To see this, first consider the solution to (34), $(c_0^c, c_1^c, c_2^c, a_1^c, a_2^c, q_1^c, q_2^c)$ and note that this solution is no longer feasible for (35) as it violates the asset supply equation constraint for this problem. This follows from Lemma 3 where it was established that $c_1^D \equiv \mathcal{C}_1(a_1^c, q_1^c, c_0^c) > c_1^c$, i.e., at $t = 1$, given $(a_1^c, q_1^c, c_0^c)$, the central bank would like to deviate to a higher level of consumption by lowering reserves. At $t = 0$, the intermediaries will anticipate this deviation, i.e., anticipating the ex-post appreciation rate to be higher, they would want to lend more at $t = 0$. In other words, the asset supply equation constraint in (35) is violated:

$$q_1^c < \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{\mathcal{C}_1(c_0^c, a_1^c, q_1^c)}{c_0^c}\right)^{\sigma} - 1\right]$$

Therefore, the solution to (34) is no longer feasible for the time-consistent problem. Intuitively, this can be thought of as the domestic bonds market clearing condition getting violated with the supply of funds exceeding the demand, i.e., there is excess lending by the intermediaries at $t = 0$

in response to the anticipated deviation in the future. This result is formally stated in Lemma 4.

**Lemma 4.** *Let* $(c_0^c, c_1^c, c_2^c, a_1^c, a_2^c, q_1^c, q_2^c)$ *be the solution to* (34) *given* $(a_0, q_0, c_{-1})$*. Let* $a_1^c, a_2^c, q_1^c > 0$*. Then* $(c_0^c, a_1^c, q_1^c)$ *is NOT a solution to* (35) *given* $(a_0, q_0, c_{-1})$*.*

*Proof.* See Appendix B.4 □

Given that the intermediaries anticipate the central bank to deviate at $t = 1$, they are willing to lend more at $t = 0$. This creates a situation of excess supply of funds: intermediaries are willing to lend more than what the consumer is willing to borrow at the current exchange rate. To restore the equilibrium, the exchange rate must appreciate at $t = 0$, i.e., even if the central bank does not change its reserve position, $a_1^c$, $c_0$ will increase to absorb the excess lending and clear the bond market. However, such an exchange rate appreciation will also increase the debt payments on previous debt, $R\left(\frac{c_0}{c_{-1}}\right)^{\sigma} q_0$. This creates a motive for the central bank to respond by increasing its reserve position and preventing excessive appreciation. In other words, the excess lending by the intermediaries is partially absorbed through an exchange rate appreciation and partially by the central bank increasing its reserves. Therefore in the time-consistent equilibrium, it is the case that $c_0^l > c_0^c$ and $a_1^l > a_1^c$. This result is formally summarized in Proposition 3.

**Proposition 3.** *Given* $(a_0, q_0, c_{-1})$*, let* $(c_0^c, c_1^c, c_2^c, a_1^c, a_2^c, q_1^c, q_2^c)$ *be the solution to* (34)*. Suppose* $a_1^c > 0, a_2^c, q_1^c > 0$*. Let* $(c_0^l, a_1^l, q_1^l, q_2^l)$ *solve* (35) *given* $(a_0, q_0, c_{-1})$*. Then* $a_1^l > a_1^c$ *and* $c_0^l > c_0^c$*.*

Figure 15 in Appendix C shows a numerical example comparing the equilibrium solutions under commitment and lack of commitment, thereby illustrating Proposition 3, in particular, $a_1^l > a_1^c$.
The argument outlined above applies to the infinite horizon model as well. In the next section, a time-consistent equilibrium for the infinite horizon policy problem is described. Furthermore, with an understanding of the analytical properties of the deterministic policy problem discussed thus far, a quantitative analysis of the stochastic policy problem is presented.

# 4 Quantitative Analysis

This section presents a quantitative analysis of the model. First, a time-consistent equilibrium for the infinite horizon stochastic policy problem is described. Second, using a test calibration, a solution to the time-consistent equilibrium is presented, illustrating the decision rules of the central bank. Third, using a simulation, the long-run equilibrium moments are compared to those of a laissez-faire equilibrium. It is shown that in the presence of volatile capital flows, reserves provide an insurance against consumption (and exchange rate) volatility. The optimal policy is to lean against the wind and build reserves in financially stable times and run down reserves in times of disruptions. As a result, a managed floating policy emerges as the optimal policy regime, with the economy maintaining both debt and reserves simultaneously in the long-run equilibrium.

### 4.1 A Time-Consistent Policy Problem

To define a time-consistent equilibrium, it is useful to set up a recursive policy problem. For the sake of computational ease, it is also useful to set up the problem with the exchange rate $e$ as the choice variable rather than consumption. Let the endogenous state variables be given by the reserve position at the beginning of a period, $a$, and the existing debt denominated in domestic currency, $\tilde{q}$.

It is assumed that $\Gamma$ is the only exogenous state variable and follows a Markov switching regime. As discussed in sub-section 3.5, a lower value of $\Gamma$ corresponds to periods of financial boom, a loosening of the credit constraint faced by the intermediaries, which results in increased capital inflows. Conversely, a higher value of $\Gamma$ corresponds to periods of financial disruption, i.e., due to a tightening of the intermediation credit constraint the economy experiences a net capital outflow.

Let $V(.)$ denote the value function associated with the consumer's utility function and let $\mathcal{E}(.)$ denote the exchange rate policy function. The control variables are choices of reserves, $a'$, domestic currency denominated borrowing, $\tilde{q}'$, and the exchange rate, $e$. The central bank's recursive policy problem is given by (36).

$$
\begin{aligned}
V\big(a, \tilde{q}, \Gamma\big) = \max_{a', \tilde{q}', e} & \left\{ \omega^{\frac{1}{\sigma}} \frac{e^{1-\frac{1}{\sigma}}}{1-\sigma} + \beta \mathbb{E}_{\Gamma'|\Gamma} \Big[ V\big(a', \tilde{q}', \Gamma'\big) \Big] \right\} \qquad \text{s.t.} \\
\omega^{\frac{1}{\sigma}} e^{1-\frac{1}{\sigma}} - \tilde{q}' + e a' &= e y - R\tilde{q} + e R^* a \\
\tilde{q}' &= \frac{1}{\Gamma} \left[ \frac{R}{R^*} \mathbb{E}_{\Gamma'|\Gamma} \left( \frac{e^2}{\mathcal{E}(a', \tilde{q}', \Gamma')} \right) - e \right] \\
a' &\geqslant 0
\end{aligned}
\tag{36}
$$

First, $c$ has been substituted out in terms of $e$ in the objective function using (13). The first constraint is the BoP equation (15) $c$ has been substituted out in terms of $e$ and $(\tilde{b}, \tilde{b}')$ substituted in terms of $(\tilde{q}, \tilde{q}')$ using (12). The second constraint is the intermediaries' asset supply equation (9). The future exchange rate in this equation is denoted by the future optimal policy $\mathcal{E}(.)$, reflecting the fact that the central bank lacks commitment and the intermediaries internalize this, i.e., they take future exchange rate policy as a given while making their lending decisions. The equilibrium is defined below.

#### 4.1.1 A Time-Consistent (Markov) Equilibrium

A time consistent (Markov) equilibrium is defined by the optimal exchange rate and borrowing decision rules $\left(\mathcal{E}(.), \tilde{Q}(.)\right)$, a reserve accumulation rule $\mathcal{A}(.)$, and a value function $V(.)$ such that:

- Given a conjectured future exchange rate policy, $\mathcal{E}(.)$, and a reserve accumulation rule $\mathcal{A}(.)$, $\left(\mathcal{E}(.), \tilde{Q}(.)\right)$ constitute a competitive equilibrium.
- Given conjectured future exchange rate policy, $\mathcal{E}(.)$, and the competitive equilibrium, $\left(\mathcal{E}(.), \tilde{Q}(.)\right)$, the central bank follows a reserve accumulation rule $\mathcal{A}(.)$ such that $V(.)$ attains a maximum.
- The conjectured future exchange rate policy is indeed correct: $\mathcal{E}(.)$.

### 4.2 Calibration

A test calibration is used to solve (36), with parameter values not significantly different from standard choices in literature. A period in the model corresponds to a year. The domestic interest rate is assumed to be 4% ($R = \beta^{-1} = 1.04$). Since it is crucial that $R^* < R$, the world interest rate is set at 2% ($R^* = 1.02$). The risk aversion parameter, $\sigma = 2$ and the tradeable good preference parameter, $\omega = 0.3$. The tradeable good endowment is normalized $y = 1$. The long-run equilibrium value of $\Gamma$ is set as $\bar{\Gamma} = 0.05$. This implies that in the deterministic steady-state, $\bar{c} = 0.9843$ and $\bar{q} = 0.392$.
It is assumed that $\ln \Gamma$ follows an $AR(1)$ structure:

$$\ln \Gamma_{t+1} = (1 - \phi) \ln \bar{\Gamma} + \phi \ln \Gamma_{t+1} + \epsilon_{t+1} \qquad \epsilon \sim \mathcal{N}(0, \sigma_\epsilon)$$

Assuming that the financial shock is persistent, $\phi$ is set at 0.95 and $\sigma_\epsilon$ at 0.5. The $AR(1)$ process is discretized to a three-state Markov chain using the Rowenhorst method.

### 4.3 Results

The problem (36) is solved using value function iteration. In this section, the optimal decision rules and the associated long-run moments for a simulated economy are presented. It is shown that in the long-run equilibrium, the economy carries both debt and reserves simultaneously. In other words, the optimal exchange rate policy is a managed float wherein the central bank actively uses reserves to manage the exchange rate. This equilibrium is compared to a laissez-faire outcome wherein the exchange rate is allowed to float freely and the central bank does not hold any reserves.

Figures 5, 6, 7 depict the policy functions for reserves, borrowing, and exchange rates respectively. In each of the figures, the left panel shows how the corresponding variable varies with the beginning of period reserves for a given level of debt. Similarly, the right panel shows how the corresponding variable varies with debt, for a given level of initial reserves. In both panels of all figures, the policy functions for the lowest and the highest state of $\Gamma$ are depicted.

Figure 5 illustrates a *leaning-against-the-wind* reserve accumulation policy: the central bank builds reserves ($a' \geqslant a$) in financially stable times, i.e., periods when $\Gamma$ is low and the economy receives higher financial inflows. Whereas, in periods of financial disruption, when $\Gamma$ is higher, the central bank finds it optimal to run down the reserves ($a' < a$). The amount of reserve accumulation/decumulation is also affected by the beginning of period debt: a low-debt economy accumulates more reserves, whereas there is little to no accumulation in a high-debt economy. In other words, reserve accumulation becomes more costly as the debt levels increase. The left panel of Figure 6 shows that borrowing is a decreasing function of the beginning of period reserves: more reserves allow for smaller borrowing. The right panel shows that the economy accumulates debt when the initial debt levels are lower whereas at higher initial debt levels, it is optimal to

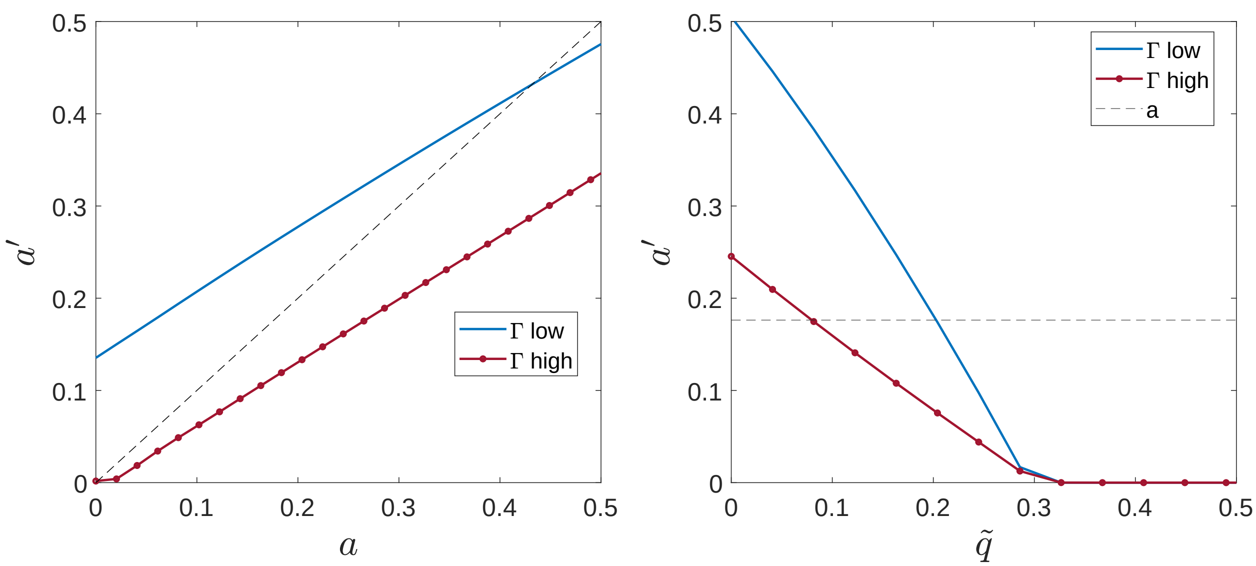



Figure 5: Policy Function- Reserves

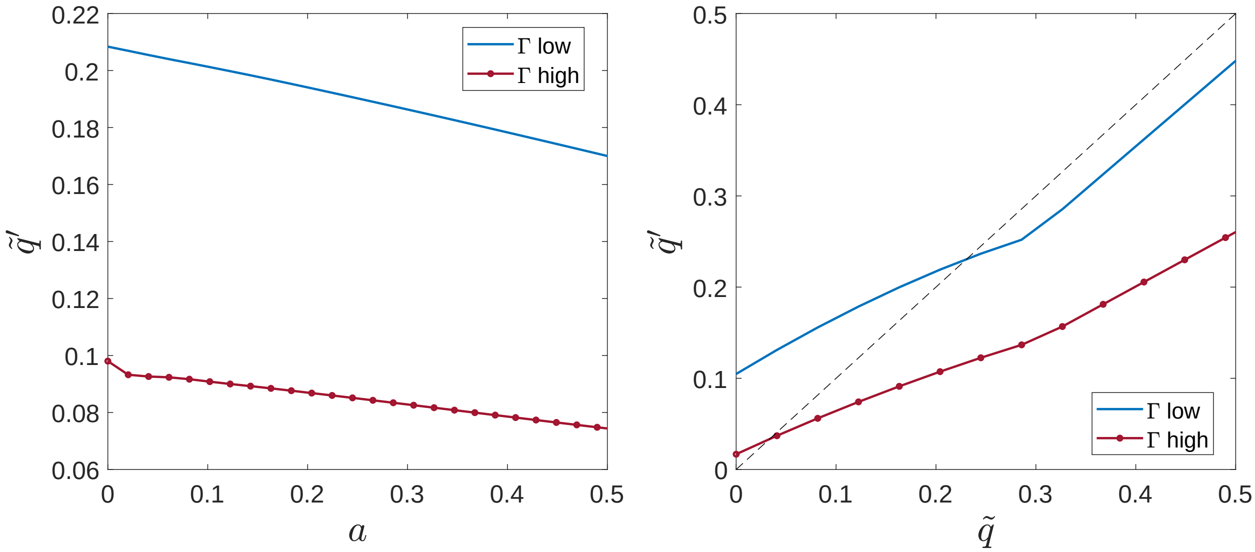



Figure 6: Policy Function- Debt

run down the debt. The exchange rate policy function illustrated in Figure 7 shows that starting a period with more reserves allows for a lower exchange rate (and higher consumption), whereas higher existing debt is associated with a higher exchange rate and hence lower consumption. The figures also show the impact of Γ on optimal choices: an increase in Γ leads to a reduction in reserves and borrowing and an exchange rate depreciation.

One can observe that in the presence of financial shocks, reserves are equivalent to a precautionary

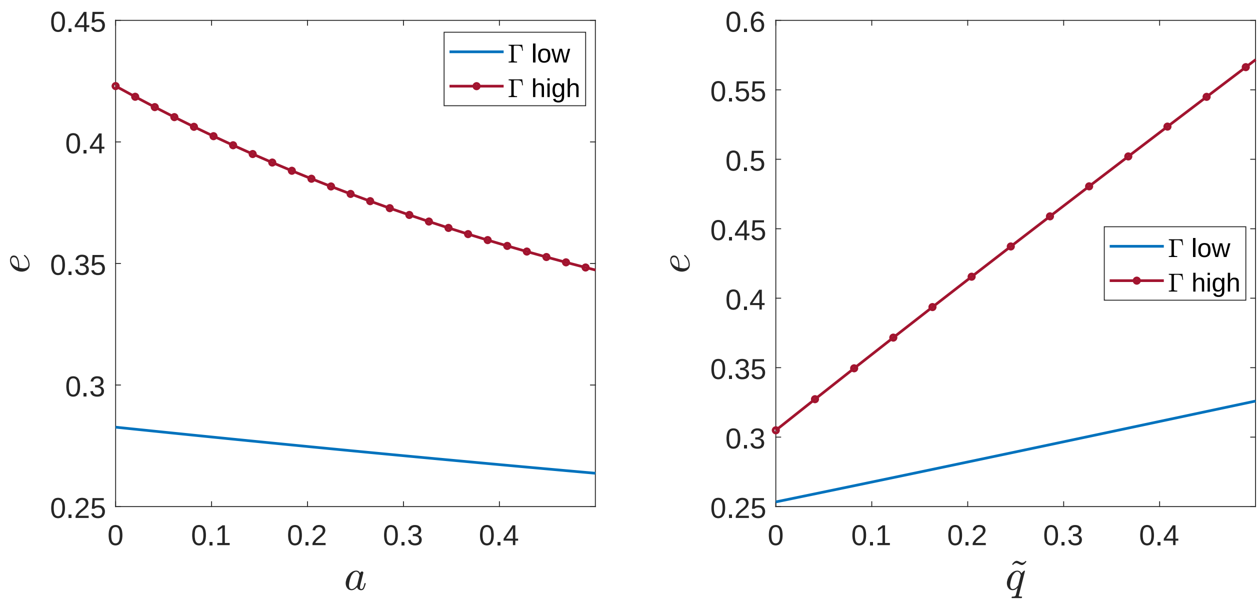



Figure 7: Policy Function- Exchange Rate

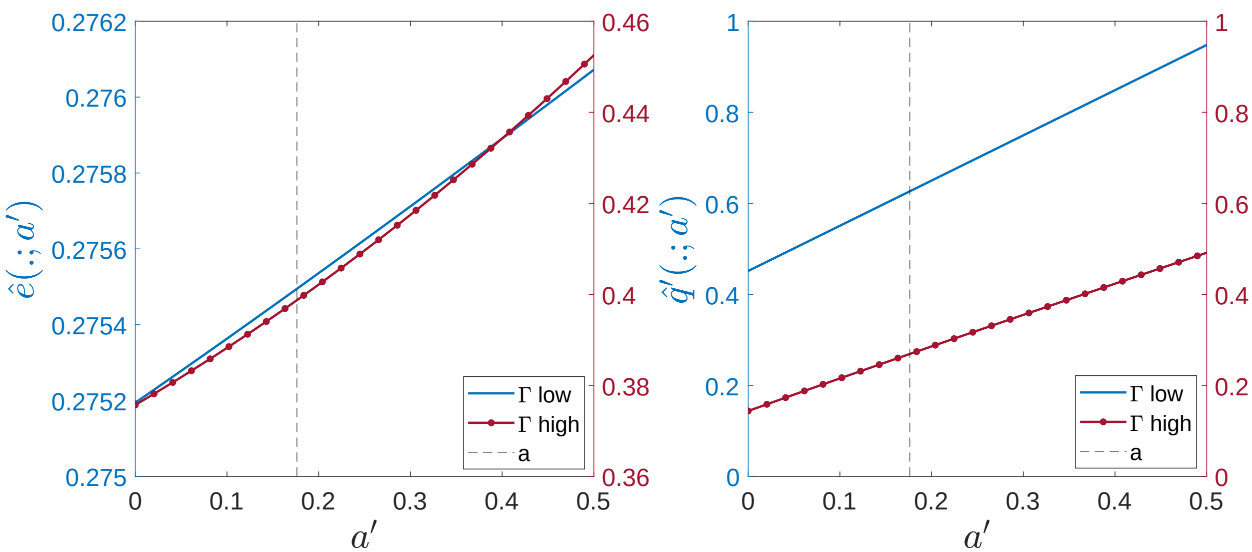



Figure 8: Competitive equilibrium variation with reserves

saving. In other words, they act as an insurance against the consumption volatility that results from exchange rate movements created by volatile capital flows. As illustrated in section 3.5, an increase in $\Gamma$ results in an exchange rate depreciation (lower consumption), and conversely a decrease leads to an appreciation (higher consumption). Accumulation of reserves in times when $\Gamma$ is lower is then equivalent to buying an insurance to mitigate the degree of exchange rate depreciation induced by a potential increase in $\Gamma$ in the future. In other words, by running down reserves, the central bank can decrease the extent of capital outflow-led depreciation during times of financial disruptions, thereby providing higher consumption.

However, the optimal insurance, i.e., the reserve accumulation in 'good' times, is influenced by two costs: exchange rate depreciation and higher debt. These costs were illustrated using the two-period model in section 3.6.1. The same is true for the infinite horizon model. To elucidate further, the following analysis demonstrates how the choice of reserves alters the competitive equilibrium. Fix an arbitrary choice of $a'$, then the competitive equilibrium can be described by two implicit functions $\hat{e}(a, \tilde{q}, \Gamma; a')$ and $\hat{\tilde{q}}'(a, \tilde{q}, \Gamma; a')$ that satisfy the BoP constraint and the asset supply equation in (36)[12]. Figure 8 plots $\hat{e}(.)$ and real debt, $\frac{\hat{\tilde{q}}'(.)}{\hat{e}(.)}$, with respect to $a'$ for a given initial state $(a, \tilde{q})$. First, note that reserve accumulation leads to an exchange rate depreciation and hence results in a lower consumption. Second, when the central bank builds reserves, consumers respond by borrowing more, thereby increasing the future debt burden. Analogous to the two-

[12]Note that given the lack of commitment, these functions take as given, the conjectured future exchange rate policy $\mathcal{E}(.)$.

period model, an exchange rate depreciation in equilibrium results in lower consumption and dilution of previous debt payments, and as a result, borrowing and reserves do not increase one-for-one. Considering these two costs, the optimal choice of reserves is thus such that the costs are equated to the marginal insurance benefits of reserves- the ability to appreciate the exchange rate in times of financial disruptions thereby providing additional resources for consumption. This reasoning is similar to the insurance role of savings in the standard consumption-savings problem with incomplete markets: reserves transfer resources from relatively abundant current states to future states where resources are relatively scarce.

Secondly, while it is clear that FX interventions are effective in this environment, it is also noteworthy that the effectiveness of the interventions is crucially dependent on $\Gamma$. As the left panel of Figure 8 shows, accumulating reserves in times when $\Gamma$ is higher is more costly as it induces a stronger exchange rate depreciation vis-à-vis times when $\Gamma$ is lower. Conversely, with higher $\Gamma$, a smaller intervention can induce a sharper response in exchange rates. In other words, the steepness of the intermediary supply rule which is dependent on the degree of friction in financial markets, $\Gamma$, is also a key determinant of the effectiveness of FX interventions.

Finally, the model is simulated using the derived decision rules and the long-run moments are presented in Table 1. The table shows the long-run average levels of debt, reserves, and consumption as percentages of the tradeable good endowment. For the sake of comparison, the long-run moments for a laissez-faire equilibrium are also shown- one where the reserves are always 0. In the long-run time-consistent equilibrium, the economy carries both debt and reserves simultaneously, i.e., the central bank actively manages the exchange rate using reserves thereby influencing the levels of debt and consumption. In the laissez-faire outcome, the economy carries a moderately smaller amount of debt and consumption is also lower on average. It is interesting to note that reserves do not have a significant effect on the long-run average debt levels. While reserves induce higher debt levels on average, the debt does not increase one-for-one with reserves. This is because, in periods when the central bank builds reserves, borrowing is higher than laissez-faire levels due to exchange rate depreciation; but in periods where it runs down reserves, an exchange

| | Reserves | 'Real' Debt | Consumption |
|---|---|---|---|
| Reserve Policy | 17.63 | 49.43 | 98.69 |
| Laissez-faire | - | 48.97 | 98.21 |

Table 1: Long-run averages

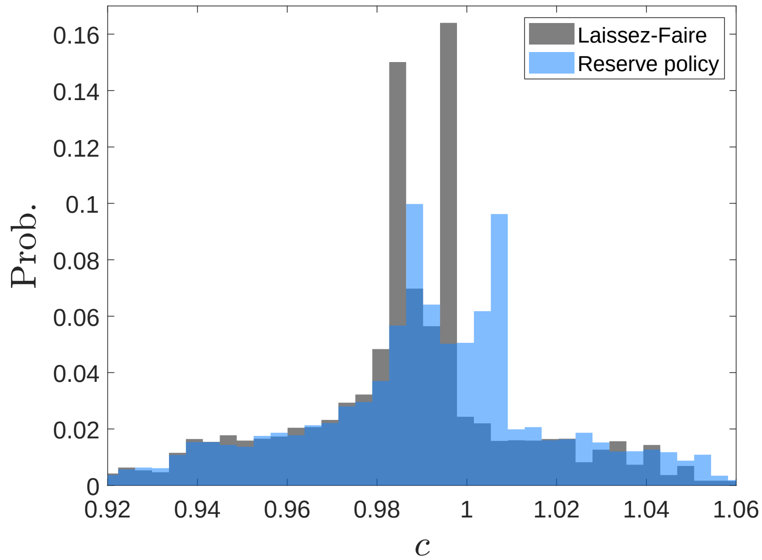



Figure 9: Consumption Density: Reserve Policy vs Laissez-faire

rate appreciation results in lower borrowing than laissez-faire levels. Similarly, while FX intervention results in higher average consumption, it is also useful to compare the long-run equilibrium distribution of consumption with the laissez-faire distribution. Figure 9 plots the consumption densities comparing the laissez-faire outcome with the optimal outcome. The use of reserves for transferring resources from abundant states to relatively scarcer states is evident from the fact that the consumption density is higher for higher levels of consumption under the intervention equilibrium vis-à-vis the laissez-faire outcome.

To conclude, this section has shown that in the presence of financial shocks, the optimal exchange rate policy is a managed float, i.e., the central bank actively manages the exchange rate using reserves. First, this result sheds light on why a free-floating exchange rate may not be optimal and offers a potential explanation for the 'Fear of Floating' phenomenon in emerging economies (Calvo and Reinhart 2002). Secondly, the analysis shows that the expansion of global financial markets and capital flow volatility has provided grounds for increasing exchange rate controls in emerging economies and an additional motive to accumulate reserves Ilzetzki, Reinhart and Rogoff (2019). Finally, this analysis provides a theoretical rationale for the observed pattern of net external debtors maintaining substantial foreign exchange reserves.

## 5 Conclusion

Motivated by the observation that net external debtor emerging economies are holding large amounts of foreign exchange reserves, this paper has proposed an explanation that rationalizes this portfolio choice by linking reserve accumulation to exchange rate controls and international capital flows, with financial market frictions playing a key role.

For an economy with existing domestic currency denominated debt, the central bank's optimal reserve accumulation policy internalizes a trade-off: while running down the reserves allows the economy to pay off its debt, such an action induces an exchange rate appreciation that increases the real value of existing debt payments. Reserve accumulation, while increasing household borrowing, generates an exchange rate depreciation that dilutes the real value of existing debt payments. This trade-off rationalizes why a debtor economy might prefer holding reserves to decreasing its debt.

Moreover, the optimal reserve accumulation policy is time-inconsistent. The central bank has an incentive to announce high future exchange rates to mitigate the external debt burden but finds it sub-optimal to implement these rates when the future period arrives. Consequently, in equilibrium, a central bank that lacks commitment holds even more reserves than under commitment.

Finally, a quantitative analysis demonstrates that in the presence of volatile capital flows, the economy optimally maintains a portfolio of external debt and reserves, with reserves essentially being an insurance against volatile capital flows, i.e., FX interventions stabilize the exchange rate and smooth consumption.

This paper has not tackled the normative questions of comparing policy instruments or analyzing their interactions. Further research is needed to compare instruments like capital controls with reserves, consider the interaction of monetary policy with exchange rate policy, and examine the role of foreign currency and government debt in the presence of financial frictions.

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# Appendix

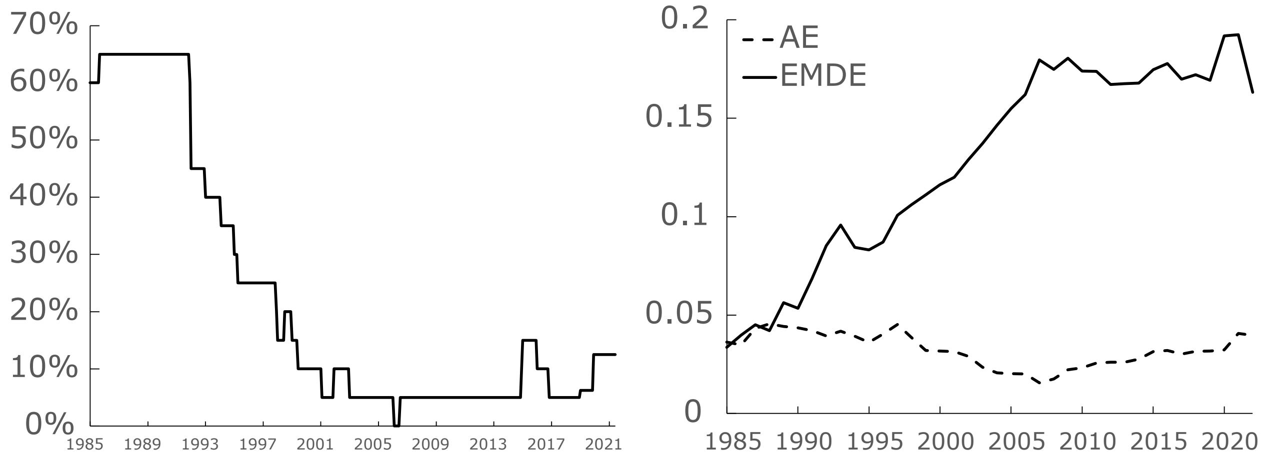
## A Insights from Data

To provide context to the theory developed in this paper, this section summarizes a few well-documented facts. First, foreign exchange reserves held by central banks in emerging economies have experienced a secular upward trend starting in the mid-1980s; and reserve holdings continue to be relatively high. Secondly, even net external debtor economies (excluding reserves) are also holding significant amounts in reserves. By contrast, most advanced economies did not jump on this reserve-building trend. Third, while reserves grew rapidly, emerging economies simultaneously eased capital controls but enforced exchange rate controls. By contrast, most advanced economy currencies are free-floating. Most of these findings are discussed extensively by Ilzetzki, Reinhart and Rogoff (2019).

A sample of the 20 largest (by 2022 GDP) emerging economies which are also persistent deficit economies and typically external debtors is considered. The countries in the sample are: Argentina, Bangladesh, Brazil, Colombia, Egypt, India, Indonesia, Korea; Republic of, Malaysia, Mexico, Nigeria, Pakistan, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, and Vietnam. The left panel of Figure 1 shows the mean and the median international investment position (IIP) of this sample and the right panel shows the official foreign exchange reserves (excluding gold and SDRs). These charts highlight that net external debtor economies have accumulated significant FX reserves: in 2022, the median IIP was -44.86% and median reserves were 16.27% in this sample.

The left panel of Figure 10 shows the IRR capital controls index (Ilzetzki, Reinhart and Rogoff 2019) for this sample of countries. It is evident that the trend in reserves is accompanied by a simultaneous trend in easing of capital controls.

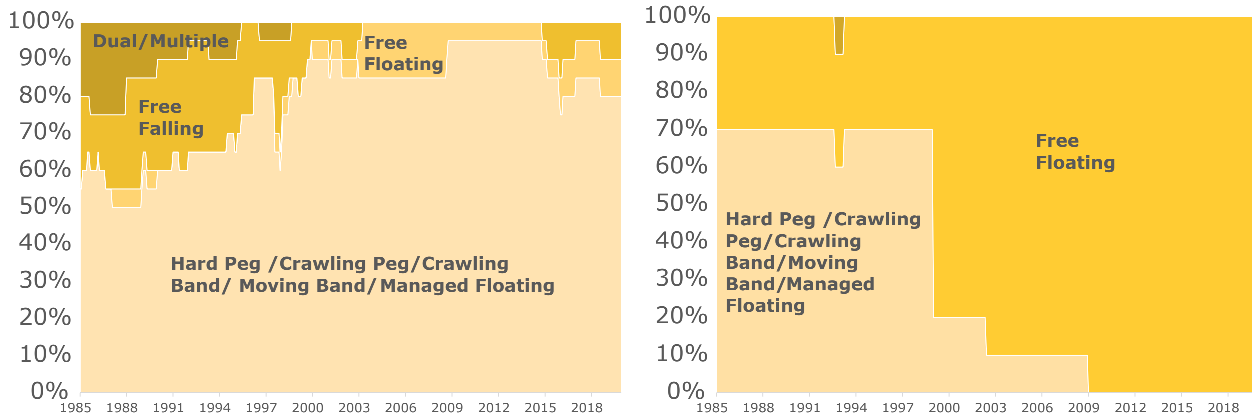
For the sake of comparison, a sample of 10 of the 12 largest advanced economies (by 2022 GDP) is also considered, namely, Australia, Belgium, Canada, France, Germany, Italy, Netherlands, Spain, the United Kingdom, and the United States. Notably Japan and Switzerland are excluded. The criteria for the selection of these countries is that not only are the currencies of these countries globally dominant, but also they are the most freely floating currencies in the world. The right panel of Figure 10 compares reserve accumulation by emerging economies sample with this sample of advanced economies. It can be observed that free-floating currency advanced economies had




(a) Easing of Capital Controls (b) (Mean) Reserves to GDP: EMDE vs AE

Notes: The left panel shows the IRR capital controls index for the sample of EMDEs. Source: Computed using the IRR Unified Market Analysis database (Ilzetzki, Reinhart and Rogoff 2019). The right panel shows the mean Reserves to GDP in the sample of EMDEs and AEs. Source: Computed using the External Wealth of Nations database (Milesi-Ferretti 2022).

Figure 10: Reserves and Capital Controls




(a) Exchange rate regimes: EMDE (b) Exchange rate regimes: AE

Notes: The left panel shows the exchange rate regimes for the sample of EMDEs and the right panel shows the regimes for the sample of AEs. Source: Computed using the IRR's coarse classification (Ilzetzki, Reinhart and Rogoff 2019).

Figure 11: Exchange rate regimes: EMDE vs AE

negligible reserve accumulation compared to emerging economies.

Finally, the left and right panels of Figure 11 compare the evolution of exchange rate regimes in these emerging and advanced economies. The regime classification follows Ilzetzki, Reinhart and Rogoff (2019). While the advanced economies in the sample are typically free-floaters, a 'Fear of Floating' phenomenon (Calvo and Reinhart 2002) is evident in the emerging economies: they have historically imposed stronger exchange rate controls and continue to do so.

Two main conclusions can be drawn from this analysis. First, the trend in reserves is accompanied by a simultaneous trend in easing of capital controls and imposition of exchange rate controls. Second, Countries that impose exchange rate controls have accumulated significantly higher reserves. These facts constitute the key motivating factors for the theory proposed in this paper.

# B Proofs

## B.1 Proof of Lemma 1

*Proof.* It suffices to establish the condition under which a perturbation to $(\hat{c}_1, \hat{c}_2, \hat{q}_2, \hat{a}_2)$ leads to a welfare gain. Consider a perturbation given by:

$q_2 = \hat{q}_2 + \Delta = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma}\right]$, where $\Delta > 0$

$c_2 = y + R^* a_2 - R^*(1 + \Gamma(\hat{q}_2 + \Delta))(\hat{q}_2 + \Delta)$

$c_1 + a_2 - (\hat{q}_2 + \Delta) = y + R^* a_1 - R\left(\frac{c_1}{c_0}\right)^{\sigma} q_1$

Differentiating wrt $\Delta$:

$\frac{dc_1}{d\Delta} + \frac{da_2}{d\Delta} - 1 + R\sigma c_1^{\sigma-1}\frac{q_1}{c_0^{\sigma}}\frac{dc_1}{d\Delta} = 0$

$\frac{dc_2}{d\Delta} = R^*\left(\frac{da_2}{d\Delta} - (1 + 2\Gamma\Delta + 2\Gamma\hat{q}_2)\right)$

$1 = \sigma\frac{1}{\Gamma}\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma}\left[\frac{1}{c_2}\frac{dc_2}{d\Delta} - \frac{1}{c_1}\frac{dc_1}{d\Delta}\right]$

Solving the above system of equations yields:

$$\frac{da_2}{d\Delta} = \frac{\frac{1}{\hat{\sigma}} + (1 + 2\Gamma\Delta + 2\Gamma\hat{q}_2)\frac{R^*}{c_2} + \frac{1}{c_1\kappa}}{\frac{R^*}{c_2} + \frac{1}{c_1\kappa}}$$

$$\frac{dc_2}{d\Delta} = R^*\left(\frac{\frac{1}{\hat{\sigma}} - \frac{1}{c_1\kappa}(2\Gamma\Delta + 2\Gamma\hat{q}_2)}{\frac{R^*}{c_2} + \frac{1}{c_1\kappa}}\right)$$

$$\frac{dc_1}{d\Delta} = -\frac{1}{\kappa}\left(\frac{\frac{1}{\hat{\sigma}} + (2\Gamma\Delta + 2\Gamma\hat{q}_2)\frac{R^*}{c_2}}{\frac{R^*}{c_2} + \frac{1}{c_1\kappa}}\right)$$

where $\kappa = 1 + \sigma R c_1^{\sigma-1}\frac{q_1}{c_0^{\sigma}}$ and $\hat{\sigma} = \sigma\left(\hat{q}_2 + \Delta + \frac{1}{\Gamma}\right) = \sigma\frac{1}{\Gamma}\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma}$.

Since $q_1 > 0$, then $\hat{\kappa} = 1 + \sigma R\hat{c}_1^{\sigma-1}\frac{q_1}{c_0^{\sigma}} > 0$.

Since $\hat{q}_2 \geqslant 0$, then $\hat{\sigma} > 0$.

Let $\hat{D} \equiv \frac{R^*}{\hat{c}_2} + \frac{1}{\hat{c}_1\hat{\kappa}}$ where $\hat{D} > 0$.

Simplify further to obtain:

$$\left.\frac{dc_2}{d\Delta}\right|_{\Delta=0} = \frac{R^*}{\hat{\kappa}\hat{D}\hat{\sigma}}\left(\frac{\hat{c}_1\hat{\kappa} - 2\Gamma\hat{q}_2\hat{\sigma}}{\hat{c}_1}\right)$$

$$\left.\frac{dc_1}{d\Delta}\right|_{\Delta=0} = -\frac{1}{\hat{\kappa}\hat{D}\hat{\sigma}}\frac{\hat{c}_1}{\hat{c}_2}\left(\frac{\hat{c}_2 + R^*2\Gamma\hat{q}_2\hat{\sigma}}{\hat{c}_1}\right)$$

Define welfare as:

$$W = \frac{\omega}{1-\sigma}\left(c_1^{1-\sigma} + \beta c_2^{1-\sigma}\right)$$

Change in welfare can be expressed as:

$$\frac{dW}{d\Delta} = \omega c_2^{-\sigma}\left(\left(\frac{c_2}{c_1}\right)^{\sigma}\frac{dc_1}{d\Delta} + \frac{1}{R}\frac{dc_2}{d\Delta}\right)$$

$$\begin{aligned}\left.\frac{dW}{d\Delta}\right|_{\Delta=0} &= \omega\hat{c}_2^{-\sigma}\left(\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma}\left.\frac{dc_1}{d\Delta}\right|_{\Delta=0} + \frac{1}{R}\left.\frac{dc_2}{d\Delta}\right|_{\Delta=0}\right)\\ &= \frac{\omega\hat{c}_2^{-\sigma}}{\hat{\kappa}\hat{D}\hat{\sigma}}\frac{1}{\hat{c}_1}\frac{R^*}{R}\left(\hat{c}_1\hat{\kappa} - 2\Gamma\hat{q}_2\hat{\sigma} - \frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1}(\hat{c}_2 + R^*2\Gamma\hat{q}_2\hat{\sigma})\right)\end{aligned}$$

Then:

$$\begin{aligned} & \left.\frac{dW}{d\Delta}\right|_{\Delta=0} > 0\\ \Leftrightarrow\quad & \left(\hat{c}_1\hat{\kappa} - 2\Gamma\hat{q}_2\hat{\sigma} - \frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1}(\hat{c}_2 + R^*2\Gamma\hat{q}_2\hat{\sigma})\right) > 0\\ \Leftrightarrow\quad & \hat{c}_1 + \sigma R\left(\frac{\hat{c}_1}{c_0}\right)^{\sigma} q_1 - 2\sigma(\hat{q}_2 + \Gamma\hat{q}_2^2) > \frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1}(\hat{c}_2 + R^*2\sigma(\hat{q}_2 + \Gamma\hat{q}_2^2))\\ \Leftrightarrow\quad & y + R^*a_1 + (\sigma-1)R\left(\frac{\hat{c}_1}{c_0}\right)^{\sigma} q_1 - 2\sigma(\hat{q}_2 + \Gamma\hat{q}_2^2) + \hat{q}_2 > \frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1}\left(y + (2\sigma-1)R^*(\hat{q}_2 + \Gamma\hat{q}_2^2)\right)\\ \Leftrightarrow\quad & R^*a_1 + (\sigma-1)R\left(\frac{\hat{c}_1}{c_0}\right)^{\sigma} q_1 > \left[\frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1} - 1\right]y + \Gamma\hat{q}_2^2 + (2\sigma-1)(\hat{q}_2 + \Gamma\hat{q}_2^2)\left(R\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1} + 1\right)\end{aligned}$$

□

### B.2 Proof of Lemma 2

*Proof.* It suffices to establish the condition under which a perturbation to $(\hat{c}_1, \hat{c}_2, \hat{q}_2, \hat{a}_2)$ leads to a welfare gain. Consider a perturbation given by:

$q_2 = \hat{q}_2 + \Delta$, where $\Delta > 0$

$c_2 = \left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}} c_1$

$a_2 = \frac{1}{R^*}\left(\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}} c_1 - y\right) + \Delta(1+\Gamma\Delta)$

$c_1 + \frac{1}{R^*}\left(\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}} c_1 - y\right) + \Gamma\Delta^2 = y + R^*a_1 - R\left(\frac{c_1}{c_0}\right)^{\sigma} q_1$

Differentiating wrt $\Delta$:

$\frac{dc_2}{d\Delta} = \frac{1}{\sigma}\frac{R^*}{R}\Gamma\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}-1} c_1 + \left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}}\frac{dc_1}{d\Delta}$

$\frac{da_2}{d\Delta} = \frac{1}{R^*}\frac{dc_2}{d\Delta} + 1 + 2\Gamma\Delta$
$\frac{dc_1}{d\Delta}\left(1 + \frac{1}{R^*}\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}} + c_1^{\sigma-1}\sigma R\frac{q_1}{c_0^\sigma}\right) = -\left(2\Gamma\Delta + \frac{1}{\sigma}\frac{R^*}{R}\Gamma\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}-1}\frac{c_1}{R^*}\right)$
which yields:

$$\frac{dc_1}{d\Delta} = -\frac{\left(2\Gamma\Delta + \frac{1}{\sigma}\frac{R^*}{R}\Gamma\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}-1}\frac{c_1}{R^*}\right)}{1 + \frac{1}{R^*}\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}} + c_1^{\sigma-1}\sigma R\frac{q_1}{c_0^\sigma}}$$

$$\left.\frac{dc_1}{d\Delta}\right|_{\Delta=0} = -\frac{\frac{1}{\sigma}\frac{R^*}{R}\Gamma\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}-1}\frac{\hat{c}_1}{R^*}}{1 + \frac{1}{R^*}\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}} + \hat{c}_1^{\sigma-1}\sigma R\frac{q_1}{c_0^\sigma}} = -\frac{\frac{1}{\sigma}\Gamma\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}}\frac{\hat{c}_1}{R^*}}{\frac{1}{\hat{c}_1}\left(\hat{c}_1 + \frac{\hat{c}_1}{R^*}\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}} + \hat{c}_1^{\sigma}\sigma R\frac{q_1}{c_0^\sigma}\right)} < 0$$

since $q_1, c_0 > 0$.

Further simplification implies:

$$\left.\frac{dc_1}{d\Delta}\right|_{\Delta=0} = -\frac{\frac{1}{\sigma}\Gamma\hat{c}_1\frac{\hat{c}_2}{R^*}}{\left(\hat{c}_1 + \frac{\hat{c}_2}{R^*} + \hat{c}_1^{\sigma}\sigma R\frac{q_1}{c_0^\sigma}\right)}$$

Similarly,

$$\begin{aligned}\left.\frac{dc_2}{d\Delta}\right|_{\Delta=0} &= \frac{1}{\sigma}\Gamma\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}}\hat{c}_1 + \left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}}\left.\frac{dc_1}{d\Delta}\right|_{\Delta=0} \\ &= \frac{1}{\sigma}\Gamma\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}}\hat{c}_1\left(\frac{\hat{c}_1 + \sigma R\hat{c}_1^{\sigma}\frac{q_1}{c_0^\sigma}}{\hat{c}_1 + \frac{\hat{c}_2}{R^*} + \hat{c}_1^{\sigma}\sigma R\frac{q_1}{c_0^\sigma}}\right) > 0\end{aligned}$$

since $\frac{q_1}{c_0^\sigma} > 0$.

Using these gives:

$$\left.\frac{da_2}{d\Delta}\right|_{\Delta=0} = \frac{1}{R^*}\left.\frac{dc_2}{d\Delta}\right|_{\Delta=0} + 1 > 0$$

Define welfare as:

$$W = \frac{\omega}{1-\sigma}\left(c_1^{1-\sigma} + \beta c_2^{1-\sigma}\right)$$

Change in welfare can be expressed as:

$$\frac{dW}{d\Delta} = \omega c_2^{-\sigma}\left(\left(\frac{c_2}{c_1}\right)^{\sigma}\frac{dc_1}{d\Delta} + \frac{1}{R}\frac{dc_2}{d\Delta}\right)$$

$$\left.\frac{dW}{d\Delta}\right|_{\Delta=0} = \omega\hat{c}_2^{-\sigma}\frac{1}{R}\left(R^*\left.\frac{dc_1}{d\Delta}\right|_{\Delta=0} + \left.\frac{dc_2}{d\Delta}\right|_{\Delta=0}\right)$$

$$
\begin{aligned}
& & \left.\frac{dW}{d\Delta}\right|_{\Delta=0} &> 0 \\
&\Leftrightarrow & \left.\frac{dc_2}{d\Delta}\right|_{\Delta=0} &> -R^* \left.\frac{dc_1}{d\Delta}\right|_{\Delta=0} \\
&\Leftrightarrow & \frac{1}{\sigma}\Gamma\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}}\hat{c}_1\left(\frac{\hat{c}_1+\sigma R\hat{c}_1^{\sigma}\frac{q_1}{c_0^{\sigma}}}{\hat{c}_1+\frac{\hat{c}_2}{R^*}+\hat{c}_1^{\sigma}\sigma R\frac{q_1}{c_0^{\sigma}}}\right) &> R^*\frac{\frac{1}{\sigma}\Gamma\hat{c}_1\frac{\hat{c}_2}{R^*}}{\left(\hat{c}_1+\frac{\hat{c}_2}{R^*}+\hat{c}_1^{\sigma}\sigma R\frac{q_1}{c_0^{\sigma}}\right)} \\
&\Leftrightarrow & \left(\hat{c}_1+\sigma R\hat{c}_1^{\sigma}\frac{q_1}{c_0^{\sigma}}\right) &> \hat{c}_1 \\
&\Leftrightarrow & q_1 &> 0
\end{aligned}
$$

□

### B.3 Proof (Sketch) of Lemma 3

*Proof.* (Sketch) Consider the problem (34). The First order conditions for $a_1, c_1$, with Lagrange multipliers $(\lambda_0, \lambda_1, \lambda_2)$ on the BoP constraints and $(\nu_1, \nu_2)$ on the non-negativity constraints are given below. FOCs for $c_0, c_2, a_2$ are omitted here for conciseness.

$a_1$:

$$\lambda_0 = \beta R^*\lambda_1 + \nu_1 \tag{37}$$

$c_1$:

$$
\begin{aligned}
&\omega c_1^{-\sigma} - \lambda_1 - \lambda_1\frac{\sigma}{c_1}R\frac{c_1^{\sigma}}{c_0^{\sigma}}\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1^{\sigma}}{c_0^{\sigma}}\right)-1\right] \\
&-\lambda_1\frac{\sigma}{c_1}\frac{1}{\Gamma}\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} \\
&+\lambda_2\frac{\sigma}{c_1}\left(\frac{c_2}{c_1}\right)^{\sigma}\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma}-1\right] \\
&+\lambda_2\frac{\sigma}{c_1}\left(\frac{c_2}{c_1}\right)^{\sigma}\left(\frac{1}{\Gamma}\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma}\right) \\
&\underbrace{-R\frac{1}{\Gamma}\frac{R}{R^*}\frac{\sigma}{c_1}\left(\frac{c_1}{c_0}\right)^{\sigma}\left(\lambda_1\left(\frac{c_1}{c_0}\right)^{\sigma}-\lambda_0\right)}_{A}
\end{aligned}
\tag{38}
$$

The expression labeled A in (38) represents the net marginal effect of $c_1$ on the quantity of borrowing in period 1, $q_1$. The marginal benefit of additional borrowing at $t = 0$ is given by $R\frac{1}{\Gamma}\frac{R}{R^*}\frac{\sigma}{c_1}\left(\frac{c_1}{c_0}\right)^{\sigma}\lambda_0$. However additional borrowing also increases the debt burden at $t = 1$. The marginal cost associated with this additional debt burden is given by $-\lambda_1 R\left(\frac{c_1}{c_0}\right)^{\sigma}\frac{1}{\Gamma}\frac{R}{R^*}\frac{\sigma}{c_1}\left(\frac{c_1}{c_0}\right)^{\sigma}$. The net marginal cost is thus given by the expression A. To show that the net effect is indeed costly at the margin, it needs to be established that A<0 at the optimum.

Since $a_1^c > 0$ by assumption, $\nu_1 = 0$. Substitute (37) into the above expression to get:

$$\begin{aligned} A &= -R\frac{1}{\Gamma}\frac{R}{R^*}\frac{\sigma}{c_1^c}\left(\frac{c_1^c}{c_0^c}\right)^{\sigma}\left(\lambda_1^c\left(\frac{c_1^c}{c_0^c}\right)^{\sigma} - \frac{R^*}{R}\lambda_1^c\right) \\ &= -\lambda_1^c R\left(\frac{c_1^c}{c_0^c}\right)^{\sigma}\frac{\sigma}{c_1^c}\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1^c}{c_0^c}\right)^{\sigma} - 1\right] \\ &= -\lambda_1^c R\left(\frac{c_1^c}{c_0^c}\right)^{\sigma}\frac{\sigma}{c_1^c}q_1^c < 0 \end{aligned}$$

Since $q_1^c > 0$ by assumption and the Lagrange multiplier on the BoP constraint, $\lambda_1^c > 0$ then $A < 0$. The idea here is that as long as $a_1 > 0$ it is always cheaper to raise resources by reducing $a_1$ rather than by increasing $q_1$ due to the resource losses involved in using the intermediation technology. Therefore, the net effect of a marginal increase in $c_1$ on $q_1$ is costly at the margin.

Now consider the first order condition for $c_1$ for the two-period sub-problem (23). The FOC is identical to (38), except for the fact that the expression A now disappears because $q_1$ is a state variable in (23): changing $c_1$ does not affect $q_1$. Intuitively, for a time-0 planner, there is an additional marginal cost associated with increasing $c_1$ whereas for the time-1 planner, this cost has disappeared. A lower marginal cost implies that the time-1 planner would like to deviate to a higher level of consumption, $c_1^D > c_1^c$. Given the discussion on the two-period model, in section 3.6.1, this deviation implies a movement along the IRC from left to right, i.e, $a_2^D < a_2^c$, $q_2^D < q_2^c$ and $c_2^D < c_2^c$. □

## B.4 Proof of Lemma 4

*Proof.* From Lemma 3, it is known that given $a_1^c, a_2^c, q_1^c > 0$ the solution to the two period problem (23) is given by:

$$c_1^D \equiv \mathcal{C}_1(c_0^c, a_1^c, q_1^c) > c_1^c$$

But since:

$$q_1^c = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1^c}{c_0^c}\right)^{\sigma} - 1\right]$$

Then:

$$q_1^c < \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{\mathcal{C}_1(c_0^c, a_1^c, q_1^c)}{c_0^c}\right)^{\sigma} - 1\right]$$

Therefore $(c_0^c, a_1^c, q_1^c)$ violates the asset supply equation constraint for (35) and is thus infeasible. □

## C Numerical Examples

In this section, numerical simulations illustrating Propositions 1, 2 and 3 are presented. The parameters used in these simulations are described in Section 4.2.

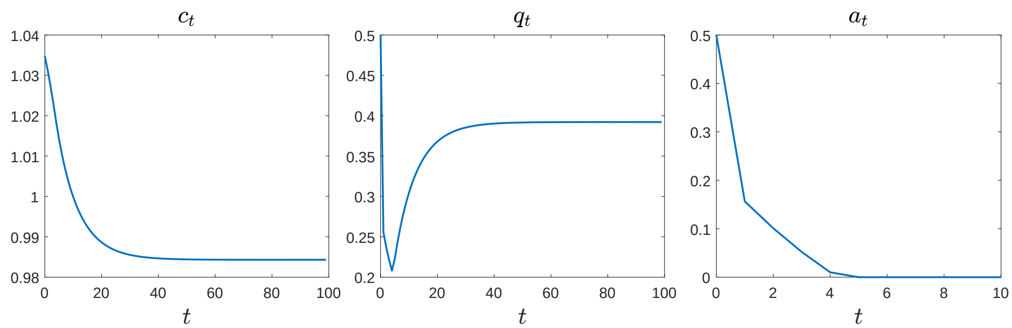



Figure 12: Proposition 1 illustrated- Example 1

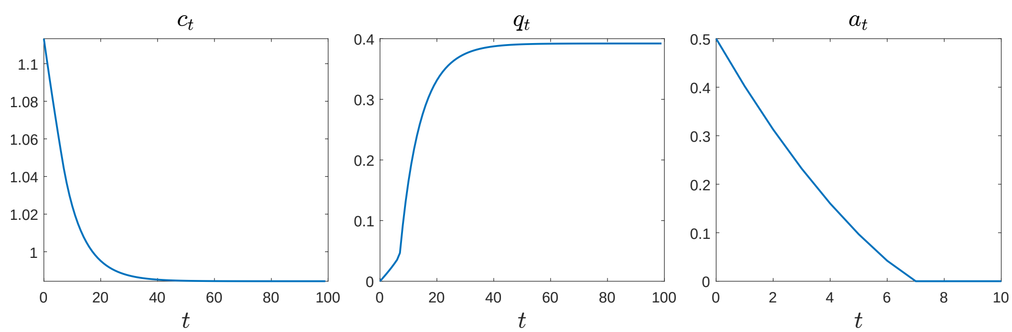



Figure 13: Proposition 1 illustrated- Example 2

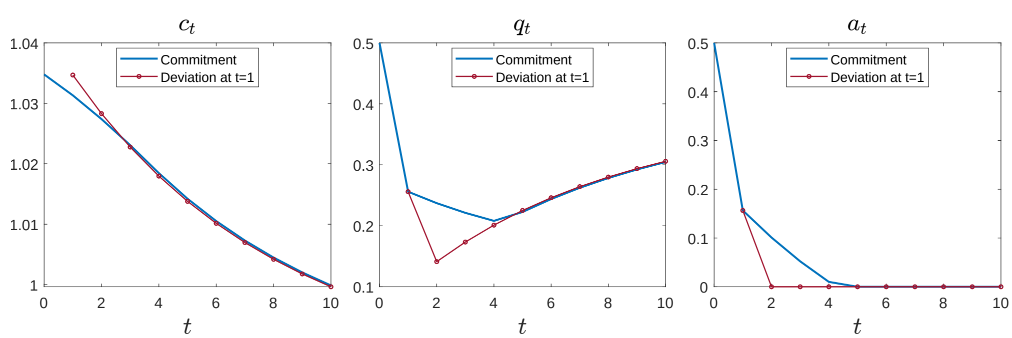



Figure 14: Proposition 2 illustrated

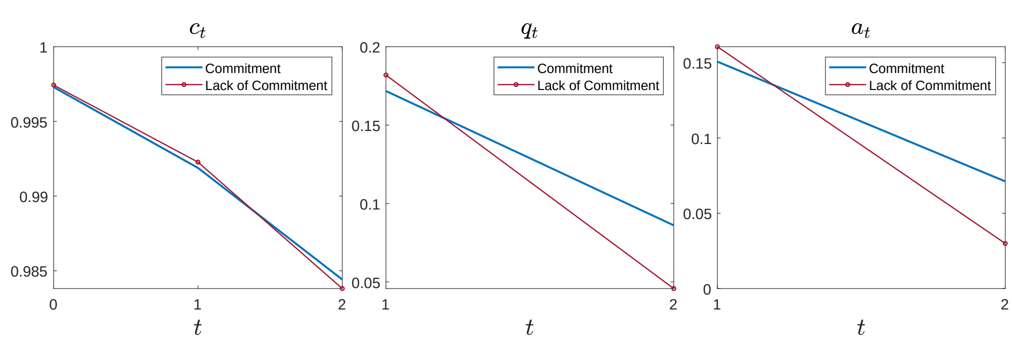



Figure 15: Proposition 3 illustrated