The Price of Quality: Demand-Driven Technology Choice and the Penn Effect

Sauhard Srivastava*

Department of Economics, University of Minnesota

August 2025

Abstract

This paper proposes a novel, demand-side explanation for the Penn effect: the observation that richer countries systematically exhibit higher price levels. We develop a general equilibrium model where income-dependent preferences lead more productive countries to produce and consume higher-quality, more resource-intensive non-tradeable goods. Our key result is that this endogenous shift toward producing superior goods, which have higher unit factor requirements, outweighs the standard cost-reducing effects of productivity growth, resulting in higher prices. The model shows that quality upgrading emerges as an equilibrium response to rising incomes and leads to higher non-tradeable prices in richer economies even in the absence of Harrod-Balassa-Samuelson (HBS) effects. Using Penn World Table data, the model replicates the empirical Penn effect, explaining about 69 percent of cross-country price variation without relying on HBS effects.

Keywords: International price differences, Quality upgrading, Penn effect, Non-tradeable goods, Endogenous technology choice, Income-dependent preferences, Purchasing power parity.

JEL Classifications: F31, F41, E31, O33, O41, D11, L15.

*sauhardsrivastava@gmail.com. The views expressed in this paper are solely mine and do not necessarily reflect the views of the institution I am affiliated with. All errors are my own.

This document was typeset using LaTeX and compiled on 2025-08-06 at 09:21:48 (UTC-05:00) using pdfTeX 1.40.27.

1 Introduction

International price comparisons reveal a striking empirical regularity: goods and services systematically cost more in rich countries than in poor ones, even after accounting for exchange rate adjustments. This phenomenon, documented extensively in purchasing power parity studies and known as the Penn effect, represents one of the most persistent deviations from the law of one price in international economics (Figure 1). The magnitude of these price differences is substantial, price levels in the richest countries can be several times higher than those in the poorest countries for seemingly identical goods and services.

The traditional explanation for this pattern, the Harrod-Balassa-Samuelson (HBS) hypothesis, attributes international price differences to productivity disparities between tradeable and non-tradeable sectors. According to this theory, countries with higher productivity in tradeable goods experience economy-wide wage increases that raise prices in the non-tradeable sector, where productivity differences are assumed to be minimal. While this mechanism provides valuable insights, it faces empirical challenges in explaining the observed magnitude of price differences and relies on increasingly questionable assumptions about sectoral productivity differences across countries. Additionally, the HBS effect struggles to account for the pervasive price increases across virtually all goods and services in wealthy countries, including those not directly tied to tradeable sector productivity, suggesting that alternative mechanisms may be at play.

We propose and formalize an alternative, demand-driven mechanism for the Penn effect centered on endogenous quality choice in non-tradeable goods. Our central premise is that economic growth involves a fundamental shift from consuming *more* to consuming *better*. As countries become more productive and incomes rise, households optimally demand superior non-tradeable goods that are more resource-intensive to produce. We argue that this preference-driven quality upgrading provides a new channel for explaining international price differences, one rooted in optimal, welfare-enhancing choices rather than exogenous technological constraints.

We construct a general equilibrium model where households exhibit income-dependent preferences for quality, and firms can choose among different quality levels for their products. The model shows that countries with higher productivity levels choose to produce and consume higher-quality non-tradeable goods. This choice reflects the optimal response to rising household incomes and the corresponding shift in demand toward superior goods. Because these superior goods are more

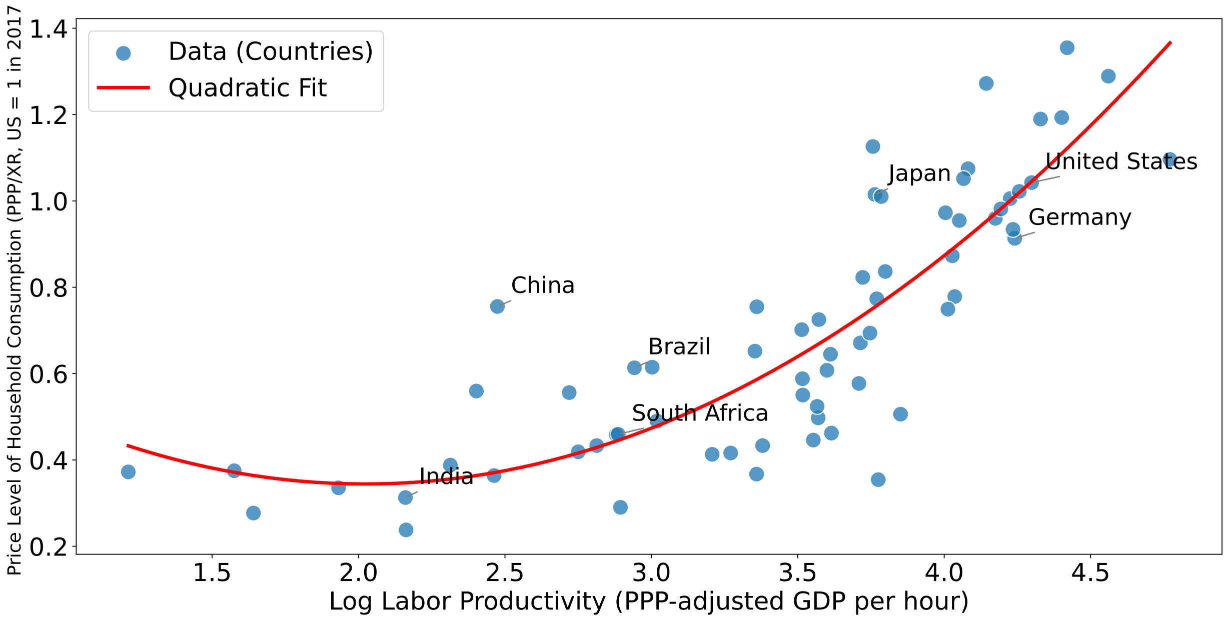


Figure 1: Penn Effect: Cross-country relationship between log productivity and price levels (2019)

Source: Scatter plot: Penn World Table 10.01 (Feenstra, Inklaar and Timmer 2015). Quadratic fit: Author's calculations.

costly to produce, the economy-wide transition to producing them leads to higher prices.

The central economic mechanism operates through two competing effects. Higher productivity generates a standard *productivity effect* that tends to lower all prices by reducing production costs. However, as incomes rise, households optimally demand higher-quality goods that require more factor inputs per unit of output, creating a *quality upgrading effect* that increases production costs. Our key finding is that the quality upgrading effect dominates the productivity effect, resulting in higher non-tradeable prices despite productivity growth. This mechanism operates entirely through demand-side effects, without requiring differences in tradeable sector productivity or barriers to technology diffusion.

Our baseline model incorporates a quality-differentiated non-tradeable sector with two quality levels: a low-quality variety and a high-quality variety. Based on income-dependent preferences, the model establishes the existence of a productivity threshold that determines whether countries specialize in low-quality or high-quality varieties. Countries with productivity below this threshold find it optimal to produce and consume the low-quality variety, while those above the threshold transition to the high-quality variety, despite its higher resource requirements. While this transition is optimal and welfare-maximizing, it generates an upward jump in non-tradeable prices, demonstrating how quality choices can create substantial price differences even in the absence of tradeable productivity differences. The mechanism illustrates that countries face a fundamental trade-off: they can either use enhanced productivity to produce more of existing goods and enjoy lower prices or upgrade to superior goods that better match their populations' evolving preferences but face higher prices.

We then generalize the baseline framework by introducing a continuum of quality levels in the non-tradeable sector. This extension allows countries to smoothly transition between quality levels as productivity increases. The model shows that countries with higher productivity levels optimally choose higher-quality varieties, leading to a continuous increase in non-tradeable prices. Importantly, the core mechanism remains unchanged: the quality upgrading effect continues to dominate the productivity effect, generating continuously rising prices as countries upgrade quality with productivity growth.

A quantitative analysis of our model validates our theoretical predictions. Using a non-linear least squares estimation, we fit our model to cross-country productivity and price level data from Penn World Table. We show that the model successfully replicates the upward-sloping portion of the Penn effect. The estimated parameters broadly capture the relationship between productivity and prices, with the model explaining approximately 69 percent of the variation in cross-country prices. Crucially, this result is achieved without invoking any traditional Harrod-Balassa-Samuelson effects, demonstrating that the quality upgrading mechanism alone can account for a substantial portion of international price differences. This exercise highlights the empirical relevance of the preference-dependent quality upgrading mechanism and provides strong evidence that demand-side factors play a crucial role in explaining cross-country price patterns.

Our framework has several advantages over existing explanations. First, it explains price differences without requiring sector-specific productivity assumptions or barriers to technology diffusion. Second, it generates realistic price patterns where virtually all goods and services cost more in rich countries, not just those in traditional identical-technology non-tradeable sectors. Third, it provides microfoundations for why quality upgrading is an equilibrium outcome of economic growth rather than an exogenous driver of it. Fourth, the mechanism operates through welfare-improving choices that reflect optimal responses to changing economic conditions, rather than market imperfections or distortions. Our analysis suggests that international price differences may be a natural and optimal result of economic growth rather than an outcome of exogenous supply-side differences.

The rest of this paper is organized as follows. Section 2 reviews the related literature on international price differences, quality differentiation, and technology choice. Section 3 develops the baseline model with two quality levels, establishing the fundamental mechanism through which endogenous quality choice generates international price differences and deriving key results on productivity thresholds and price discontinuities. Section 4 extends the framework to a continuum of quality levels in an analytically tractable manner, and presents a quantitative analysis of our model that validates our theoretical predictions. Section 5 concludes.

2 Related Literature

Our framework builds on and integrates several key areas in international and growth economics. The foundation for understanding cross-country price differences was established by Harrod (1933), Balassa (1964), and Samuelson (1964), what came to be known as the Harrod-Balassa-Samuelson hypothesis. Their seminal works shows how productivity differences in tradeable sectors can lead to economy-wide price level differences through wage spillovers to non-tradeable sectors. This mechanism has been widely studied and refined, with microeconomic foundations provided by Asea and Corden (1994) and modern open economy frameworks developed by Obstfeld and Rogoff (1996).

Empirical investigations of the HBS effect have yielded mixed results. Bergstrand (1991) and Asea and Mendoza (1994) find support for the mechanism using OECD data, while Ito, Isard and Symansky (1999) show that the relationship varies considerably across regions and time periods. Rogoff (1996) highlights the "PPP puzzle", i.e., the slow convergence of international prices, which traditional HBS models struggle to explain fully. These empirical challenges motivate the search for alternative mechanisms that can generate substantial price differences.

Beyond the HBS framework, other theories have been proposed to explain international price deviations. One prominent strand of literature emphasizes the role of the non-tradeable distribution sector. Burstein, Neves and Rebelo (2003), for example, show that distribution costs, which are intensive in local labor, can account for a significant portion of final goods prices and explain real exchange rate puzzles. Another set of theories focuses on market integration and trade costs. Alessandria and Kaboski (2011), for instance, develop a model where pricing-to-market and international trade costs can generate systematic deviations from the law of one price. Our paper contributes to this search for alternative explanations by proposing a novel mechanism rooted in demand-side quality choice, which complements these existing theories.

Our work is motivated by a rich literature documenting quality differentiation across income levels. In international trade, Linder (1961) first proposed that countries with similar income levels trade more intensively due to demand similarities. Subsequent work by Schott (2004), Hummels and Klenow (2005), and Hallak (2006) provides compelling evidence that richer countries both produce and export higher-quality, higher-priced goods. While this literature focuses on traded goods, the same phenomenon has been rigorously documented within countries. Aguiar and Bils (2015) provide crucial empirical support for our paper's central mechanism. By analyzing detailed household expenditure data, they show that high-income households pay systematically higher unit prices for goods within the same consumption category. They interpret this as a shift to higher quality. Their work provides strong evidence that quality upgrading is a fundamental feature of consumption behavior as income rises. Our paper builds directly on this insight, proposing a theoretical model where this empirically-documented quality choice at the micro level aggregates up to explain macro-level price differences across countries.

The literature on endogenous technology choice provides crucial theoretical underpinnings for our model. Basu and Weil (1998) develop a framework where productivity depends on factor endowments, while Acemoglu (2001) examines how skill availability influences technology adoption. Caselli and Coleman (2006) analyzes how factor endowments affect technology choice across countries. We extend these insights by focusing specifically on quality-enhancing technologies and their implications for price levels rather than factor returns or growth rates.

Research on non-homothetic preferences offers important insights for understanding quality choice. Matsuyama (2000) shows how non-homothetic preferences can generate demand-driven comparative advantage, while Fieler (2011) shows how such preferences explain trade patterns between countries at different development stages. Fajgelbaum, Grossman and Helpman (2011) develop a framework where income differences drive quality choice in differentiated products, generating predictions for trade flows. This line of research has been extended into large-scale quantitative models by Fajgelbaum and Khandelwal (2016) to assess the distributional gains from trade. While this literature primarily uses non-homothetic preferences to explain trade patterns and their welfare effects, our model applies these insights to explain domestic price levels.

Recent studies on price setting and markups also contribute to our understanding of price differences. Klenow and Malin (2010) and Gopinath, Itskhoki and Rigobon (2011) show that pricing behavior varies across countries and products. While these studies focus on nominal rigidities and market structure, we explore how quality choices affect production costs and lead to price differences.

Our contribution synthesizes these various research streams into a coherent framework that explains international price differences through endogenous quality choice. Unlike previous work that treats quality differences as exogenous or focuses primarily on traded goods, we develop a general equilibrium model where quality choices arise from income-dependent preferences and affect economy-wide prices. This mechanism complements traditional HBS effects and can generate substantial price differences without requiring sector-specific productivity assumptions or barriers to technology diffusion. The framework provides new insights into why quality upgrading represents an equilibrium response to economic growth rather than merely an input to the growth process.

3 Baseline Model: Two Quality Levels

This section presents a static general equilibrium model showing how endogenous quality choice in the non-tradeable sector drives international price differences. The model consists of countries that differ only by a single productivity parameter, A, in a non-tradeable sector with two quality-differentiated varieties, a low- and a high-quality variety. In order to isolate the impact of quality choice on prices, we shut down any HBS effects that arise from tradeable sector productivity differences across countries.

The section proceeds by first defining the economic environment, the optimization problems for households and firms, and deriving the competitive equilibrium conditions. Our baseline model establishes the core mechanism that governs a country's choice between producing and consuming a low- or a high-quality variety of the non-tradeable good and how this drives price differences across countries. First, we prove the existence of a unique productivity threshold, $\hat{A}$, above which countries switch from specializing in the low-quality to the high-quality variety (Proposition 1). Second, we show that this quality upgrading channel dominates the standard productivity growth channel and induces an upward jump in the non-tradeable price level at the threshold (Proposition 2).

3.1 Economic Environment

The world consists of N countries, each populated by identical households. Households in every country consume a tradeable good, c_T, and two non-tradeable goods: $\tilde{c}_N$ and c_N. While c_N is the standard-HBS type-non-tradeable good, $\tilde{c}_N$, is a quality-differentiated non-tradeable good, differentiated into two varieties: a low-quality variety, c_L, and a high-quality variety, c_H.

We assume that all countries are identical with respect to the tradeable sector, with households in each country being endowed with one unit of the tradeable good, $y_T = 1$. This assumption shuts down non-tradeable sector price differences arising due to HBS effects, i.e., through tradeable sector productivity differences across countries. We assume that the tradeable good is the numeraire in every country. The law of one price for tradeable goods implies that the nominal exchange rate between any two countries is also 1[1].

Let $y_{N,i}$ denote the output of the standard non-tradeable good and let $p_{N,i}$ denote its price in units of the tradeable good. This good is produced by competitive firms using a production technology given by:

$$y_{N,i} = A_N \ell_{N,i}$$

where $\ell_{N,i}$ represents the labor input and A_N represents the productivity. This good, like the standard HBS type non-tradeable good, is assumed to have an identical production technology across all countries, i.e., A_N is the same for all countries.

Let $(y_{L,i}, y_{H,i})$ denote the outputs of the low-quality and high-quality varieties of the quality-differentiated non-tradeable good, and let $(p_{L,i}, p_{H,i})$ denote their prices in units of the tradeable good, respectively. Both the varieties are produced by competitive firms using production technologies given by:

$$y_{L,i} = A_i A_L \ell_{L,i}$$

$$y_{H,i} = A_i A_H \ell_{H,i}$$

where $\ell_{L,i}, \ell_{H,i}$ denote labor inputs, A_i represents a country specific productivity term and A_L, A_H represent variety specific productivity terms. The productivity parameters A_L and A_H capture product quality differences in a reduced-form manner: we assume that the high-quality variety

[1]Let $p_{T,i}$ denote the local currency price of the tradeable good in a country i. The law of one price holds globally for the tradeable good so that for any country pair (i, j), $p_{T,i} = e_{i,j} p_{T,j}$, where $e_{i,j}$ is the nominal exchange rate between the currencies of countries i and j expressed in units of country i's currency. Since $p_{T,i}$ is normalized to 1 in every country, the law of one price then implies that $e_{i,j} = 1$ for all (i, j).

has a higher unit labor requirement: $A_H < A_L$, for every country. Notably, we assume complete technological diffusion with respect to production technologies for high- as well as low-quality varieties, i.e., (A_L, A_H) are identical for all countries. In other other words, for any given amount of labor, a country with a higher A is better at producing both high- and low-quality varieties in absolute terms, however, all countries are identical with respect to their ability to produce the high-quality variety relative to the low-quality variety.

The households are also endowed with one unit of labor, which is supplied inelastically. Labor is assumed to be perfectly mobile across the three production sectors within a country. However there is no mobility of labor across countries. Let w_i denote the wage rate in units of the tradeable good. In what follows, we consider the optimization problems of agents in a country with a given productivity level A, and suppress the country index i for notational simplicity.

3.2 Household Problem

The representative household derives utility from consuming the tradeable good, c_T, the two varieties of the quality-differentiated non-tradeable good, (c_H, c_L), and the standard non-tradeable good, c_N. In addition, the household considers the two varieties, (c_H, c_L), to be perfect substitutes. We assume that the preferences over the three goods are additively separable, while allowing them to be non-homothetic. The household's preferences are given by the following utility function:

$$U = \frac{\phi_1}{1-\sigma}(c_L + \theta c_H)^{1-\sigma} + \phi_2 \log c_N + \log c_T \tag{1}$$

where $(\phi_1, \phi_2, \theta) > 0$ signify utility preference weights and $1 - \sigma$ is the concavity parameter for the utility derived from the consumption of the quality-differentiated good. Given this functional form and model assumptions, $\frac{1}{\sigma}$ signifies the price elasticity of demand for the quality-differentiated non-tradeable good[2]. We assume that the demand is relatively elastic for this good: $0 < \sigma < 1$.

Moreover, we assume that households have income-dependent preferences over the two varieties of the quality-differentiated good. In particular, the relative preference for the high-quality variety, denoted by θ, is modeled as a continuous, strictly increasing function of the household's labor income, w. For analytical tractability, we assume a constant elasticity form:

$$\theta(w) = \vartheta w^k$$

[2]See Appendix A for the derivation.

with $\vartheta > 0$ and $0 < k \leqslant \frac{\sigma}{1-\sigma}$, so that the elasticity with respect to w is given by: $\frac{w\,\theta'(w)}{\theta(w)} = k$.

Under this specification, richer households place a higher relative value on the high-quality variety c_H compared to the low-quality variety c_L. In other words, even if two households face the same relative prices, a richer household will opt for the high-quality variety while a poorer household may choose only the low-quality variety. Consequently, the high-quality good c_H is a 'normal' or a 'superior' good with a positive income elasticity of demand, whereas c_L is a relatively 'inferior' good. The model structure implies that $\frac{k(1-\sigma)}{\sigma}$ signifies the income elasticity of demand for the high-quality variety, c_H[2]. The assumption that $k \leqslant \frac{\sigma}{(1-\sigma)}$ implies that the income elasticity of the high-quality variety is at most 1: i.e. the high-quality variety while a superior good, does not necessarily represent a luxury good.

The household's problem is to maximize utility, (1), subject to their budget constraint:

$$p_L c_L + p_H c_H + p_N c_N + c_T = w + y_T \tag{2}$$

3.3 Firm Problem

The firms in all the production sectors hire workers at the competitive wage w and choose labor inputs to maximize profits. The profit maximization problem for the firms producing the variety $x \in \{L, H\}$ of the quality-differentiated good is given by:

$$\max_{\ell_x} p_x A A_x \ell_x - w\ell_x \tag{3}$$

Similarly, the profit maximization problem for the firms producing the standard non-tradeable good is given by:

$$\max_{\ell_N} p_N A_N \ell_N - w\ell_N \tag{4}$$

3.4 Competitive Equilibrium

A competitive equilibrium consists of the consumption decisions, $\{c_L, c_H, c_N, c_T\}$, labor allocations, $\{\ell_L, \ell_H, \ell_N\}$, and prices, $\{p_L, p_H, p_N, w\}$, for each country, such that:

- Given prices, households make consumption decisions by maximizing utility, (1), subject to their budget constraint, (2).
- Given prices, firms in every sector make production decisions by solving (3) or (4) for their

respective sectors.

- Markets clear:
  - Labor market:
$$\ell_L + \ell_H + \ell_N = 1 \tag{5}$$
  - Non-tradeable goods markets:
$$c_N = A_N \ell_N \tag{6}$$
$$c_L = A A_L \ell_L \tag{7}$$
$$c_H = A A_H \ell_L \tag{8}$$
  - Tradeable goods market:
$$\sum_{i=1}^{N} c_{T,i} = \sum_{i=1}^{N} y_{T,i} = N \tag{9}$$

In Appendix A, we derive the first order conditions of the households and firms and combine them with market clearing conditions to obtain a system of equations (49)-(61) that characterize the competitive equilibrium in $\{c_T, c_N, c_L, c_H, p_N, p_L, p_H, w, \ell_N, \ell_L, \ell_H\}$ together with non-negativity lagrange multipliers (μ_L, μ_H) on (c_L, c_H) respectively.

3.5 Optimal Endogenous Quality Choice

In this sub-section we establish that countries with different non-tradeable sector productivity levels, A, make different choices with respect to the variety of the quality-differentiated non-tradeable good they produce and consume- c_H, c_L. We show that the optimal choice is endogenously determined by the consumer's relative preference for the high-quality variety, $\theta(w)$, and its relative price, $\frac{p_H}{p_L}$. The assumption that the relative productivity of the firms producing these varieties is constant across countries, implies that the relative price is also constant. From the firm's first order conditions, (53), (54), we get:
$$\frac{p_H}{p_L} = \frac{A_L}{A_H}$$
While the relative price is constant, the relative preference for the high quality variety, $\theta(w)$, is a strictly increasing function of the country's non-tradeable sector income, w. This dynamic creates three distinct equilibria, which are formally derived in A.4-A.6 of Appendix A:

- A country has a corner equilibrium with $c_H = 0, c_L > 0$ if and only if:

$$\theta(w) \leqslant \frac{p_H}{p_L} = \frac{A_L}{A_H}$$

  In other words, if the relative price of the high-quality variety, $\frac{p_H}{p_L}$, is higher than the consumer's relative preference for this variety, $\theta(w)$, then the country specializes in producing and consuming only the low quality variety.

- A country has a corner equilibrium with $c_L = 0, c_H > 0$ if and only if:

$$\theta(w) \geqslant \frac{p_H}{p_L} = \frac{A_L}{A_H}$$

  In other words, if the relative price of the high-quality variety, $\frac{p_H}{p_L}$, is lower than the consumer's relative preference for this variety, $\theta(w)$, then the country specializes in producing and consuming only the high-quality variety.

- A country has an interior equilibrium $c_H > 0, c_L > 0$ if and only if:

$$\theta(w) = \frac{p_H}{p_L} = \frac{A_L}{A_H}$$

  In other words, if the relative price of the high-quality variety, $\frac{p_H}{p_L}$, is equal to the consumer's relative preference for this variety, $\theta(w)$, then the country is indifferent between producing and consuming the low and high quality varieties.

Since the equilibrium wage, w, is itself a function of the country's underlying productivity, A, there is a direct link between productivity and optimal quality choice. We establish that there exists a productivity threshold, $\hat{A}$, which determines whether a country will specialize in producing and consuming the low-quality or high-quality variety of the non-tradeable good. This threshold is derived through a sequence of results presented in Lemmas 1, 2, and 3, which establish the conditions for equilibrium outcomes under varying productivity levels. Proposition 1 consolidates these findings, summarizing how a country's optimal quality choice emerges endogenously based on its productivity level relative to the threshold, $\hat{A}$.

Lemma 1. *Assume that $\theta(w)$ is continuous, strictly increasing in w and $\theta^{-1}(\frac{A_L}{A_H}) > \phi_2$. Then there exists a threshold productivity level, $\hat{A} > 0$, such that, for any country with a productivity level A, producing only the low-quality variety: $c_L > 0, c_H = 0$ is an equilibrium if and only if $A \leqslant \hat{A}$. Furthermore, this threshold* *productivity level is unique and given by:*

$$\hat{A} = \frac{1}{\phi_1^{\frac{1}{1-\sigma}} A_L} \left[\theta^{-1}\left(\frac{A_L}{A_H}\right) - \phi_2\right]^{\frac{\sigma}{1-\sigma}} \theta^{-1}\left(\frac{A_L}{A_H}\right) > 0 \tag{10}$$

Proof. See Appendix B. □

Lemma 2. *Assume that $\theta(w)$ is continuous, strictly increasing in w and $\theta^{-1}(\frac{A_L}{A_H}) > \phi_2$. For any country with a productivity level A, producing both the low- and high-quality varieties: $c_L > 0, c_H > 0$, is an equilibrium if and only if $A = \hat{A}$, where $\hat{A}$ is given by (10). Furthermore, the country is indifferent between both the varieties so that there are infinitely many competitive equilibria with*

$$c_H \in \left[0, \hat{A}A_H\left(1 - \frac{\phi_2}{\hat{w}}\right)\right], \quad c_L \in \left[0, \hat{A}A_L\left(1 - \frac{\phi_2}{\hat{w}}\right)\right]$$

such that

$$\frac{c_H}{A_H} + \frac{c_L}{A_L} = \hat{A}\left(1 - \frac{\phi_2}{\hat{w}}\right)$$

where $\hat{w} = \theta^{-1}\left(\frac{A_L}{A_H}\right)$.

Proof. See Appendix B. □

Lemma 3. *Assume $\theta(w) = \vartheta w^k$ with $\vartheta > 0$, $0 < k \leqslant \frac{\sigma}{1-\sigma}$, and $\left(\frac{1}{\vartheta}\frac{A_L}{A_H}\right)^{\frac{1}{k}} > \phi_2$. Then for any country with a productivity level A, producing only the high-quality variety: $c_H > 0, c_L = 0$ is an equilibrium if and only if $A \geqslant \hat{A}$ where $\hat{A}$ is given by (10).*

Proof. See Appendix B. □

Together, these lemmas establish the conditions under which a country transitions between low-quality and high-quality production and consumption. Building on these results, Proposition 1 summarizes the optimal quality choice based on the productivity threshold $\hat{A}$.

Proposition 1 (Optimal quality choice)**.** *Assume $\theta(w) = \vartheta w^k$ with $\vartheta > 0$, $0 < k < \frac{\sigma}{1-\sigma}$, and $\left(\frac{1}{\vartheta}\frac{A_L}{A_H}\right)^{\frac{1}{k}} > \phi_2$. Let $\hat{A}$ be given by (10). Then:*

- *For a country with a productivity level $0 < A < \hat{A}$, the unique equilibrium is to produce and consume only the low-quality variety: $c_L > 0, c_H = 0$.*
- *For a country with a productivity level $A > \hat{A}$, the unique equilibrium is to produce and consume only the high-quality variety: $c_H > 0, c_L = 0$.*

- *A country with a productivity level $A = \hat{A}$ is indifferent between producing and consuming the low- and high-quality varieties with infinitely many competitive equilibria.*

Proof. Follows as a Corollary from Lemmas 1, 2, and 3. □

Proposition 1 formalizes a fundamental economic phenomenon: as productivity and income rise, consumption patterns shift from emphasizing quantity to prioritizing quality. At the individual level, this reflects the tendency for consumers to consume better quality goods as their income grows. The model captures this through the income-dependent preferences $\theta(w)$, where a higher wage endows the household with a stronger relative preference for the superior variety.

This proposition scales this micro-level behavior to the macroeconomy. Countries with productivity below the threshold $A < \hat{A}$ operate in a low-quality equilibrium, where the population's income-dependent preferences for quality are insufficient to justify incurring the higher costs of superior varieties (reflected in the relative price of the superior good (A_L/A_H)). For them, growth is initially about quantity; productivity gains are channeled into producing more of the essential, basic/low-quality varieties.

However, as a country's productivity approaches and crosses the threshold $\hat{A}$, its economy faces a trade-off: it can either use its enhanced productive capacity to produce an even larger quantity of the basic good, or it can upgrade to a higher-quality variety even if the effective quantity produced decreases. The model shows that the second path is optimal. This transition is driven by the endogenous increase in wages and the corresponding shift in aggregate preferences, making quality upgrading welfare-enhancing despite its higher unit factor requirements. The model shows that richer countries $(A > \hat{A})$ choose to reallocate resources to the high-quality good, despite a potential reduction in physical output (because $A_H < A_L$).

This quality-biased technology adoption illuminates not only the dynamic evolution of economies over time but also explains cross-sectional differences in consumption patterns observed among countries with varying productivity levels. Notably, the model illustrates that this variation arises endogenously as an optimal response to differences in incomes across countries, rather than being driven by exogenous technological limitations. This model provides a unified framework for understanding quality differentiation both within a country's growth trajectory and across countries at different stages of economic development.

We now turn to examining the price implications of this quality differentiation. The following subsection shows how this endogenous quality choice generates systematic price differences in the non-tradeable sector. Specifically, we show that as countries cross the productivity threshold $\hat{A}$ and shift toward quality-enhanced production, this transition results in higher equilibrium prices for non-tradeable goods.

3.6 Quality choice and price level differences

In this subsection we show how quality choice generates systematic differences in non-tradeable sector price levels across countries with varying productivity levels. For a country with a given productivity level A, let the competitive equilibrium wages and prices be given by functions $\left\{w^*(A), p^*_N(A), p^*_L(A), p^*_H(A)\right\}$.

First, Lemma 4 establishes that the equilibrium wage, $w^*(A)$, is a continuous and strictly increasing function of A. Second, Lemma 5 shows that the equilibrium prices of the non-tradeable goods, $p^*_N(A), p^*_L(A), p^*_H(A)$, are continuous functions of A, with $p^*_N(A)$ strictly increasing and $p^*_L(A), p^*_H(A)$ strictly decreasing in A. Finally, we define a non-tradeable sector price index, $P^*_{NT}(A)$, as a weighted arithmetic mean of these prices and in Proposition 2 we establish that there is a discontinuous increase in this price index at the quality threshold $\hat{A}$, thereby illustrating quality choice induced increase in non-tradeable sector prices.

The following lemmas establish the properties of the equilibrium wage and prices in the non-tradeable sector as a function of the country's productivity level A.

Lemma 4. *For any country with a productivity level A, the equilibrium wage, $w^*(A)$, is a continuous, strictly increasing function in A and is given by:*

$$w^*(A) = \begin{cases} w^L(A), & \text{if } A \leqslant \hat{A} \\ w^H(A), & \text{if } A \geqslant \hat{A} \end{cases}$$

with $w^L(A), w^H(A)$ being continuous, strictly increasing, strictly concave functions in A, where:

- $w^L(A)$ *solves the labor market clearing equation:*

$$\frac{\phi_1^{\frac{1}{\sigma}} (AA_L)^{\frac{1-\sigma}{\sigma}}}{w^{1/\sigma}} + \frac{\phi_2}{w} = 1 \tag{11}$$

- $w^H(A)$ *solves the labor market clearing equation:*

$$\frac{\phi_1^{\frac{1}{\sigma}}\theta(w)^{\frac{1-\sigma}{\sigma}}(AA_H)^{\frac{1-\sigma}{\sigma}}}{w^{1/\sigma}} + \frac{\phi_2}{w} = 1 \tag{12}$$

- *For* $A < \hat{A} : w^H(A) < w^L(A)$
- *For* $A > \hat{A} : w^H(A) > w^L(A)$
- *For* $A = \hat{A} : w^L(A) = w^H(A) = \hat{w} = \theta^{-1}\left(\frac{A_L}{A_H}\right)$

Proof. See Appendix B. □

Lemma 5. *For any country with a productivity level* A*:*

- *The equilibrium prices of the two varieties of the quality-differentiated good,* $p_L^*(A), p_H^*(A)$ *are given by:*

$$p_L^*(A) = \frac{1}{A_L}\frac{w^*(A)}{A} \tag{13}$$
$$p_H^*(A) = \frac{1}{A_H}\frac{w^*(A)}{A} \tag{14}$$

  with $p_L^*(A), p_H^*(A)$ *being continuous, strictly decreasing functions in* A*.*

- *The equilibrium price of the standard non-tradeable good,* $p_N^*(A)$*, is given by:*

$$p_N^*(A) = \frac{w^*(A)}{A_N} \tag{15}$$

  with $p_N^*(A)$ *being a continuous, strictly increasing function in* A*.*

Proof. See Appendix B. □

Figure 2a illustrates the equilibrium wage, $w^*(A)$, as a function of productivity A. The wage is continuous and strictly increasing in A, with a kink at the quality threshold, $\hat{A}$, as illustrated in Lemma 4. While the kink at the quality threshold, $\hat{A}$, is associated with the quality switch, the fact that the wages are strictly increasing in A is crucially dependent on the assumption that the demand for the quality differentiated good is relatively price elastic: $0 < \sigma < 1$. This is because when productivity, A, increases, the marginal cost of producing the quality-differentiated good falls, leading to a lower price for this good relative to the standard non-tradeable good. However, because demand is relatively price elastic, the reduction in price triggers a more than proportional increase in demand. This increases demand for labor in the quality-differentiated sector and draws workers away from the standard non-tradeable sector. The resulting competition for labor between sectors bids up the equilibrium wage. Thus, higher productivity in the quality-differentiated sector ultimately raises wages economy-wide. This mechanism relies critically on the price elasticity of demand: if demand were inelastic, the increase in quantity demanded would be less than proportional, and the wage effect would be muted or even reversed.

Figures 2b and 2c illustrate the equilibrium prices of the standard non-tradeable good $p^*_N(A)$ and the quality-differentiated varieties $p^*_L(A), p^*_H(A)$ respectively. The price of the standard non-tradeable good is continuous and strictly increasing in A, while the prices of the quality-differentiated varieties are continuous and strictly decreasing in A, as established in Lemma 5. Moreover because $w^*(A)$ is kinked at $\hat{A}$ due to the switch from the low- to high-quality variety, the prices are also kinked at $\hat{A}$.

It is notable that $p^*_L(A)$ and $p^*_H(A)$ are decreasing with respect to A occurs even though the wage $w^*(A)$ is increasing, because the ratio $w^*(A)/A$ is strictly decreasing in A.[3]. While an increase in productivity, increases demand more than proportionately in the quality-differentiated sector leading firms to bid up wages to attract labor away from the standard non-tradeable sector, the resulting net increase in the equilibrium wage is less than proportional to the productivity gain. The primary reason is again that demand for the quality-differentiated good is price elastic ($0 < \sigma < 1$). Even a small increase in wages raises prices, which in turn causes demand to fall more than proportionately which in turn reduces the upward pressure on wages that the initial productivity increase created.

The second reason is that the income elasticity of demand for both quality-differentiated varieties is less than or equal to one: it is zero for the low-quality variety and $\frac{k(1-\sigma)}{\sigma} \leqslant 1$ for the high-quality variety[4]. Together these imply that the net increase in equilibrium wages is not strong enough to offset the direct effect of increase in A in the denominator of $p^*_L(A)$ and $p^*_H(A)$. Effectively, productivity gains lead to lower marginal costs and thus lower prices for these goods.

The result in Lemma 5 that the relative price of the standard non-tradeable good p_N is increasing in A, also illustrated in 2b, is reminiscent of the standard HBS result: richer countries have higher

[3] In other words $w^*(A)$ is piece-wise strictly concave in A: $w^L(A)$ is strictly concave in A for $A < \hat{A}$ and $w^H(A)$ is strictly concave in A for $A > \hat{A}$. See Lemma 12 in Appendix B for proof.

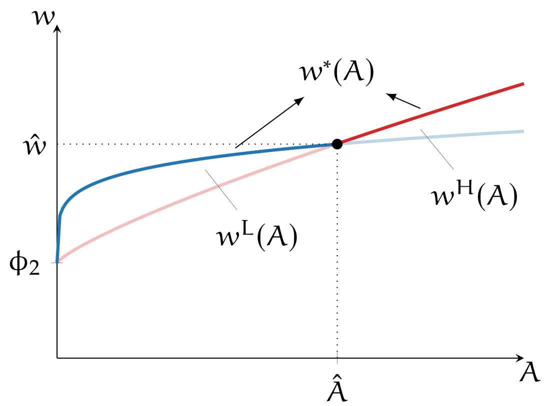
[4] If income elasticity is greater than 1, the increase in wages could be more than proportionate to an increase in productivity due to demand side income effects dominating. The proof of Lemma 12 establishes conditions under which $\frac{w^*(A)}{A}$ could be strictly increasing in A. This implies that prices would rise with increase in productivity. Further discussion on the behavior of wages and prices of such 'luxury goods' is beyond the scope of this paper.

relative prices of non-tradeable goods. In the HBS model, this happens because of differences in traded sector productivity. However, the underlying mechanism is this model is distinct from the traditional HBS explanation.

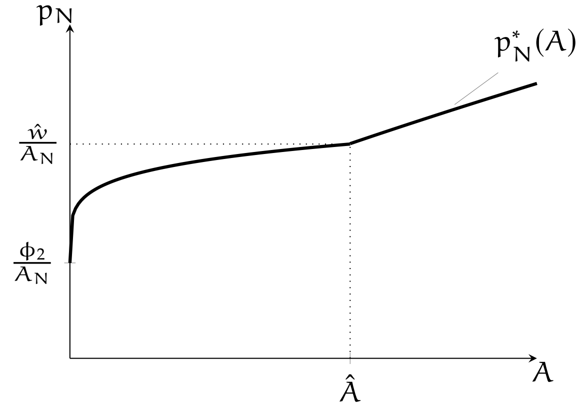
In this model, a rise in productivity A lowers the effective cost of producing the quality-differentiated good (both varieties). Because demand for this good is relatively price-elastic ($0 < \sigma < 1$), the resulting lower price leads to a more than one-for-one increase in demand, driving up the economy-wide wages, w^* as explained above. This, in turn, leads to an increase in the standard non-tradeable sector's price p_N. Thus, higher productivity in the quality differentiated non-tradeable good together with a relatively price elastic demand leads to higher prices in the the standard non-tradeable sector- which has identical production technology across countries. This is in contrast to the HBS model, where the relative price of non-tradeable goods is driven by differences in traded sector productivity.

However, this price increase in the standard non-tradeable sector is secondary to the paper's main argument- it is true even if endogenous quality choice between c_L, c_H is eliminated from the model. The core contribution is to elucidate how the endogenous quality switch itself generates an increase in the non-tradeable price level. Figure 2c highlights this mechanism: because $A_H < A_L$, the equilibrium price of the high-quality non-tradeable good, $p^*_H(A)$, always exceeds that of the low-quality good, $p^*_L(A)$, for any given A. The darker shaded regions in the figure represent the actual prices faced by countries, depending on whether they produce and consume the low- or high-quality variety. Specifically, countries with productivity levels just below the threshold $\hat{A}$ specialize in the low-quality variety and thus face lower prices, while those just above $\hat{A}$ specialize in the high-quality variety and face higher prices. The lighter shaded regions indicate the shadow prices of varieties not produced or consumed by a given country. The price of the quality-differentiated non-tradeable good is thus represented by only the darker shaded regions of both the curves. The discontinuous upward jump in non-tradeable prices at the quality threshold created as the result of countries switching from low- to high-quality production illustrates the key argument of this paper.

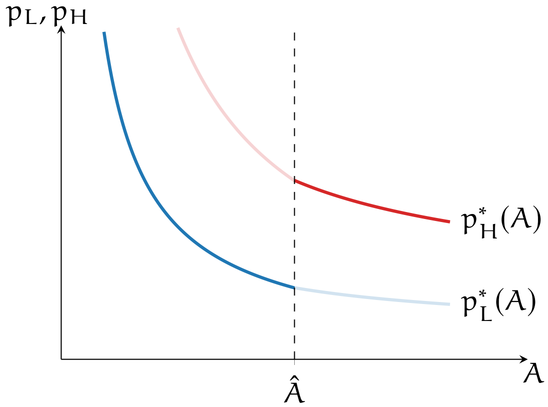
In order to formalize this idea further, we construct a non-tradeable sector's price index, P_{NT}, defined as the weighted arithmetic mean of the prices $\{p_N, p_L, p_H\}$, where the weights are the expenditure shares, i.e., share of non-tradeable sector income, w, spent on the corresponding sector:



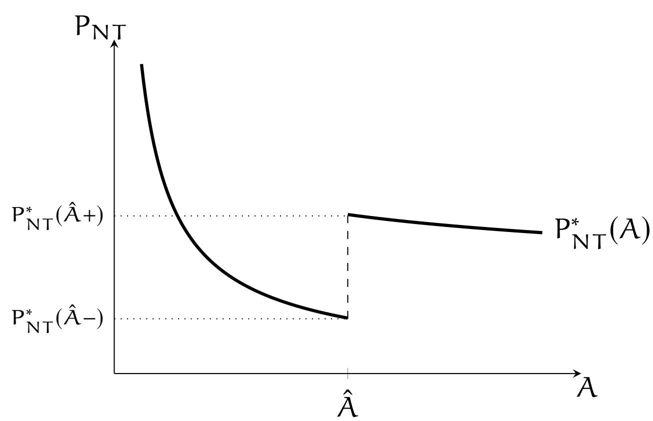
(a) Equilibrium Wage $w^*(A)$



(b) Price of standard non-tradeable $p^*_N(A)$



(c) Prices of quality-differentiated varieties



(d) Non-tradeable price index $P^*_{NT}(A)$

Figure 2: Equilibrium wages and prices as a function of A

$\left\{\frac{p_N c_N}{w}, \frac{p_L c_L}{w}, \frac{p_H c_H}{w}\right\}$[5]. The price index is given by:

$$P_{NT} = \left(\frac{p_N c_N}{w}\right) p_N + \left(\frac{p_L c_L}{w}\right) p_L + \left(\frac{p_H c_H}{w}\right) p_H$$

From equations (50)-(51), we have that $p_N c_N = \phi_2$. For $A < \hat{A}$, we have $p_H c_H = 0$, then:

$$P^*_{NT}(A) = \left(\frac{\phi_2}{w^*(A)}\right) p^*_N(A) + \left(1 - \frac{\phi_2}{w^*(A)}\right) p^*_L(A)$$

Similarly for $A > \hat{A}$, we have $p_L c_L = 0$, then:

$$P^*_{NT}(A) = \left(\frac{\phi_2}{w^*(A)}\right) p^*_N(A) + \left(1 - \frac{\phi_2}{w^*(A)}\right) p^*_H(A)$$

For $A = \hat{A}$ expenditure shares $\left\{\frac{p_L c_L}{w}, \frac{p_H c_H}{w}\right\}$ are non-unique since there are infinitely many

[5] The consumer's budget constraint implies that $p_N c_N + p_L c_L + p_H c_H = w$

solutions with respect to (c_H, c_L) as established in Lemma 2. For $c_L \in \left[0, \hat{A}A_L\left(1-\frac{\phi_2}{\hat{w}}\right)\right]$, we have $\frac{p_L c_L}{w} \in \left[0, 1-\frac{\phi_2}{\hat{w}}\right]$. Let $\frac{p_L c_L}{w} \equiv z$, then:

$$P^*_{NT}(\hat{A}) = \frac{\phi_2}{\hat{w}} p^*_N(\hat{A}) + z(p^*_L(\hat{A}) - p^*_H(\hat{A})) + \left(1-\frac{\phi_2}{\hat{w}}\right) p^*_H(\hat{A}) \qquad \forall z \in \left[0, 1-\frac{\phi_2}{\hat{w}}\right]$$

where $p^*_N(\hat{A}), p^*_L(\hat{A}), p^*_H(\hat{A})$ are unique as established in Lemma 5. The above implies that as long as $p^*_L(\hat{A}) \neq p^*_H(\hat{A})$, the price index $P^*_{NT}(A)$ is non-unique at $A = \hat{A}$.

Therefore, the price index for the non-tradeable sector can be summarized as:

$$P^*_{NT}(A) = \begin{cases} \left(\frac{\phi_2}{w^*(A)}\right) p^*_N(A) + \left(1-\frac{\phi_2}{w^*(A)}\right) p^*_L(A), & \text{if } A < \hat{A} \\ \left(\frac{\phi_2}{\hat{w}}\right) p^*_N(\hat{A}) + z(p^*_L(\hat{A}) - p^*_H(\hat{A})) + \left(1-\frac{\phi_2}{\hat{w}}\right) p^*_H(\hat{A}), & \text{if } A = \hat{A},\ \forall z \in \left[0, 1-\frac{\phi_2}{\hat{w}}\right] \\ \left(\frac{\phi_2}{w^*(A)}\right) p^*_N(A) + \left(1-\frac{\phi_2}{w^*(A)}\right) p^*_H(A), & \text{if } A > \hat{A} \end{cases} \tag{16}$$

The following proposition establishes that there is a discontinuous increase in the non-tradeable price index at the quality threshold $\hat{A}$ if and only if the the low-quality variety is more factor intensive than the high-quality variety: $A_H < A_L$. The magnitude of this jump is strictly increasing in the productivity gap between the two varieties, $(A_L - A_H)$.

Proposition 2 (Price level discontinuity at the quality threshold)**.** *Let the non-tradeable sector price level be given by the index $P^*_{NT}(A)$ as defined in equation (16). Let $\hat{A}$ be the unique productivity threshold at which a country is indifferent between producing and consuming the low- and high-quality varieties given by (10). Then:*

- *There is a discontinuous jump in the price level at the quality threshold $\hat{A}$ if and only if $A_H \neq A_L$. More specifically,*

$$A_H < A_L \Leftrightarrow \lim_{A \to \hat{A}^+} P^*_{NT}(A) > \lim_{A \to \hat{A}^-} P^*_{NT}(A)$$

- *The magnitude of this price level discontinuity is a strictly increasing function of the productivity gap between the low- and high-quality varieties, $(A_L - A_H)$. The jump is given by:*

$$\lim_{A \to \hat{A}^+} P^*_{NT}(A) - \lim_{A \to \hat{A}^-} P^*_{NT}(A) = \left(1-\frac{\phi_2}{\hat{w}}\right) \frac{\hat{w}}{\hat{A}} \left(\frac{A_L - A_H}{A_L A_H}\right)$$

where $\hat{w} = w^*(\hat{A}) = \theta^{-1}\left(\frac{A_L}{A_H}\right)$. This jump is strictly positive if and only if $A_L > A_H$ and is equal to zero if $A_L = A_H$.

Proof. See Appendix B. □

Proposition 2 provides a formal foundation for this paper's central argument: the higher price levels observed in richer countries can be explained by an endogenous shift toward higher-quality, more costly non-tradeable goods. The mechanism operates through a clear causal chain: higher country-wide productivity (A) leads to higher household wages (w). These higher wages, in turn, strengthen the population's preference for quality ($\theta(w)$). At the threshold $\hat{A}$, this preference becomes strong enough to overcome the higher relative price of the superior good, inducing the economy to switch production from the low- to the high-quality variety.

The resulting price level increase is best understood as the outcome of two competing forces. On one hand, the underlying increase in productivity (A) acts as a *productivity effect*, which tends to lower the prices. On the other hand, the switch to the high-quality variety, which is by assumption more factor-intensive ($A_H < A_L$), constitutes a *quality upgrading effect* that raises the cost of production. Proposition 2 establishes that at the quality threshold, this second effect dominates, causing an increase in the non-tradeable price index. Figure 2d illustrates this: prices fall with productivity until the threshold (productivity effect), then jump upward as the country switches to a higher quality good (quality upgrading effect).

The results in this section formalize the quality-quantity trade-off that countries face: initially, productivity growth allows for a greater quantity of basic goods, but past the threshold, the economy optimally chooses to produce a potentially smaller quantity of a superior good to meet the demands of a more affluent society. Such a choice is welfare improving, even if it leads to higher prices. Although prices rise, households are better off because they enjoy higher-quality goods that better match their preferences.

Ultimately, the model shows that the Penn effect is not just a byproduct of productivity differences in tradeables, but also a result of income-driven quality choices in non-tradeables. Richer countries have higher prices, in part, because they are rationally choosing to consume better, more sophisticated, and more resource-intensive non-tradeable goods and services.

Note that since the model presented in this section allows only a discrete choice between two qualities, it predicts a discontinuous jump in prices at the threshold $\hat{A}$. To better match with the continuous increase in non-tradeable prices observed in reality and to empirically validate the model, the next section generalizes this framework to a continuum of quality levels, showing how rising productivity leads to a smooth process of quality upgrading and a continuous rise in non-tradeable prices.

4 Extended Model: Continuum of Quality Levels

This section generalizes our baseline model by allowing countries to choose from a continuum of quality levels indexed by $q \in [0, 1]$. By introducing a continuum of quality levels, the extended model captures the smooth evolution of quality choices as productivity increases, generating continuous changes in price levels.

The section proceeds by first outlining the economic environment, then formulating the household and firm problems, and finally deriving the equilibrium conditions. We then illustrate the behavior of equilibrium prices and wages as a function of country productivity, A, and show how the model generates a continuous increase in non-tradeable prices as countries transition to higher quality varieties. Proposition 3 establishes the central result: as productivity rises, countries optimally upgrade to superior quality varieties, and analogous to the previous section's intuition, the quality upgrading effect dominates the cost-reducing productivity effect, generating higher equilibrium prices for non-tradeable goods. The section concludes by presenting a quantitative analysis to validate the theoretical predictions of the model and illustrate its empirical relevance in explaining international price differences.

4.1 Economic Environment

The economic environment parallels our baseline model but with an essential generalization. As before, households in each country consume a tradeable good, c_T, a standard non-tradeable good, c_N, and a quality-differentiated non-tradeable good. However, we now assume that the quality-differentiated good exists in a continuum of varieties indexed by quality level $q \in [0, 1]$, where quality increases monotonically in q such that $q = 0$ represents the lowest possible quality and $q = 1$ represents the highest possible quality.

The production technology for a variety q is given by:

$$y(q) = A \cdot A_H(q) \cdot \ell_H(q) \tag{17}$$

where $\ell_H(q)$ represents the labor input, A represents a country-specific productivity level, and

$A_H(q)$ represents the variety-specific productivity. As in the two-variety model, we assume complete technological diffusion across countries, meaning that firms in all countries face the same $A_H(q)$ function. This implies that while a country with higher productivity level A enjoys absolute advantages in producing all varieties, the relative productivity differences across varieties are identical for every country, ensuring that all countries face the same relative prices for different quality levels.

Crucially, we assume that $A_H(q)$ is continuous and strictly decreasing in q. This captures the intuitive notion that higher quality varieties require more labor per unit of output-a key assumption that drives our results. This generalizes our two-variety specification where we had $A_H(q_L) = A_L$ and $A_H(q_H) = A_H$ with $A_H < A_L$.

For analytical tractability, we assume that $A_H(q)$ decreases linearly in q[6]:

$$A_H(q) = 1 - \alpha_0 q \tag{18}$$

where $0 < \alpha_0 < 1$. This specification ensures that $A_H(0) = 1$ represents the highest possible productivity (for the lowest quality variety) and $A_H(1) = 1 - \alpha_0$ represents the lowest productivity (for the highest quality variety). The parameter α_0 thus measures the magnitude of the quality-productivity trade-off, larger values of α_0 indicate that quality upgrading requires proportionally more labor.

All other assumptions regarding the tradeable good c_T, the standard non-tradeable good c_N, and the labor market remain identical to our baseline model. However, to emphasize the role of quality choice in generating international price differences and to obtain closed-form solutions, we make one simplifying assumption: household consumption of the standard non-tradeable good is fixed at $\bar{c}_N$, implying perfectly inelastic demand for this good. While this assumption is not essential for our core results, it considerably simplifies the algebra while preserving the key economic mechanisms.

4.2 Household Problem

The household's utility function generalizes naturally to accommodate the continuum of quality levels. As in our baseline model, households treat different quality varieties as perfect substitutes, allowing for corner solutions where they specialize in consuming a single quality level. The utility

[6] Sub-section 4.8 generalizes this assumption by allowing for a more flexible functional form that captures the empirical relationship between quality and productivity.

function is given by:

$$U = \frac{\phi_1}{1-\sigma}\left(\int_0^1 \theta(w,q)c_H(q)dq\right)^{1-\sigma} + \phi_2 \log \bar{c}_N + \log c_T \tag{19}$$

The key innovation lies in the preference weight function $\theta(w, q)$, which captures how household valuation of quality variety q depends on the household's labor income w. This function embodies our assumption about income-dependent preferences for quality: wealthier households place relatively higher value on superior quality varieties.

Following our baseline specification but allowing for quality-dependent income elasticities, we assume:

$$\theta(q, w) = w^{k(q)} \tag{20}$$

where $k(q)$ represents the elasticity of variety q with respect to non-tradeable sector income w. We assume that $k(q)$ is continuous and strictly increasing in q, $k : [0, 1] \to \left[0, \frac{\sigma}{1-\sigma}\right]$. The assumption that $k(q)$ is strictly increasing in q implies that, given any level of income w, the household places a higher weight on a higher quality variety. Moreover, since $k(q) \geqslant 0$, then $\theta(w, q)$ is also increasing in w. This implies that with an increase in income, the relative preference for a higher quality variety increases more than that for a lower quality variety. In this sense, a good indexed by q is relatively superior to all varieties $[0, q)$. Note that analogous to the two-variety model, $\frac{1-\sigma}{\sigma}k(q)$ represents the income elasticity of demand for variety q where the variety $q = 0$ has an income elasticity of 0 and the variety $q = 1$ has an income elasticity equal to 1. This implies that while all varieties represent normal goods, with each variety q being relatively superior to the varieties $[0, q)$, they do not necessarily represent luxury goods. For analytical tractability, we assume a linear functional form for $k(q)$:

$$k(q) = \frac{\sigma}{1-\sigma} q \tag{21}$$

This specification ensures that the income elasticity increases linearly from zero (for the lowest quality) to one (for the highest quality), providing a parsimonious yet realistic representation of quality-dependent preferences.

The household's budget constraint generalizes to:

$$\int_0^1 p_H(q)c_H(q)dq + p_N\bar{c}_N + c_T = w + y_T \tag{22}$$

where $p_H(q)$ is the price of variety q. The household's optimization problem is to maximize utility

(19) subject to the budget constraint (22) with respect to $\{c_T, c_H(q)\}$ for all $q \in [0,1]$.

4.3 Firm Problem

The production side of the economy remains broadly unchanged, with perfectly competitive firms operating in each quality segment. Each quality level q is produced by firms who choose their labor inputs to maximize profits, taking the wage rate and output price as given. The firm's problem for quality level q is:

$$\max_{\ell_H(q)} p_H(q) A A_H(q) \ell_H(q) - w \ell_H(q) \tag{23}$$

The firm's problem for the standard non-tradeable good remains the same as (4).

4.4 Competitive Equilibrium: Definition

A competitive equilibrium consists of:

- Consumption decisions $\{c_H(q) \text{ for all } q \in [0,1], \bar{c}_N, c_T\}$
- Labor allocations $\{\ell_H(q) \text{ for all } q \in [0,1], \ell_N\}$
- Prices $\{p_H(q) \text{ for all } q \in [0,1], p_N, w\}$

such that:

- Given prices, households make consumption decisions by maximizing utility, (19) subject to their budget constraint, (22).
- Given prices, firms in every non-tradeable sector make production decisions by solving (23) or (4) for their respective sector.
- Markets clear:
  - Labor market:
$$\int_0^1 \ell_H(q) dq + \ell_N = 1 \tag{24}$$
  - Non-tradeable goods markets:

$$\bar{c}_N = A_N \ell_N \tag{25}$$

$$c_H(q) = A A_H(q) \ell_H(q) \quad \forall q \in [0,1] \tag{26}$$

– Tradeable goods market: (9).

4.5 Competitive Equilibrium: Characterization

In Appendix C, we derive the first order conditions for the household's problem and the firms' problem. We consider only corner solution cases where the country produces and consumes only a variety q^* that maximizes the household's utility[7]. We then obtain a system of equations (83)-(91) that characterize the competitive equilibrium in $\{c_T, c_H(q), p_N, p_H(q), w, \ell_N, \ell_H(q), q^*\}$ for a country with a given level of productivity A.

The equilibrium conditions can be condensed into two key relationships that jointly determine the optimal quality choice q^* and the equilibrium wage w for any given productivity level A. First, substitute (85), (87), and (88) into (91) to obtain:

$$\frac{c_H(q^*)}{AA_H(q^*)} = 1 - \frac{\bar{c}_N}{A_N}$$

Let $1 - \frac{\bar{c}_N}{A_N} \equiv \bar{\ell}$ denote the net labor supply for the quality differentiated product. Further, using (89) we get an equation in (w, q^*) for any given A:

$$\phi_1^{\frac{1}{\sigma}} \theta(q^*, w)^{\frac{1-\sigma}{\sigma}} (AA_H(q^*))^{\frac{1-\sigma}{\sigma}} w^{-\frac{1}{\sigma}} = \bar{\ell} \tag{27}$$

Moreover, combining (86) with (90) we get another equation in (w, q^*):

$$q^* \in \text{argmax}_{q' \in [0,1]} \frac{A\theta(q', w)A_H(q)}{w} \tag{28}$$

These two equations- (27) and (28)- constitute the core of our equilibrium system. For any given productivity level A, they jointly determine the optimal quality choice $q^*(A)$ and equilibrium wage $w(A)$. The remaining equilibrium variables are then solved using equations (83)-(89).

While equation (27) represents a resource constraint on labor, equation (28) represents the quality optimization condition: households choose the quality level that maximizes their utility per unit of expenditure, which reduces to maximizing the product of the preference weight $\theta(q, w)$ and the productivity level $A_H(q)$.

[7] Given the assumption of perfect substitutability between varieties, there always exists at least one such variety q^*.

4.6 Quality Choice and Wage Determination

We first establish the equilibrium relationship between the optimal quality choice q^* and the household income level, w, in Lemma 6. The lemma establishes that for any given level of income, $w > 0$, there exists a unique quality level, $q^*(w) \in [0,1]$, that solves (28), i.e., maximizes the household's utility per unit of expenditure, and this quality level is non-decreasing in w. The functional forms assumed for the preference weight $\theta(w,q)$ and the productivity function $A_H(q)$ facilitate the existence and uniqueness of this solution. In other words, the functional forms imply that the optimal q^* is always a corner solution and that this solution is unique for any given income level w.

Lemma 6. *Assume* $\theta(w,q) = w^{k(q)}$, *where* $k(q) = \frac{\sigma}{1-\sigma}q$, *and* $A_H(q) = 1 - a_0 q$, *where* $0 < a_0 < 1$. *For every* $w > 0$, *there exists a unique* $q^*(w) \in [0,1]$ *that solves* (28). $q^*(w)$ *is continuous in* w, *and there exist thresholds* $0 \leqslant \underline{w} < \overline{w} \leqslant \infty$ *such that:*

$$q^*(w) = \begin{cases} 0 & \textit{if } w \leqslant \underline{w}, \\ \frac{1}{a_0} - \frac{1-\sigma}{\sigma \ln w} & \textit{if } \underline{w} < w < \overline{w}, \\ 1 & \textit{if } w \geqslant \overline{w}, \end{cases} \tag{29}$$

where $q^*(w)$ *is strictly increasing in* w *for* $\underline{w} < w < \overline{w}$.

Proof. See Appendix D. □

This result shows that as incomes increase, countries transit through three distinct phases of quality specialization. At low income levels ($w \leqslant \underline{w}$), the income is insufficient to overcome the higher production costs associated with quality upgrading, leading countries to specialize in the lowest quality variety. As incomes rise beyond the threshold $\underline{w}$, the income-dependent preference for quality becomes strong enough to justify the higher production costs, and countries choose to consume higher quality varieties. This quality upgrading process continues smoothly until the income reaches the upper threshold $\overline{w}$, beyond which countries specialize in the highest quality variety. Figure 3 diagramatically illustrates this relationship between non tradeable sector income, w, and optimal quality choice, $q^*(w)$.

Given the relationship between optimal quality level, q^* and incomes, w, as defined by the function (29), we can now derive the equilibrium optimal quality choice $q^*(A)$ and hence the equilibrium

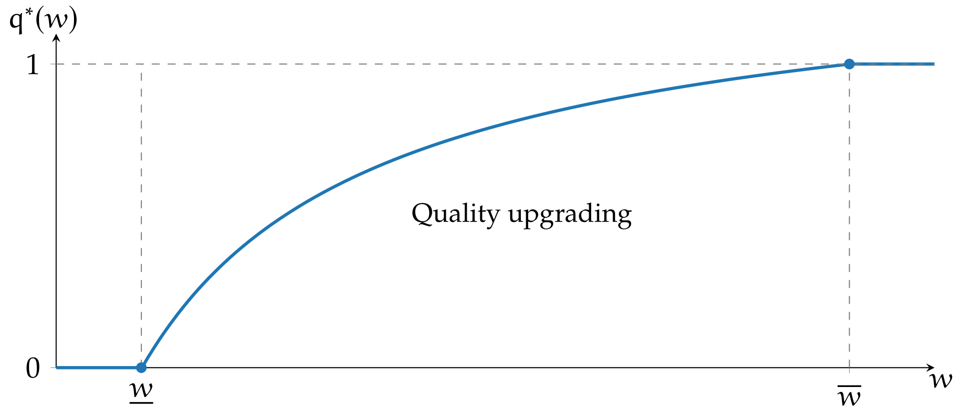


Figure 3: Optimal quality choice as a function of non-tradeable sector income

wages $w(q^*(A))$ as a function of productivity A. Using (29) and (27), Lemma 7 derives these functions and shows the existence and uniqueness of the competitive equilibrium for any productivity level $A > 0$ while characterizing the evolution of chosen quality levels and incomes with increase in productivity.

Lemma 7. *Assume* $\theta(w, q) = w^{k(q)}$ *with* $k(q) = \frac{\sigma}{1-\sigma}q$, *and* $A_H(q) = 1 - a_0 q$, *where* $0 < a_0 < 1$. *For every* $A > 0$, *there exist unique* $q^*(A) \in [0, 1]$ *and* $w(q^*(A)) \in (0, \infty)$ *that solve* (27) *and* (28). *Moreover, there exist thresholds* $0 < \underline{A} < \infty$ *and* $\underline{A} < \overline{A} < \infty$ *such that:*

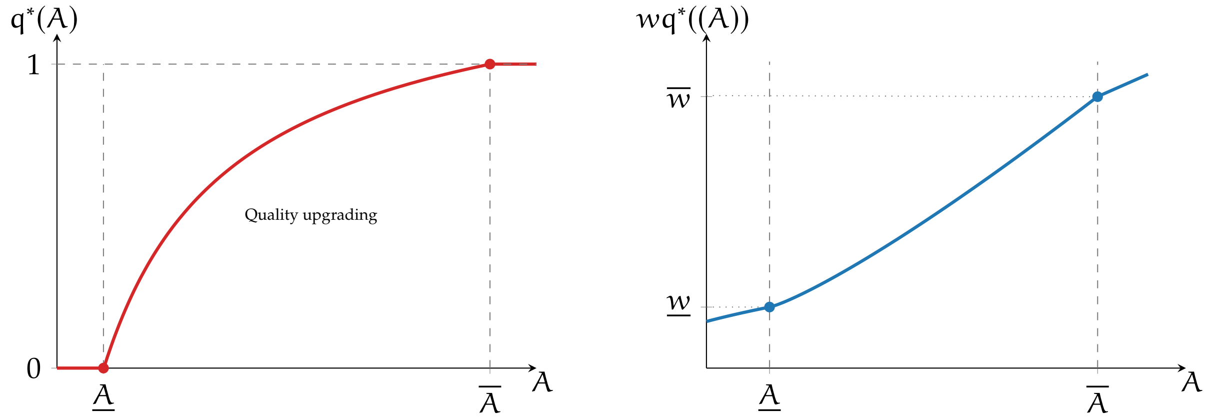
- $q^*(A) = 0$ *for* $A \leqslant \underline{A}$,
- $q^*(A) = 1$ *for* $A \geqslant \overline{A}$,
- $q^*(A)$ *is strictly increasing in* A *for* $\underline{A} < A < \overline{A}$ *and is given by the solution to the equation:*

$$A = \frac{1}{1 - a_0 q^*(A)} \left[\exp\left(\frac{a_0(1 - \sigma q^*(A))}{\sigma(1 - a_0 q^*(A))} \right) \left(\frac{\bar{\ell}^\sigma}{\phi_1} \right)^{\frac{1}{1-\sigma}} \right] \tag{30}$$

- $w(q^*(A))$ *is continuously and strictly increasing in* A *for all* $A > 0$; *and is given by:*

$$w(q^*(A)) = \begin{cases} \left(\frac{\phi_1}{\bar{\ell}^\sigma}\right)(A)^{1-\sigma}, & \text{if } A \leqslant \underline{A} \\ \exp\left(\frac{a_0(1-\sigma)}{\sigma(1-a_0 q^*(A))}\right), & \text{if } \underline{A} \leqslant A \leqslant \overline{A}, \\ \left(\frac{\phi_1}{\bar{\ell}^\sigma}\right)^{\frac{1}{1-\sigma}} (A(1 - a_0)), & \text{if } A \geqslant \overline{A} \end{cases} \tag{31}$$

where $q^*(A)$ *solves* (30) *for* $A \in [\underline{A}, \overline{A}]$.



(a) Optimal quality choice $q^*(A)$ (b) Equilibrium wage $w(q^*(A))$

Figure 4: Quality choice and wages as functions of productivity

Proof. See Appendix D. □

This Lemma shows that our result from 1 in the baseline framework, i.e., endogenous quality choice driven by income-dependent preferences extends naturally to the case of quality continuum. First, countries with very low productivity levels ($A \leqslant \underline{A}$) find it optimal to specialize in the lowest quality variety, reflecting the fact that their income levels are insufficient to justify the higher costs associated with quality upgrading. Second, countries with very high productivity levels ($A \geqslant \overline{A}$) optimally choose the highest quality variety, as their high incomes make superior quality attractive despite its higher resource requirements. Third, and most significantly, countries with intermediate productivity levels smoothly upgrade their quality choice as their productivity increases, generating a continuous relationship between productivity and quality specialization.

The strict monotonicity of both $q^*(A)$ and $w(q^*(A))$ in A implies that growth in this model is characterized by simultaneous increases in wages and quality sophistication. The result shows that more productive countries not only have higher wages/incomes but also produce and consume higher-quality goods and services.

Figure 4 illustrates Lemma 7 graphically. Panel (a) shows how optimal quality choice $q^*(A)$ evolves with productivity, exhibiting the same pattern as the income-quality relationship but now as a function of the fundamental productivity parameter. Panel (b) shows the monotonic relationship between productivity and equilibrium wages $w(q^*(A))$, which underlies the quality upgrading process.

These results establish that the equilibrium system (27)-(28) has a unique solution for any productivity level, providing the basis for analyzing how productivity differences translate into international price differences through the quality choice mechanism, which is the focus of the next sub-section.

4.7 Quality Choice and Price-Level Differences

In this sub-section we establish the relationship between productivity levels, quality choices, and equilibrium prices for the quality-differentiated good. We show that the equilibrium price of the quality-differentiated good is a continuous, strictly increasing function of productivity in the quality upgrading range $[\underline{A}, \overline{A}]$. This result directly extends Proposition 2 from the baseline framework to the case of a continuum of quality varieties thereby providing an explanation for how quality upgrading leads to a continuous increase in non-tradeable prices.

First, note from equation (86) that the equilibrium price of any variety, $q \in [0, 1]$ as a function of productivity, A, is given by:

$$p_H(q, A) = \frac{w(q^*(A))}{AA_H(q)} \quad \forall q \in [0, 1] \tag{32}$$

Lemma 8 establishes that $\frac{w(q^*(A))}{A}$ is a continuous and strictly decreasing function of productivity A for all $A > 0$. Analogous to the explanation in the baseline framework, this result is dependent on two crucial but plausible assumptions. First, the fact that the demand for the quality-differentiated good is relatively price elastic: $0 < \sigma < 1$. Second, the fact that the income elasticity of the quality-differentiated varieties is at most 1: $k(q) \leqslant 1$.

Lemma 8. *Let $w(q^*(A))$ be given by (31) where $q^*(A)$ solves (30). Then $\frac{w(q^*(A))}{A}$ is continuous and strictly decreasing in A for all $A > 0$.*

Proof. See Appendix D. □

Lemma 8 implies that the price of any variety $q \in [0, 1]$ is also strictly decreasing in A. This illustrates the *productivity effect* on prices: as productivity increases, the marginal cost of production decreases, leading to lower prices for every quality level $q \in [0, 1]$.

Furthermore, equation (32) implies that the price of the optimal variety, $p_H(q^*(A), A)$ is a function of $\frac{w(q^*(A))}{A}$ and $\frac{1}{A_H(q^*(A))}$. While the former is strictly decreasing in A, the latter is strictly increasing in A. The latter reflects the *quality upgrading effect*: as countries upgrade to higher quality varieties, the marginal cost of production increases due to the higher labor intensity associated with superior quality varieties. The net effect on prices, therefore, depends on the relative strength of the productivity effect vis-a-vis the quality upgrading effect.

Lemma 9 formally shows that in the quality-upgrading range, $(\underline{A}, \overline{A})$ the quality upgrading effect dominates the productivity effect causing $p_H(q^*(A), A)$ to increase with A.

Lemma 9. *Assume* $\theta(w, q) = w^{k(q)}$, *where* $k(q) = \frac{\sigma}{1-\sigma} q$, *and* $A_H(q) = 1 - a_0 q$, *where* $0 < a_0 < 1$. *For every* $A > 0$, *the equilibrium price for optimally chosen variety of the quality differentiated good,* $p_H(q^*(A), A)$, *is a continuous function of* A. *It is:*

- *strictly decreasing in* A *for* $A \leqslant \underline{A}$ *and is given by:*

$$p_H(0, A) = \left(\frac{\phi_1}{\bar{\ell}^\sigma}\right)(A)^{-\sigma} \tag{33}$$

- *strictly increasing in* A *for* $\underline{A} < A < \overline{A}$ *and is given by:*

$$p_H(q^*(A), A) = \left(\frac{\phi_1}{\bar{\ell}^\sigma}\right)^{\frac{1}{1-\sigma}} e^{\frac{a_0(q^*(A)-1)}{(1-a_0 q^*(A))}} \tag{34}$$

  where $q^*(A)$ *solves* (30) *for* $A \in [\underline{A}, \overline{A}]$.

- *constant for* $A \geqslant \overline{A}$ *and is given by:*

$$p_H(1, A) = \left(\frac{\phi_1}{\bar{\ell}^\sigma}\right)^{\frac{1}{1-\sigma}} \tag{35}$$

Proof. See Appendix D. □

Lemma 9 shows that despite higher productivity levels, countries that endogenously upgrade to superior quality varieties face higher equilibrium prices for the quality-differentiated good. On one hand, higher productivity A directly reduces the marginal cost of production, which would normally lead to lower prices. On the other hand, countries endogenously shift toward higher-quality varieties that are more labor-intensive to produce (lower $A_H(q)$), which increases prices. The dominance of the latter effect implies that $p_H(q^*(A), A)$ increases with A in the quality-upgrading range $(\underline{A}, \overline{A})$

In the low productivity range ($A \leqslant \underline{A}$), countries specialize in the lowest quality variety, so productivity improvements translate directly into lower prices through the standard cost-reduction channel. However, once countries enter the intermediate productivity range ($\underline{A} < A < \overline{A}$), the quality upgrading effect dominates. Countries progressively shift toward higher-quality varieties, and the increased labor intensity more than offsets the direct productivity gains, leading to rising

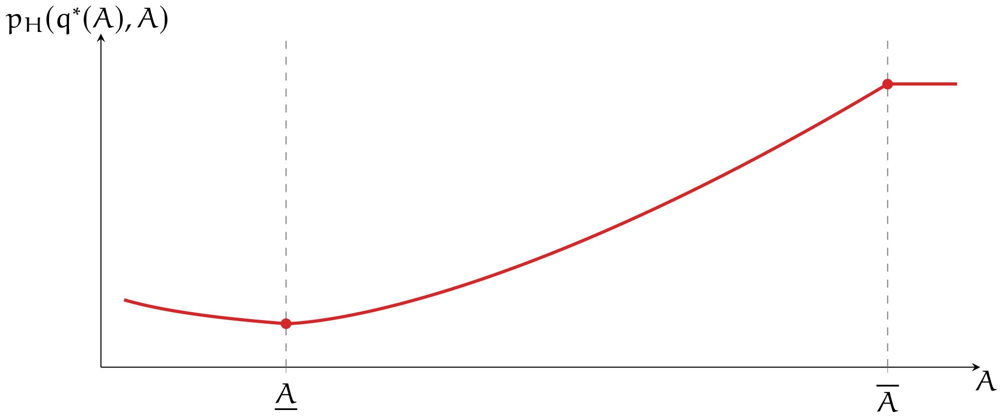


Figure 5: Evolution of Quality-Differentiated Good Prices with Productivity

prices. Finally, in the high productivity range ($A \geqslant \overline{A}$), countries have reached the quality frontier and further productivity gains manifest as constant prices since quality choice no longer responds to productivity changes. Figure 5 illustrates this U-shaped relationship between productivity and prices.

This pattern of price evolution carries significant implications for understanding international price differences. It is no coincidence that the U-shaped relationship between non-tradeable prices and productivity implied by the model matches the U-shaped relationship observed in data, as shown by the quadratic fit in Figure 1. This suggests that the Penn effect can be attributed not merely to tradeable sector productivity disparities but to the equilibrium quality choices that emerge during a country's growth process. Specifically, countries with higher productivity levels optimally select to produce and consume higher-quality varieties of non-tradeable goods. These higher-quality varieties are associated with higher prices due to their greater factor intensity, even when overall productivity levels are higher.

Importantly, such choices are endogenous and welfare-enhancing: countries with higher productivity rationally choose to produce and consume higher-quality varieties because these varieties better align with the preferences of their increasingly affluent populations. This occurs even when these countries face higher prices for the superior varieties and even when they have the ability to produce larger quantities of lower-quality varieties. The shift toward higher-quality goods reflects an optimal trade-off between quantity and quality, where the welfare gains from consuming better goods outweigh the potential benefits of consuming more of the lower-quality alternatives. Proposition 3 synthesizes our main findings and establishes the central result of the extended model: endogenous quality-biased technology choice provides a mechanism for explaining international price differences without appealing to the HBS hypothesis.

Proposition 3 (Quality choice and non-tradeable prices)**.** *Assume* $\theta(w, q) = w^{k(q)}$ *with* $k(q) = \frac{\sigma}{1-\sigma} q$, *where* $0 < \sigma < 1$, *and* $A_H(q) = 1 - a_0 q$, *where* $0 < a_0 < 1$. *Let* $\underline{A}$ *and* $\overline{A}$ *be given by the thresholds in Lemma 7. Then:*

- *For a country with a productivity level* $\underline{A} < A < \overline{A}$, *the unique equilibrium is to produce and consume an intermediate quality variety:* $0 < q^*(A) < 1$, *with* $q^*(A)$ *strictly increasing in* A.
- *The equilibrium price* $p_H(q^*(A), A)$ *is continuous and strictly increasing in* A *for* $\underline{A} < A < \overline{A}$.

Proof. Follows as a corollary from Lemmas 6, 7, and 9. □

This section has established that productivity growth is associated with a continuous process of quality upgrading. An increase in productivity induces a country to shift its production and consumption to higher quality varieties $q^*(A)$. This quality specialization is an endogenous outcome driven by rising wages, which in turn strengthens household preferences for superior quality.

Second, this section has provided a novel explanation for the Penn effect. It demonstrates that despite the cost-reducing effects of higher productivity, the equilibrium price of the non-tradeable good, $p_H(q^*(A), A)$, is strictly increasing in A throughout this quality-upgrading process. This occurs because the shift toward more resource-intensive, higher-quality varieties (the quality upgrading effect) dominates the direct productivity gains. This result offers an alternative to the traditional Harrod-Balassa-Samuelson hypothesis by showing how international price differences can arise endogenously from quality choices, even with identical technological access.

Third, although this quality upgrading leads to higher prices, the transition is welfare-enhancing. The equilibrium outcome reflects the optimal choice of households who, as their incomes rise, derive greater utility from consuming higher-quality goods. The willingness to pay higher prices for superior varieties is a rational response to evolving preferences, emphasizing that economic growth involves not just an increase in the quantity of goods consumed, but a welfare-improving shift in their quality.

Finally, the result highlights the importance of income-dependent preferences in shaping production patterns and international trade. The assumption that wealthier households place relatively higher value on quality creates a natural demand-side driver for quality upgrading that operates independently of supply-side productivity differences.

In the next-subsection, we turn to a quantitative analysis of the model's predictions using cross-country data on productivity and prices. We estimate the model parameters to fit the data and validate the theoretical predictions established in this section.

4.8 Quantitative Analysis

In this section, we conduct a quantitative analysis to empirically validate the theoretical predictions of our model. We utilize cross-country data on productivity and prices, to discipline the model parameters and fit the data to the model. We use Penn World Table (PWT) 10.01 data (Feenstra, Inklaar and Timmer 2015) on PPP-adjusted GDP per hour as the proxy for cross country non-tradeable productivity, A, and the exchange rate based price level of household consumption as the proxy for the price of quality adjusted non-tradeable good, $p_H(q^*(A))$.

As established in Proposition 3, the model generates a positive relationship between productivity, A, and the price of the quality-adjusted non-tradable good, $p_H(q^*(A))$, through endogenous quality choice $q^*(A)$. The calibration exercise in this sub-section validates that this theoretical mapping can quantitatively replicate the shape and magnitude of the empirical Penn effect (Figure 1) even in the absence of tradeable-sector productivity differences across countries.

While the analytical results established above assume linear functional forms for $A_H(q), k(q)$ for tractability, these results continue to hold for a wide range of assumptions on these functions. Recall that the competitive equilibrium is characterized by equations (27), (28), and equations (83)-(89). From equation (28), an interior optimum implies that the equilibrium wage is given by:

$$w(q^*(A)) = exp\left(-\frac{A'_H(q^*(A))}{A_H(q^*(A))\, k'(q^*(A))}\right) \tag{36}$$

Furthermore, combining equations (27) and (36), the optimal quality level $q^*(A)$ solves:

$$A = \frac{1}{A_H(q^*(A))} exp\left[\left(-\frac{A'_H(q^*(A))}{A_H(q^*(A))\, k'(q^*(A))}\right)\left(\frac{1}{1-\sigma} - k(q^*(A))\right)\right]\left(\frac{\bar{\ell}^{\sigma}}{\phi_1}\right)^{\frac{1}{1-\sigma}} \tag{37}$$

From equation (86), the equilibrium price of any variety $q \in [0, 1]$ is given by:

$$p_H(q, A) = \frac{w(q^*(A))}{A\, A_H(q)} \quad \text{for all } q \in [0, 1] \tag{38}$$

Therefore, given a productivity level A, the price of the quality-differentiated non-tradeable good equals the price of the optimal variety $q^*(A)$:

$$p_H(q^*(A), A) = \frac{w(q^*(A))}{A\, A_H(q^*(A))} \tag{39}$$

In the calibrated model, we continue to assume a linear functional form for the preference weight elasticity $k(q)$:

$$k(q) = \frac{\sigma}{1-\sigma} q$$

However, we generalize the variety-specific productivity function $A_H(q)$ beyond the linear case to allow for greater flexibility in matching the data. Specifically, we adopt the functional form:

$$A_H(q) = (1 - a_0 q)^{a_1}, \quad 0 < a_0 < 1, \quad a_1 > 0$$

This specification preserves the essential property that $A_H(q)$ is strictly decreasing in q, capturing the fact that higher quality varieties require more resources per unit of output. The function normalizes productivity for the base variety ($q = 0$) to 1. The parameter a_1 controls the curvature of the quality-productivity relationship, with $a_1 = 1$ corresponding to the linear analytical case. Both parameters a_0 and a_1 determine the sensitivity of variety-specific productivity to quality level q, thereby governing the extent to which quality upgrading increases resource intensity.

In addition to a_0, a_1, a third essential model parameter is σ, which represents the price elasticity of demand for the quality-differentiated good. We estimate these three parameters, $\{a_0, a_1, \sigma\}$, using nonlinear least squares, i.e., by minimizing the sum of squared residuals between model-implied prices (equation (39)) and observed price levels in the data[8]. Table 1 presents the estimated parameter values.

Figure 6 displays the model-generated relationship between log productivity ($\log A$) and non-tradeable prices ($p_H(q^*(A), A)$) alongside the empirical data. The figure demonstrates that the calibrated model successfully captures the upward-sloping portion of the observed productivity-price relationship, providing strong evidence that the quality choice mechanism can quantitatively account for the Penn effect without invoking traditional HBS effects from tradeable sector productivity differences.

To assess the model's quantitative fit, we report three goodness-of-fit statistics in Table 2. The

[8]For appropriate scaling between model and data values, we estimate $\log \underline{A}_{data}$ and $\underline{p}_{data}$ by fitting a parabola to the relationship between log labor productivity and the consumption price index, as shown in Figure 1. The vertex of the fitted parabola provides $\log \underline{A}_{data}$ and $\underline{p}_{data}$. The data is then scaled relative to the model-implied values $(\underline{A}, p_H(q^*(\underline{A})))$. Only data points in the upward-sloping region of the parabola, specifically those with $A \geqslant \underline{A}_{data}$, are used in parameter estimation.

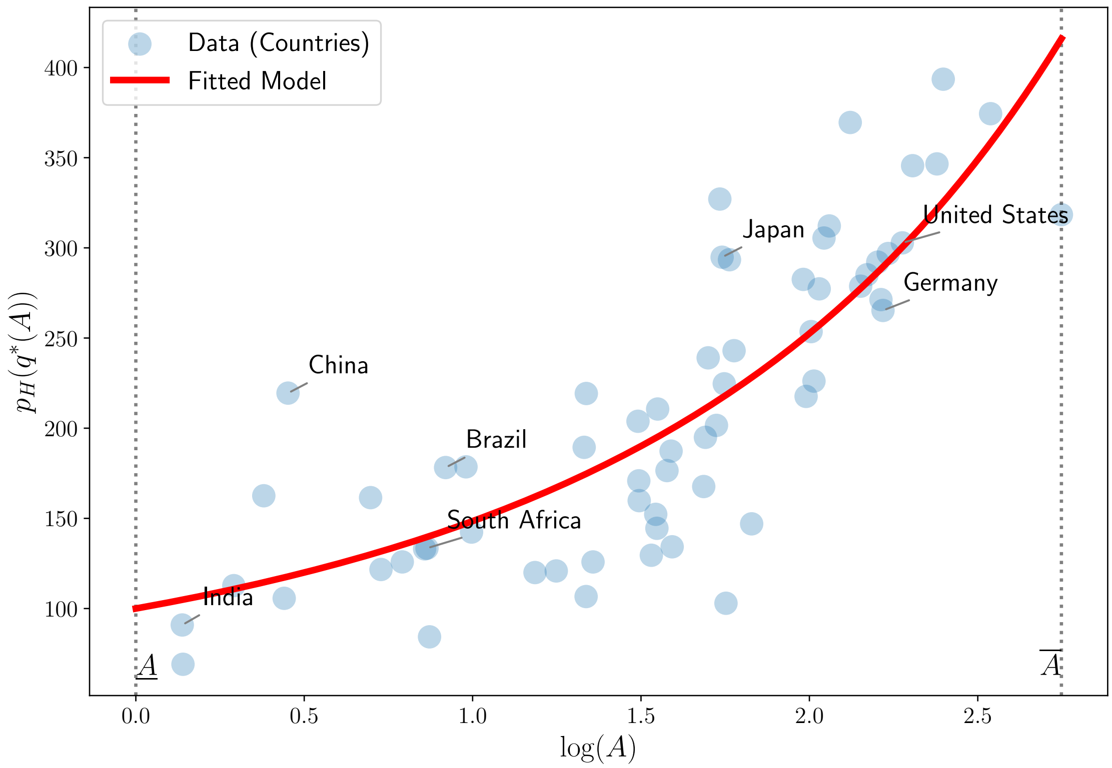


Figure 6: Model-generated relationship vs. empirical Penn effect

Notes: Data is scaled to match the calibrated model with $\underline{A} = 1$ and $p_H(0, \underline{A}) = 100$. Source: Scatter plot: Penn World Table 10.01 (Feenstra, Inklaar and Timmer 2015). Fitted Model: Author's calculations.

Table 1: Estimated Model Parameters (Non-linear least squares)

Parameter	**Value**
a_0	0.6170
a_1	2.3103
σ	0.5400

Table 2: Model Fit Statistics

Statistic	**Value**
Coefficient of determination (R^2)	0.6884
Root mean squared error (RMSE)	0.0278
Correlation coef. between fitted residuals and A	-0.0390

coefficient of determination ($R^2 = 0.6884$) indicates that the model explains approximately 69 percent of the variation in cross-country price levels. The root mean squared error (RMSE) is 0.0278, which implies an average prediction error of only 2.78 percent relative to the baseline price level. Finally, the correlation coefficient between fitted residuals and productivity is close to zero

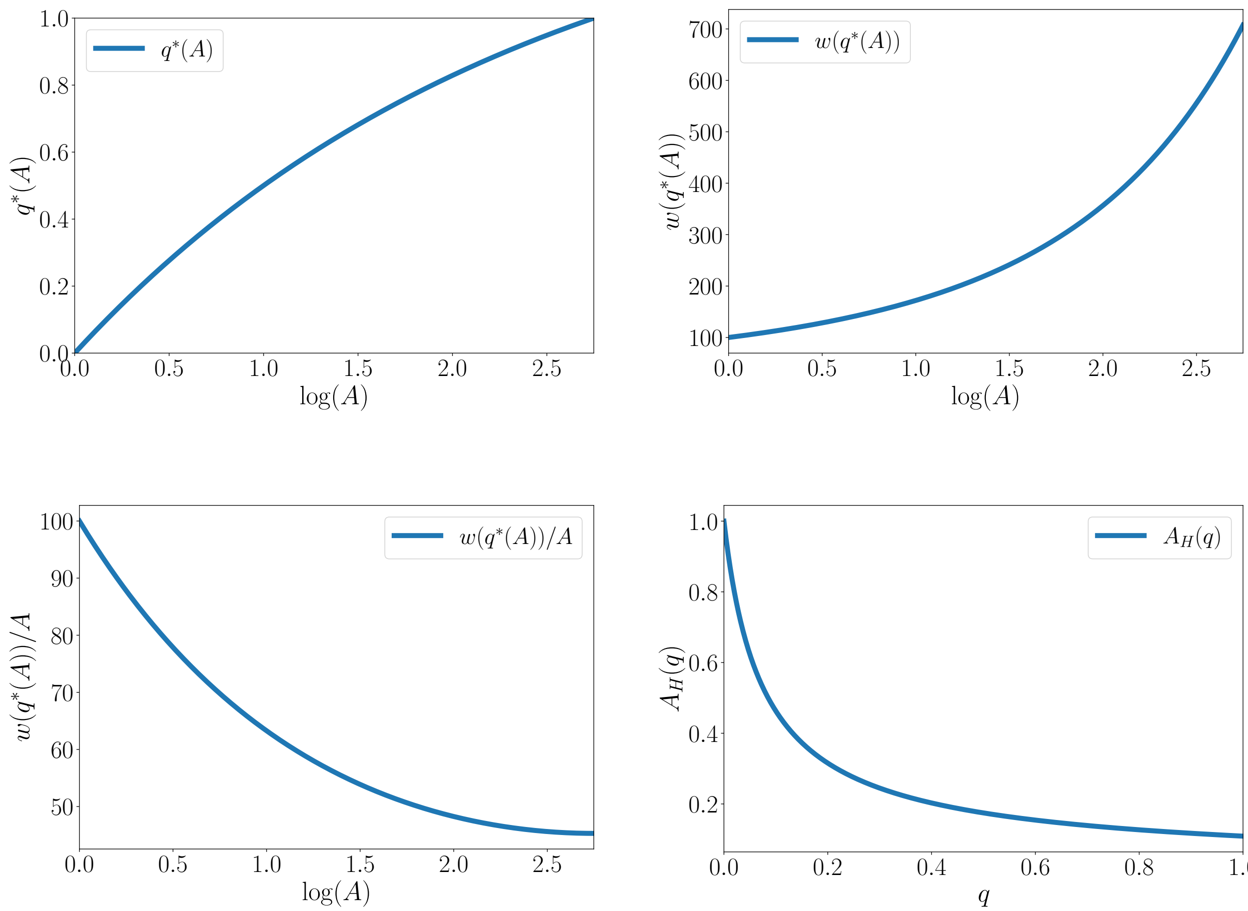


Figure 7: Equilibrium variables: (a) $q^*(A)$, (b) $w(q^*(A))$, (c) $\frac{w(q^*(A))}{A}$, (d) $A_H(q)$

(−0.0390), indicating the absence of systematic bias in the model's predictions across the productivity distribution. These statistics collectively confirm that the quality upgrading mechanism provides a robust quantitative explanation for international price differences.

To illustrate the underlying economic mechanism, Figure 7 decomposes the equilibrium price relationship into its constituent components. Panel (a) shows that the optimal quality level $q^*(A)$ increases monotonically with productivity, confirming the theoretical prediction that richer countries endogenously choose higher quality varieties. Panel (b) shows that the equilibrium wage $w(q^*(A))$ also rises with productivity, reflecting the income effects that drive quality upgrading. Panel (c) presents a crucial result: the wage-to-productivity ratio $w(q^*(A))/A$ decreases with productivity, capturing the direct productivity effect that would normally lead to lower prices. Panel (d) displays the function $A_H(q)$, which decreases strictly in quality, representing the higher resource requirements of superior varieties.

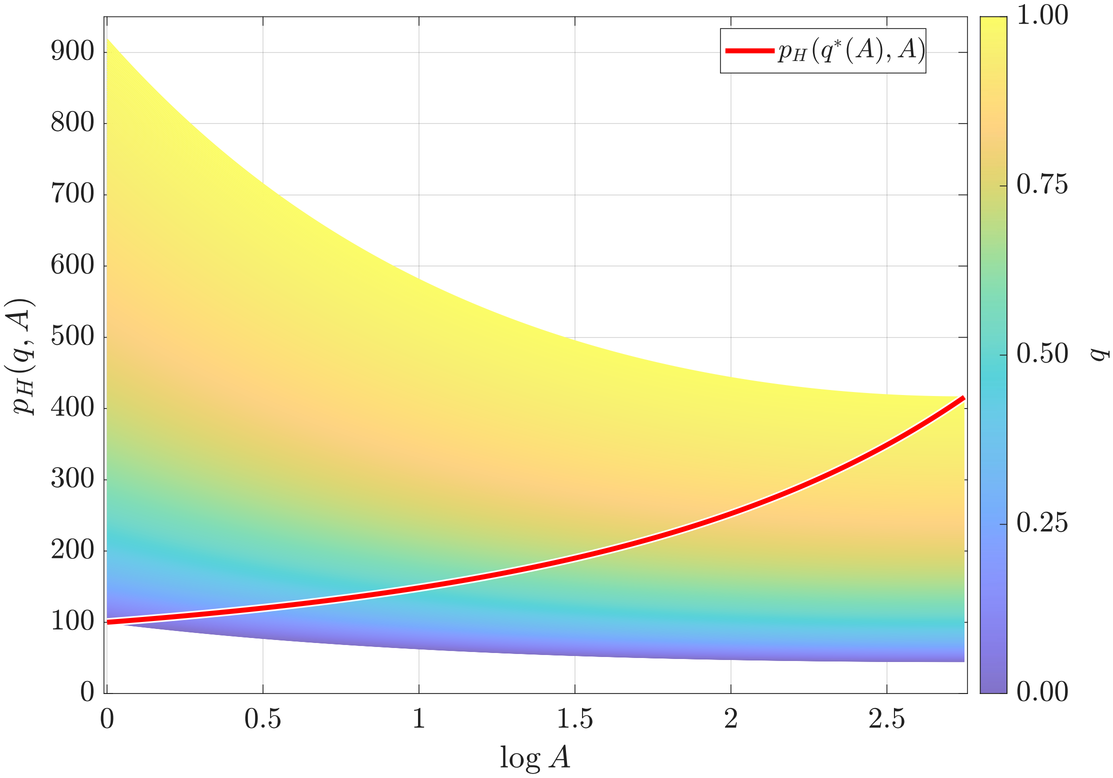


Figure 8: Color map of $p_H(q, A)$

The fact that $\frac{w(q^*(A))}{A}$ is strictly decreasing in productivity, A, implies that the equilibrium price of any variety $q \in [0, 1]$ is also strictly decreasing in A. This is illustrated by the downward-sloping color map in Figure 8 where each color represents a different quality level, q. This reflects a unsurprising result: productivity growth reduces production costs and lowers prices for all varieties. However, the fitted model curve from Figure 6 is superimposed on this color map. This curve represents the locus of prices for the optimal variety, $p_H(q^*(A))$, that a country with a given productivity level, A, chooses to produce and consume. Crucially, this curve is strictly increasing in A. This is because, while $\frac{w(q^*(A))}{A}$ is strictly decreasing, $\frac{1}{A_H(q^*(A))}$ is strictly decreasing in A, and the latter effect dominates, thereby resulting in higher effective prices.

The economic intuition for this result is as follows: while higher productivity reduces the cost of producing any given quality level (the productivity effect), with increase in productivity and incomes, countries optimally respond by upgrading to higher quality varieties that require more labor per unit of output (the quality upgrading effect). The quality upgrading effect dominates the productivity effect, and as a result, countries with higher productivity levels face higher equilibrium prices for non-tradeable goods. This mechanism provides a novel explanation for the Penn effect that operates through demand-side preferences rather than tradeable sector productivity differences across countries.

Our quantitative analysis establishes that the quality choice mechanism represents a compelling alternative to traditional explanations of international price differences. The model successfully replicates the empirical Penn effect without relying on sector-specific productivity assumptions or barriers to technology diffusion. Instead, the framework shows how quality upgrading emerges as an equilibrium response to rising incomes, thereby explaining why goods and services systematically cost more in wealthy economies. The robustness of these results across different specifications and the strong empirical fit support the view that quality-biased technology choice constitutes a fundamental driver of cross-country price patterns.

The quantitative validation presented in this section reinforces the paper's key contribution: quality choice provides a novel and empirically grounded mechanism for understanding the Penn effect. Unlike traditional theories that require differential productivity growth across tradeable sectors, our model generates realistic price differences through the endogenous adoption of quality-enhancing technologies. International price differences, in this framework, are not mere artifacts of exogenous productivity disparities but arise as rational, welfare-maximizing responses to economic growth.

5 Conclusion

This paper has developed a novel theoretical framework for understanding international price differences through a quality-biased technology choice. We show that as countries become more productive and incomes rise, they endogenously shift to producing higher-quality, more resource-intensive non-tradeable goods. This preference-driven quality upgrading exerts upward pressure on prices, generating the Penn effect even when there are no productivity differences in the tradeable sector.

Our baseline model with two quality levels establishes the insight that countries below a threshold productivity level specialize in low-quality varieties, while those above the threshold switch to high-quality varieties that require more labor per unit of output. This quality switch creates a discontinuous upward jump in the price level at the quality threshold. We hypothesize this jump as the source of increasing non-tradeable prices in richer countries, despite productivity being higher.

The extended model with a continuum of quality levels demonstrates the robustness and generalizability of this mechanism. It shows that countries undergo smooth quality upgrading as they develop, with quality choices increasing monotonically with productivity. Analogous to the baseline framework, this quality upgrading can lead to rising prices for non-tradeable goods despite productivity improvements, as the shift toward more labor-intensive quality varieties more than offsets the direct cost-reduction effects of higher productivity. A quantitative analysis of our framework confirms these predictions and we show that approximately 69 percent of the variation in non-tradeable prices can be explained by quality upgrading effects without invoking HBS effects.

Our framework offers several advantages over existing explanations of international price differences. First, it does not require implausibly large non-tradeable sectors to generate significant price differences, as the quality upgrading mechanism can produce substantial price effects even in relatively small sectors. Second, it provides a natural explanation for why price differences persist and even increase as countries grow, rather than converging toward equality as traditional trade theory might suggest. Third, it captures the realistic observation that economic growth involves not just producing more goods but producing better goods that match the evolving preferences of wealthier populations.

The model also raises important questions about welfare measurement and international comparisons. If countries at different development levels consume different quality varieties, standard price indices may not accurately capture welfare differences across countries. This suggests that international poverty comparisons and purchasing power parity calculations should account for quality differences in addition to pure price differences.

Several avenues for future research emerge from our study. First, empirical testing using detailed price and quality data across countries could provide further insights into the mechanisms driving quality upgrading. Second, extending the model to include trade in quality-differentiated goods could illuminate how quality specialization affects trade patterns and comparative advantage. Third, incorporating endogenous innovation and quality improvement over time could provide insights into the dynamic relationship between technological progress and quality evolution.

References

Acemoglu, Daron. 2001. "Directed technical change." *The Review of Economic Studies*, 68(4): 781–809.

Aguiar, Mark, and Mark Bils. 2015. "Has consumption inequality mirrored income inequality?" *American Economic Review*, 105(9): 2725–56.

Alessandria, George, and Joseph P Kaboski. 2011. "Explaining the border effect: The role of exchange rate variability, shipping costs, and geography." *Journal of International Economics*, 84(1): 1–15.

Asea, Patrick K, and Enrique G Mendoza. 1994. "Balassa-Samuelson vs. the Harrod-Balassa-Samuelson hypothesis: an answer." *Review of International Economics*, 2(1): 20–29.

Asea, Patrick K, and W Max Corden. 1994. "The Balassa-Samuelson model: an overview." *Review of International Economics*, 2(3): 191–200.

Balassa, Bela. 1964. "The purchasing-power parity doctrine: a reappraisal." *Journal of Political Economy*, 72(6): 584–596.

Basu, Susanto, and David N Weil. 1998. "Appropriate technology and growth." *The Quarterly Journal of Economics*, 113(4): 1025–1054.

Bergstrand, Jeffrey H. 1991. "Structural determinants of real exchange rates and national price levels: Some empirical evidence." *American Economic Review*, 81(1): 325–334.

Burstein, Ariel T, João C Neves, and Sergio Rebelo. 2003. "Distribution costs and real exchange rate dynamics during exchange-rate-based stabilizations." *Journal of Monetary Economics*, 50(6): 1189–1214.

Caselli, Francesco, and Wilbur John Coleman. 2006. "The world technology frontier." *American Economic Review*, 96(3): 499–522.

Fajgelbaum, Pablo D, and Amit K Khandelwal. 2016. "Measuring the unequal gains from trade." *The Quarterly Journal of Economics*, 131(3): 1103–1180.

Fajgelbaum, Pablo D, Gene M Grossman, and Elhanan Helpman. 2011. "Income distribution, product quality, and international trade." *Journal of Political Economy*, 119(4): 721–765.

Feenstra, Robert C, Robert Inklaar, and Marcel P Timmer. 2015. "The Next Generation of the Penn World Table." *American Economic Review*, 105(10): 3150–3182.

Fieler, Ana Cecilia. 2011. "Nonhomotheticity and bilateral trade: evidence and a quantitative explanation." *Econometrica*, 79(4): 1069–1101.

Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon. 2011. "International prices, costs, and markup differences." *American Economic Review*, 101(6): 2450–2486.

Hallak, Juan Carlos. 2006. "Product quality and the direction of trade." *Journal of International Economics*, 68(1): 238–265.

Harrod, Roy F. 1933. "International economics."

Hummels, David, and Peter J Klenow. 2005. "The variety and quality of a nation's exports." *American Economic Review*, 95(3): 704–723.

Ito, Takatoshi, Peter Isard, and Steven Symansky. 1999. "Purchasing power parity and East Asian currencies in the 1990s." *IMF Staff Papers*, 46(1): 42–83.

Klenow, Peter J, and Benjamin A Malin. 2010. "Microeconomic evidence on price-setting." *Handbook of Monetary Economics*, 3: 231–284.

Linder, Staffan Burenstam. 1961. "An essay on trade and transformation."

Matsuyama, Kiminori. 2000. "A Ricardian model with a continuum of goods under nonhomothetic preferences: demand complementarities, income distribution, and North-South trade." *Journal of Political Economy*, 108(6): 1093–1120.

Obstfeld, Maurice, and Kenneth Rogoff. 1996. *Foundations of international macroeconomics.* MIT Press.

Rogoff, Kenneth. 1996. "The purchasing power parity puzzle." *Journal of Economic Literature*, 34(2): 647–668.

Samuelson, Paul A. 1964. "Theoretical notes on trade problems." *The Review of Economics and Statistics*, 46(2): 145–154.

Schott, Peter K. 2004. "Across product within country differences in export quality." *The Quarterly Journal of Economics*, 119(2): 647–697.

Appendix

A Baseline Model: Competitive Equilibrium Solution

A.1 Household optimality

Household optimization involves maximizing utility, (1), with respect to $\{c_T, c_N, c_H, c_N\}$, subject to the budget constraint, (2) and non-negativity constraints, $(c_L, c_H) \geqslant 0$. The optimization problem yields the following first-order conditions:

$$c_T = \frac{1}{\lambda} \tag{40}$$

$$c_N = \frac{\phi_2}{\lambda p_N} \tag{41}$$

$$\phi_1(c_L + \theta(w)c_H)^{-\sigma} + \mu_L = \lambda p_L \tag{42}$$

$$\phi_1(c_L + \theta(w)c_H)^{-\sigma}\theta(w) + \mu_H = \lambda p_H \tag{43}$$

$$\mu_H c_H = 0, \quad c_H, \mu_H \geqslant 0 \tag{44}$$

$$\mu_L c_L = 0, \quad c_L, \mu_L \geqslant 0 \tag{45}$$

where λ, μ_L, μ_H are Lagrange multipliers.

A.2 Firm optimality

The firms' optimization problems, (3), (4), yield the following optimality conditions:

$$p_N = \frac{w}{A_N} \tag{46}$$

$$p_L = \frac{w}{AA_L} \tag{47}$$

$$p_H = \frac{w}{AA_H} \tag{48}$$

A.3 Equilibrium solution

Finally, we combine the household optimality conditions, (40)-(45), with firm optimality conditions, (46)-(48), and market clearing conditions, (5)-(9), to obtain a system of equations that characterize the competitive equilibrium.

First note that, the non-traded goods market clearing conditions, (6)-(8) when combined with the firms' zero profit conditions and the household's budget constraint, (2), imply that the equilibrium outcome is an autarky, i.e., $c_T = y_T = 1$ for every country. The competitive equilibrium in $\{c_T, c_N, c_L, c_H, p_N, p_L, p_H, w, \ell_N, \ell_L, \ell_H\}$ together with Lagrange multipliers (μ_L, μ_H) is characterized by the following system of 13 equations:

$$c_T = 1 \tag{49}$$

$$p_N = \frac{w}{A_N} \tag{50}$$

$$c_N = \frac{A_N \phi_2}{w} \tag{51}$$

$$\ell_N = \frac{\phi_2}{w} \tag{52}$$

$$p_L = \frac{w}{AA_L} \tag{53}$$

$$p_H = \frac{w}{AA_H} \tag{54}$$

$$\ell_L = \frac{c_L}{AA_L} \tag{55}$$

$$\ell_H = \frac{c_H}{AA_H} \tag{56}$$

$$\frac{c_L}{AA_L} + \frac{c_H}{AA_H} + \frac{\phi_2}{w} = 1 \tag{57}$$

$$\phi_1(c_L + \theta(w)c_H)^{-\sigma} + \mu_L = p_L \tag{58}$$

$$\phi_1(c_L + \theta(w)c_H)^{-\sigma}\theta(w) + \mu_H = p_H \tag{59}$$

$$\mu_H c_H = 0, \quad c_H, \mu_H \geqslant 0 \tag{60}$$

$$\mu_L c_L = 0, \quad c_L, \mu_L \geqslant 0 \tag{61}$$

A.4 Corner solution case: $c_L > 0, c_H = 0$

Consider the case where $c_L > 0, c_H = 0$. From, (61) we have that $\mu_L = 0$. From (58) we have:

$$c_L = \phi_1^{\frac{1}{\sigma}} p_L^{-\frac{1}{\sigma}} \tag{62}$$

Note the resemblance of (62) with a constant elasticity of demand function, where the price elasticity of demand is $\frac{1}{\sigma}$ and income elasticity is 0.

Then from (59), (60), and (62) we have that:

$$\mu_H = p_H - \theta(w)p_l \geqslant 0 \tag{63}$$

Finally, combining (53) and (62) with (57) we have that:

$$\frac{\phi_1^{\frac{1}{\sigma}}(AA_L)^{\frac{1-\sigma}{\sigma}}}{w^{1/\sigma}} + \frac{\phi_2}{w} = 1 \tag{64}$$

This corner solution in $\{c_T, c_N, c_L, c_H, p_N, p_L, p_H, w, \ell_N, \ell_L, \ell_H\}$ is then characterized by (49)-(56) and (62), (64) with $c_H = 0$, and this solution is a competitive equilibrium if and only if:

$$\theta(w) \leqslant \frac{p_H}{p_L} = \frac{A_L}{A_H}$$

A.5 Corner solution case: $c_H > 0, c_L = 0$

Consider the case where $c_H > 0, c_L = 0$. From, (60) we have that $\mu_H = 0$. From (60) we have that:

$$c_H = \phi_1^{\frac{1}{\sigma}} \theta(w)^{\frac{1-\sigma}{\sigma}} p_H^{-\frac{1}{\sigma}} \tag{65}$$

Note the resemblance of (65) with a constant elasticity of demand function, where the price elasticity of demand is $\frac{1}{\sigma}$ and given $\theta(w) = \vartheta w^k$, income elasticity is $\frac{k(1-\sigma)}{\sigma}$.

Then from (58), (61), and (65) we have that:

$$\mu_L = p_L - \frac{p_H}{\theta(w)} \geqslant 0 \tag{66}$$

Finally, combining (54) and (65) with (57) we have that:

$$\frac{\phi_1^{\frac{1}{\sigma}} \theta(w)^{\frac{1-\sigma}{\sigma}} (AA_H)^{\frac{1-\sigma}{\sigma}}}{w^{1/\sigma}} + \frac{\phi_2}{w} = 1 \tag{67}$$

This corner solution in $\{c_T, c_N, c_L, c_H, p_N, p_L, p_H, w, \ell_N, \ell_L, \ell_H\}$ is then characterized by (49)-(56) and (65), (67) with $c_L = 0$, and this solution is a competitive equilibrium if and only if:

$$\theta(w) \geqslant \frac{p_H}{p_L} = \frac{A_L}{A_H}$$

A.6 Interior solution case: $c_L > 0, c_H > 0$

Consider the case where $c_L > 0, c_H > 0$. From (60), (61) we have that $\mu_H = \mu_L = 0$. From (58) and (59) we have that:

$$\phi_1(c_L + \theta(w)c_H)^{-\sigma} = p_L \tag{68}$$

$$\phi_1(c_L + \theta(w)c_H)^{-\sigma}\theta(w) = p_H \tag{69}$$

Combining these we get:

$$\theta(w) = \frac{p_H}{p_L} = \frac{A_L}{A_H} \tag{70}$$

Moreover, using (68) and (53) we have that:

$$\frac{c_H}{A_H} + \frac{c_L}{A_L} = \phi_1^{\frac{1}{\sigma}} \frac{1}{A_L} \left(\frac{w}{AA_L} \right)^{-\frac{1}{\sigma}} \tag{71}$$

Finally, combining (71) with (57) gives us:

$$\frac{\phi_1{}^{\frac{1}{\sigma}} (AA_L)^{\frac{1-\sigma}{\sigma}}}{w^{1/\sigma}} + \frac{\phi_2}{w} = 1 \tag{72}$$

This interior competitive equilibrium solution in $\{c_T, c_N, c_L, c_H, p_N, p_L, p_H, w, \ell_N, \ell_L, \ell_H\}$ is characterized by (49)-(57) and (70) and (72).

B Baseline Model: Proofs

Proof of Lemma 1:

Proof. In subsection A.4 of Appendix A we derive the system of equations, (49)-(56) and (62), (64) that characterize the corner solution with $c_H = 0$ and $c_L > 0$ and this solution is an equilibrium if and only if:

$$\theta(w) \leqslant \frac{p_H}{p_L} = \frac{A_L}{A_H}$$

Specifically, consider the equation, (64), which solves for w as a function of A in this corner case:

$$\frac{\phi_1{}^{\frac{1}{\sigma}} (AA_L)^{\frac{1-\sigma}{\sigma}}}{w^{1/\sigma}} + \frac{\phi_2}{w} = 1$$

This equation can further be expressed as:

$$A = \tilde{\phi}_L (w - \phi_2)^{\frac{\sigma}{1-\sigma}} w \tag{73}$$

where $\tilde{\phi}_L = \frac{1}{\phi_1^{\frac{1}{1-\sigma}} A_L}$.

Since $\theta(w)$ is continuous and strictly increasing in w, for $w = \theta^{-1}\left(\frac{A_L}{A_H}\right) \equiv \hat{w} > 0$ we have:

$$A = \frac{1}{\phi_1^{\frac{1}{1-\sigma}} A_L} \left[\theta^{-1}\left(\frac{A_L}{A_H}\right) - \phi_2 \right]^{\frac{\sigma}{1-\sigma}} \theta^{-1}\left(\frac{A_L}{A_H}\right) \equiv \hat{A} > 0$$

where $0 < \phi_2 < \theta^{-1}\left(\frac{A_L}{A_H}\right)$.

Furthermore, using (73), lemma 10 in Appendix B shows that for any $A \geqslant 0$, there exists a unique $w^L \geqslant \phi_2$ which solves (73) and that w^L is a strictly increasing function of A. Let this function be denoted by $w^L(A)$. Then for any $A \leqslant \hat{A}$ we have that:

$$A \leqslant \hat{A} \Leftrightarrow w^L(A) \equiv w \leqslant \hat{w} \Leftrightarrow \theta(w) \leqslant \frac{A_L}{A_H}$$

In other words, the corner case $c_H = 0, c_L > 0$ is a competitive equilibrium if and only if $A \leqslant \hat{A}$. Moreover, since $w^L(A)$ is strictly increasing in A, $\hat{A}$ is unique. □

Proof of Lemma 2:

Proof. In subsection A.6 of Appendix A we derive the system of equations, (49)-(57), (70) and (72) that characterize an interior solution which is an equilibrium if and only if:

$$\theta(w) = \frac{p_H}{p_L} = \frac{A_L}{A_H}$$

Since $\theta(w)$ is strictly increasing in w, we have that $w = \hat{w} = \theta^{-1}\left(\frac{A_L}{A_H}\right)$. Substituting this in equation (72) implies that it must be the case that $A = \hat{A}$. Then:

$$\theta(w) = \frac{A_L}{A_H} \Leftrightarrow w = \theta^{-1}\left(\frac{A_L}{A_H}\right) \Leftrightarrow A = \hat{A}$$

Moreover, from (57), any value of $c_H \in \left[0, \hat{A}A_H\left(1 - \frac{\phi_2}{\hat{w}}\right)\right], \quad c_L \in \left[0, \hat{A}A_L\left(1 - \frac{\phi_2}{\hat{w}}\right)\right]$, such that (57) holds, is a competitive equilibrium. □

Proof of Lemma 3:

Proof. In subsection A.5 of Appendix A we derive the system of equations, (49)-(56) and (65), (67) that characterize the corner solution with $c_L = 0$ and $c_H > 0$ and this solution is an equilibrium if and only if:

$$\theta(w) \geqslant \frac{p_H}{p_L} = \frac{A_L}{A_H}$$

Specifically, consider the equation, (67), which solves for w as a function of A in this corner case:

$$\frac{\phi_1^{\frac{1}{\sigma}}\theta(w)^{\frac{1-\sigma}{\sigma}}(AA_H)^{\frac{1-\sigma}{\sigma}}}{w^{1/\sigma}} + \frac{\phi_2}{w} = 1$$

which can further be expressed as:

$$A = \tilde{\phi}_H (w - \phi_2)^{\frac{\sigma}{1-\sigma}} \frac{w}{\theta(w)} \tag{74}$$

where $\tilde{\phi}_H = \frac{1}{\phi_1^{\frac{1}{1-\sigma}} A_H}$.

Since $\theta(w)$ is continuous and strictly increasing in w, for $w = \theta^{-1}\left(\frac{A_L}{A_H}\right) \equiv \hat{w} > 0$ we have:

$$A = \hat{A} = \frac{1}{\phi_1^{\frac{1}{1-\sigma}} A_L} \left[\theta^{-1}\left(\frac{A_L}{A_H}\right) - \phi_2\right]^{\frac{\sigma}{1-\sigma}} \theta^{-1}\left(\frac{A_L}{A_H}\right) > 0$$

where $0 < \phi_2 < \theta^{-1}\left(\frac{A_L}{A_H}\right)$

Under the assumption that $\theta(w)$ has a constant elasticity structure, i.e., $\theta(w) = \vartheta w^k$ where $\vartheta, k > 0$ and $k = \frac{w\theta'(w)}{\theta(w)} > 0$ is the elasticity of $\theta(w)$ with respect to w; using (74), lemma 11 in Appendix B shows that if $k < 1 + \frac{\sigma}{1-\sigma}$, then, there exists a unique $w^H \geqslant \phi_2$ which solves (74) and that w^H is a strictly increasing function of A. Let this function be denoted by $w^H(A)$. Since we assume that $0 < k \leqslant \frac{\sigma}{1-\sigma} < 1 + \frac{\sigma}{1-\sigma}$ this result holds for all k in the assumed bounds. Then for any $A \geqslant \hat{A}$ we have that:

$$A \geqslant \hat{A} \Leftrightarrow w^H(A) \equiv w \geqslant \hat{w} \Leftrightarrow \theta(w) \geqslant \frac{A_L}{A_H}$$

In other words, the corner case $c_L = 0, c_H > 0$ is a competitive equilibrium if and only if $A \geqslant \hat{A}$.

□

Proof of Lemma 4:

Proof. These results follow from Lemmas 1, 2 and 3.

In Lemma 1 we showed that for every A, there exists a unique w^L that solves (11) and that $w^L(A)$ exists and is a continuous, strictly increasing, strictly concave function of A. Moreover, since this corner case is an equilibrium for $A \leqslant \hat{A}$, then $w^*(A) = w^L(A)$ for $A \leqslant \hat{A}$.

Similarly, in Lemma 3 we showed that under the assumption that $\theta(w) = \vartheta w^k$, with $\vartheta > 0$ and $0 < k \leqslant \frac{\sigma}{1-\sigma}$, for every A, there exists a unique w^H that solves (12) and that $w^H(A)$ exists and is a continuous, strictly increasing, strictly concave function of A. Moreover, since this corner case is an equilibrium from $A \geqslant \hat{A}$, then $w^*(A) = w^H(A)$ for $A \geqslant \hat{A}$.

Furthermore, by Lemma 2, since for $A = \hat{A}$ we have an interior equilibrium with $\theta(w) = \frac{A_L}{A_H}$, equations 11 and 12 are identical, hence $w^L(\hat{A}) = w^H(\hat{A})$. Then $w^*(A)$ is a continuous, strictly increasing function in A. Note, however, that $w^*(A)$ has a kink at $A = \hat{A}$.

Finally since for $A < \hat{A}$ we have $\theta(w) < \frac{A_L}{A_H}$ then $w^H(A) < w^L(A)$, and for $A > \hat{A}$ we have $\theta(w) > \frac{A_L}{A_H}$ then $w^H(A) > w^L(A)$.

□

Proof of Lemma 5:

Proof. The conditions, (13), (14) and (15) follow from the competitive equilibrium conditions (53), (54), and (50) respectively, i.e., the first order conditions for firms.

From Lemma 4, since $w^*(A)$ continuous in A then so are $p^*_L(A), p^*_H(A), p^*_N(A)$. Since $w^*(A)$ is strictly increasing in A then so is $p^*_N(A)$.

Moreover, Lemma 12 in Appendix B establishes that $\frac{w^*(A)}{A}$ is strictly decreasing in A. Then $p^*_L(A), p^*_H(A)$ are strictly decreasing in A.

□

Proof of Proposition 2:

Proof. For any $A \leqslant \hat{A}$, a country produces only the low-quality variety ($c_L > 0, c_H = 0$). From (16), the price index is:

$$P^*_{NT}(A) = \left(\frac{\phi_2}{w^*(A)}\right) p^*_N(A) + \left(1 - \frac{\phi_2}{w^*(A)}\right) p^*_L(A)$$

By Lemmas 4 and 5, since the equilibrium wage and price functions are continuous, the left-hand limit is:

$$\lim_{A \to \hat{A}^-} P^*_{NT}(A) = \left(\frac{\phi_2}{w^*(\hat{A})}\right) p^*_N(\hat{A}) + \left(1 - \frac{\phi_2}{w^*(\hat{A})}\right) p^*_L(\hat{A})$$

For any $A \geqslant \hat{A}$, a country produces only the high-quality variety ($c_H > 0, c_L = 0$). The price index is:

$$P^*_{NT}(A) = \left(\frac{\phi_2}{w^*(A)}\right) p^*_N(A) + \left(1 - \frac{\phi_2}{w^*(A)}\right) p^*_H(A)$$

The right-hand limit is therefore:

$$\lim_{A \to \hat{A}^+} P^*_{NT}(A) = \left(\frac{\phi_2}{w^*(\hat{A})}\right) p^*_N(\hat{A}) + \left(1 - \frac{\phi_2}{w^*(\hat{A})}\right) p^*_H(\hat{A})$$

Let $\hat{w} = w^*(\hat{A})$. From the firms' first-order conditions, also summarized in Lemma 5, we have $p_L^*(\hat{A}) = \frac{\hat{w}}{\hat{A}A_L}$ and $p_H^*(\hat{A}) = \frac{\hat{w}}{\hat{A}A_H}$. Then

$$A_H < A_L, \Leftrightarrow p_H^*(\hat{A}) > p_L^*(\hat{A})$$

Since the expenditure share on the quality-differentiated good, $(1 - \phi_2/\hat{w})$, is positive, it follows that:

$$\lim_{A \to \hat{A}^+} P_{NT}^*(A) > \lim_{A \to \hat{A}^-} P_{NT}^*(A)$$

The magnitude of the price jump is the difference between the right-hand and left-hand limits:

$$\begin{aligned}
\lim_{A \to \hat{A}^+} P_{NT}^*(A) - \lim_{A \to \hat{A}^-} P_{NT}^*(A) &= \left[\left(\frac{\phi_2}{\hat{w}}\right) p_N^*(\hat{A}) + \left(1 - \frac{\phi_2}{\hat{w}}\right) p_H^*(\hat{A})\right] - \left[\left(\frac{\phi_2}{\hat{w}}\right) p_N^*(\hat{A}) + \left(1 - \frac{\phi_2}{\hat{w}}\right) p_L^*(\hat{A})\right] \\
&= \left(1 - \frac{\phi_2}{\hat{w}}\right)\left(p_H^*(\hat{A}) - p_L^*(\hat{A})\right) \\
&= \left(1 - \frac{\phi_2}{\hat{w}}\right)\frac{\hat{w}}{\hat{A}}\left(\frac{A_L - A_H}{A_L A_H}\right)
\end{aligned}$$

This expression shows that the jump is strictly positive if and only if $A_L > A_H$. The magnitude is directly and positively related to the productivity difference $(A_L - A_H)$. If $A_L = A_H$, the jump is zero, and the price level is continuous at $\hat{A}$. □

Lemma 10. *Consider the equation* $A = \tilde{\phi}_L(w - \phi_2)^{\frac{\sigma}{1-\sigma}} w \equiv f(w)$, *where* $\tilde{\phi}_L > 0$, $\phi_2 > 0$, *and* $0 < \sigma < 1$. *For every* $A \geqslant 0$, *there exists a unique* $w^L \geqslant \phi_2$ *such that* $f(w^L) = A$, *and* $w^L = f^{-1}(A)$ *is continuous and strictly increasing in A.*

Proof. We show that $f(w)$ is a bijection from $[\phi_2, \infty)$ to $[0, \infty)$.

First, $f(w)$ is continuous on $[\phi_2, \infty)$. At $w = \phi_2$, $f(\phi_2) = 0$, and as $w \to \infty$, $f(w) \approx \tilde{\phi}_L w^{\frac{1}{1-\sigma}} \to +\infty$ since $\frac{1}{1-\sigma} > 1$ (given $0 < \sigma < 1$). Thus, $f(w)$ covers $[0, \infty)$.

Next, $f(w)$ is strictly increasing. For $w > \phi_2$, the derivative is:

$$f'(w) = \tilde{\phi}_L(w - \phi_2)^{\frac{2\sigma-1}{1-\sigma}}\left[\frac{1}{1-\sigma}w - \phi_2\right].$$

Since $w > \phi_2$, $\frac{1}{1-\sigma}w - \phi_2 > 0$ (as $\frac{1}{1-\sigma} > 1$), and $(w - \phi_2)^{\frac{2\sigma-1}{1-\sigma}} > 0$, so $f'(w) > 0$. At $w = \phi_2$, $f(w) = 0$ uniquely, and $f(w)$ increases thereafter.

Since $f(w)$ is continuous and strictly increasing, it is a bijection from $[\phi_2, \infty)$ to $[0, \infty)$. Thus, for every $A \geqslant 0$, there exists a unique $w^L \geqslant \phi_2$ such that $A = f(w^L)$.

Finally, since $f(w)$ is continuous and strictly increasing on $[\phi_2, \infty)$, and differentiable with $f'(w) > 0$ for $w > \phi_2$, the inverse function theorem implies that $w^L = f^{-1}(A)$ exists and is continuous and strictly increasing on $[0, \infty)$. □

Lemma 11. *Consider the equation* $A = \tilde{\phi}_H (w - \phi_2)^{\frac{\sigma}{1-\sigma}} \frac{w}{\theta(w)} \equiv g(w)$, *where* $\tilde{\phi}_H > 0$, $0 < \phi_2 < 1$, $0 < \sigma < 1$, *and* $\theta(w) = \vartheta w^k$ *with* $\vartheta > 0$ *and* $k > 0$. *For every* $A \geqslant 0$, *there exists a unique* $w^H \geqslant \phi_2$ *such that* $g(w^H) = A$, *and* $w^H = g^{-1}(A)$ *is continuous and strictly increasing in* A, *provided* $k < 1 + \frac{\sigma}{1-\sigma}$.

Proof. Substituting $\theta(w)$ into $g(w)$ we have that $g(w) = \frac{\tilde{\phi}_H}{\vartheta}(w - \phi_2)^{\frac{\sigma}{1-\sigma}} w^{1-k}$ for $w \geqslant \phi_2$. We show that $f(w)$ is a bijection from $[\phi_2, \infty)$ to $[0, \infty)$ if $k < 1 + \frac{\sigma}{1-\sigma}$.

Since $\phi_2 > 0$, $k > 0$, and $\vartheta > 0$, $g(w)$ is continuous on $[\phi_2, \infty)$.

Evaluate the range:

- At $w = \phi_2$, $g(\phi_2) = \frac{\tilde{\phi}_H}{\vartheta}(0)^{\frac{\sigma}{1-\sigma}} \phi_2^{1-k} = 0$.
- As $w \to \infty$, $g(w) \approx \frac{\tilde{\phi}_H}{\vartheta} w^{\frac{\sigma}{1-\sigma}+1-k}$. The exponent $\frac{\sigma}{1-\sigma} + 1 - k = \frac{1-k(1-\sigma)}{1-\sigma} > 0$ if $k < 1 + \frac{\sigma}{1-\sigma}$, so $g(w) \to +\infty$.

Check monotonicity:

$$g'(w) = \frac{\tilde{\phi}_H}{\vartheta}(w - \phi_2)^{\frac{\sigma}{1-\sigma}-1} w^{-k} \left[\frac{\sigma}{1-\sigma} w + (1-k)(w - \phi_2) \right].$$

Since $\frac{\tilde{\phi}_H}{\vartheta}(w - \phi_2)^{\frac{\sigma}{1-\sigma}-1} w^{-k} > 0$ for $w > \phi_2$, $g'(w) > 0$ if:

$$\frac{\sigma}{1-\sigma} w + (1-k)(w - \phi_2) > 0.$$

For large w, this approximates $\frac{\sigma}{1-\sigma} + 1 - k = \frac{1-k(1-\sigma)}{1-\sigma} > 0$, which holds when $k < 1 + \frac{\sigma}{1-\sigma}$. Near $w = \phi_2^+$, the term remains positive under this bound.

Thus, if $k < 1 + \frac{\sigma}{1-\sigma}$, $g(w)$ is continuous, strictly increasing, it is a bijection from $[\phi_2, \infty)$ to $[0, \infty)$. Thus, for every $A \geqslant 0$, there exists a unique $w^H \geqslant \phi_2$ such that $A = g(w^H)$.

Finally, if $k < 1 + \frac{\sigma}{1-\sigma}$, $g(w)$ is continuous and strictly increasing on $[\phi_2, \infty)$, and differentiable with $g'(w) > 0$ for $w > \phi_2$, the inverse function theorem implies that $w^H = g^{-1}(A)$ exists and is continuous and strictly increasing on $[0, \infty)$. □

Lemma 12. $\frac{w^*(A)}{A}$ *is strictly decreasing in* A.

Proof. For $A \leqslant \hat{A}$, $w^*(A) = w^L(A) \equiv w$, where w solves:

$$A = \tilde{\phi}_L (w - \phi_2)^{\frac{\sigma}{1-\sigma}} w$$

Taking log both sides and differentiating wrt A:

$$\begin{aligned}
\ln A &= \ln \tilde{\phi}_L + \ln w + \frac{\sigma}{1-\sigma} \ln(w - \phi_2), \\
\frac{1}{A} &= \frac{1}{w}\frac{dw}{dA} + \frac{\sigma}{(1-\sigma)(w-\phi_2)}\frac{dw}{dA}, \\
\frac{dw}{dA} &= \frac{1}{A\left(\frac{1}{w} + \frac{\sigma}{(1-\sigma)(w-\phi_2)}\right)}, \\
\frac{dw}{dA} &= \frac{w(1-\sigma)(w-\phi_2)}{A(w-(1-\sigma)\phi_2)}.
\end{aligned}$$

Solving for $\frac{d}{dA}\left(\frac{w}{A}\right)$:

$$\begin{aligned}
\frac{d}{dA}\left(\frac{w}{A}\right) &= \frac{A\frac{dw}{dA} - w}{A^2} = \frac{\left(\frac{w(1-\sigma)(w-\phi_2)}{w-(1-\sigma)\phi_2}\right) - w}{A^2}, \\
&= \frac{-\sigma w^2}{A^2(w-(1-\sigma)\phi_2)}
\end{aligned}$$

Since $w > \phi_2$, $(1 - \sigma < 1)$, the denominator $A^2(w - (1-\sigma)\phi_2)$ is positive, while the numerator $-\sigma w^2$ is negative. Therefore:

$$\frac{d}{dA}\left(\frac{w}{A}\right) < 0.$$

Then $\frac{w^*(A)}{A}$ is strictly decreasing for $0 < A \leqslant \hat{A}$.

For $A \geqslant \hat{A}$, $w^*(A) = w^H(A) \equiv w$, where w solves:

$$A = \frac{\tilde{\phi}_H}{\vartheta}(w - \phi_2)^{\frac{\sigma}{1-\sigma}} w^{1-k}$$

Taking logs both sides and differentiating wrt A:

$$\begin{aligned}
\ln A &= \ln C + \frac{\sigma}{1-\sigma}\ln(w-\phi_2) + (1-k)\ln w, \\
\frac{1}{A} &= \frac{\sigma}{1-\sigma}\frac{1}{w-\phi_2}\frac{dw}{dA} + (1-k)\frac{1}{w}\frac{dw}{dA}, \\
\frac{dw}{dA} &= \frac{1}{A\left(\frac{\sigma}{(1-\sigma)(w-\phi_2)} + \frac{1-k}{w}\right)}.
\end{aligned}$$

Solving for $\frac{d}{dA}\left(\frac{w}{A}\right)$:

$$\frac{d}{dA}\left(\frac{w}{A}\right) = \frac{A\frac{dw}{dA} - w}{A^2} = \frac{\frac{1}{\left(\frac{\sigma}{(1-\sigma)(w-\phi_2)} + \frac{1-k}{w}\right)} - w}{A^2},$$
$$= \frac{w\left(-\sigma w + k(1-\sigma)(w-\phi_2)\right)}{A^2\left(\sigma w + (1-k)(1-\sigma)(w-\phi_2)\right)}.$$

The denominator is always positive if:

$$A^2\left(\sigma w + (1-k)(1-\sigma)(w-\phi_2)\right) > 0,$$

$$k < 1 + \frac{\sigma}{1-\sigma}\frac{w}{w-\phi_2}.$$

Since by assumption $0 < k < \frac{\sigma}{1-\sigma}$ and we have that $w > \phi_2$ then the denominator is always positive. The sign of $\frac{d}{dA}\left(\frac{w}{A}\right)$ depends on the numerator:

$$N(w) = -\sigma w + k(1-\sigma)(w-\phi_2).$$

Then for $0 < k \leqslant \frac{\sigma}{1-\sigma}$, $N(w) < 0$. Therefore:

$$\frac{d}{dA}\left(\frac{w}{A}\right) < 0.$$

Then $\frac{w^*(A)}{A}$ is strictly decreasing for $\hat{A} \leqslant A$. □

C Quality Continuum Model: Competitive Equilibrium Solution

C.1 Household optimality

Household optimization involves maximizing utility, (19), with respect to $\{c_T, c_H(q)\}$, subject to the budget constraint, (22) and non-negativity constraints, $c_H(q) \geqslant 0$. The optimization problem yields the following first-order conditions:

$$c_T = \frac{1}{\lambda} \tag{75}$$

$$\phi_1\left(\int_0^1 \theta(q',w)c_H(q)dq'\right)^{-\sigma}\theta(q,w) + \mu_H(q) = \lambda p_H(q) \quad \forall q \in [0,1] \tag{76}$$

$$\mu_H(q)c_H(q) = 0, \quad c_H(q), \mu_H(q) \geqslant 0 \quad \forall q \in [0,1] \tag{77}$$

where $\lambda, \mu_H(q)$ are Lagrange multipliers. We can combine the first order conditions for varieties $(q, s) \in [0, 1]$ and express them as:

$$\frac{\theta(q, w)}{p_H(q)} + \tilde{\mu}_H(q) = \frac{\theta(s, w)}{p_H(s)} + \tilde{\mu}_H(s)$$

for all $(q, s) \in [0, 1]$, where $\tilde{\mu}_H(.)$ is the scaled Lagrange multiplier so that $\tilde{\mu}_H(q)c_H(q) = 0$ for all q. We consider a corner solution where the consumer consumes only variety q^* so that[9]:

$$c_H(q) = c_H(q^*)\delta(q - q^*) \quad \forall q \in [0, 1] \tag{78}$$

where $\delta(q - q^*)$ is a Dirac delta function. Since $c_H(q^*) > 0$, $\tilde{\mu}_H(q^*) = 0$ so that q^* satisfies:

$$q^* \in \text{argmax}_{q \in [0,1]} \frac{\theta(q, w)}{p_H(q)} \tag{79}$$

and

$$\phi_1 c_H(q^*)^{-\sigma} \theta(q^*, w)^{1-\sigma} = \lambda p_H(q^*) \tag{80}$$

C.2 Firm optimality

The firms' optimization problems, (4), (23), yield the following optimality conditions:

$$p_N = \frac{w}{A_N} \tag{81}$$

$$p_H(q) = \frac{w}{AA_H(q)} \quad \forall q \in [0, 1] \tag{82}$$

C.3 Equilibrium solution

Finally, we combine the household optimality conditions, (75), (78)-(80), with firm optimality conditions, (81), (82), and market clearing conditions, (24)-(26), (9), to obtain a system of equations that characterize the competitive equilibrium.

As in the baseline model, the non-traded goods market clearing conditions, (25)-(26) when combined with the firms' zero profit conditions and the household's budget constraint, (22), imply that the equilibrium outcome is an autarky, i.e., $c_T = y_T = 1$ for every country. Given, $\bar{c}_N$, the competitive equilibrium in $\{c_T, c_H(q), p_N, p_H(q), w, \ell_N, \ell_H(q), q^*\}$ is characterized by the following system of

[9] Given the perfect substitutability between varieties, given any income, w, and prices, $p_H(q)$, there always exists a corner solution where the household consumer only one variety that maximizes $\frac{\theta(q,w)}{p_H(q)}$, even though such a corner solution may not be unique.

equations:

$$c_T = 1 \tag{83}$$

$$p_N = \frac{w}{A_N} \tag{84}$$

$$\ell_N = \frac{\bar{c}_N}{A_N} \tag{85}$$

$$p_H(q) = \frac{w}{AA_H(q)} \tag{86}$$

$$\ell_H(q) = \frac{c_H(q)}{AA_H(q)} \tag{87}$$

$$c_H(q) = c_H(q^*)\delta(q - q^*) \tag{88}$$

$$\phi_1 c_H(q^*)^{-\sigma}\theta(q^*, w)^{1-\sigma} = p_H(q^*) \tag{89}$$

$$q^* \in \text{argmax}_{q' \in [0,1]} \frac{\theta(q', w)}{p_H(q')} \tag{90}$$

$$\int_0^1 \ell_H(q')dq' + \ell_N = 1 \tag{91}$$

D Quality Continuum Model: Proofs

Proof of Lemma 6:

Proof. Under the functional forms assumed, from (28), define the objective function as:

$$g(w, q) = w^{\frac{\sigma}{1-\sigma}q - 1}(1 - a_0 q),$$

which is continuous on $q \in [0, 1]$, a compact set, so a maximum exists for each $w > 0$. We find the maximizer by examining the interior and boundaries.

For $q \in (0, 1)$, the FOC is:

$$w^{\frac{\sigma}{1-\sigma}q-1} \ln w \frac{\sigma}{1-\sigma}(1 - a_0 q) - w^{\frac{\sigma}{1-\sigma}q-1}(a_0) = 0.$$

which yields

$$q = \frac{1}{a_0} - \frac{(1-\sigma)}{\sigma \ln w}$$

The FOC holds for all $q \in (0, 1)$, i.e., $\frac{1}{a_0} - \frac{(1-\sigma)}{\sigma \ln w} \in (0, 1)$ which holds for $\underline{w} < w < \overline{w}$.

- Lower bound:

$$\frac{1}{a_0} - \frac{1-\sigma}{\sigma \ln w} = 0,$$

$$\underline{w} = e^{\frac{a_0(1-\sigma)}{\sigma}} > 1.$$

- Upper bound:

$$\frac{1}{a_0} - \frac{1-\sigma}{\sigma \ln w} = 1,$$

$$\overline{w} = e^{\frac{a_0(1-\sigma)}{\sigma(1-a_0)}}.$$

Since $1 - a_0 < 1$, $\frac{a_0(1-\sigma)}{\sigma(1-a_0)} > \frac{a_0(1-\sigma)}{\sigma}$, so $\overline{w} > \underline{w}$.

Second derivative check:

$$\frac{\partial^2 g}{\partial q^2} = w^{\frac{\sigma}{1-\sigma}q-1}\left(-\frac{\sigma}{1-\sigma}\ln w a_0\right)$$

Since $\ln w > 0$ for $1 < \underline{w} < \overline{w}$, $\frac{\partial^2 g}{\partial q^2} < 0$ which confirms a unique maximum in the interior.

For $w \leqslant \underline{w}$, $\frac{\partial g}{\partial q} = w^{\frac{\sigma}{1-\sigma}q-1}\left[\frac{\sigma}{1-\sigma}\ln w(1-a_0 q) - a_0\right] < 0$ for all $0 < q < 1$ when $\frac{\sigma}{1-\sigma}\ln w(1-a_0 q) < a_0$. Since $w \leqslant \underline{w}$, $\ln w \leqslant \frac{a_0(1-\sigma)}{\sigma}$, and for $q > 0$:

$$\frac{\sigma}{1-\sigma}\ln w(1-a_0 q) \leqslant \frac{\sigma}{1-\sigma} \cdot \frac{a_0(1-\sigma)}{\sigma}(1-a_0 q) = a_0(1-a_0 q) < a_0,$$

so $\frac{\partial g}{\partial q} < 0$. Thus, $g(w, q)$ is strictly decreasing between $g(w, 0)$ and $g(w, 1)$ for all $w \leqslant \underline{w}$, so that $q^* = 0$ is the unique maximizer.

For $w \geqslant \overline{w}$, $\frac{\partial g}{\partial q} = w^{\frac{\sigma}{1-\sigma}q-1}\left[\frac{\sigma}{1-\sigma}\ln w(1-a_0 q) - a_0\right] > 0$ for all $0 < q < 1$ when $\frac{\sigma}{1-\sigma}\ln w(1-a_0 q) > a_0$. At $w = \overline{w}$ and $q = 1 - \epsilon$:

$$\frac{\sigma}{1-\sigma} \cdot \frac{a_0(1-\sigma)}{\sigma(1-a_0)}(1 - a_0(1-\epsilon)) = a_0 + \frac{a_0^2 \epsilon}{1-a_0} > a_0,$$

for small $\epsilon > 0$, Thus, $g(w, q)$ is strictly increasing between $g(w, 0)$ and $g(w, 1)$ for all $w \geqslant \overline{w}$, so that $q^* = 1$ is the unique maximizer.

Therefore:

$$q^*(w) = \begin{cases} 0 & \text{if } w \leqslant \underline{w}, \\ \frac{1}{a_0} - \frac{1-\sigma}{\sigma \ln w} & \text{if } \underline{w} < w < \overline{w}, \\ 1 & \text{if } w \geqslant \overline{w}, \end{cases} \tag{92}$$

where $\underline{w} = e^{\frac{a_0(1-\sigma)}{\sigma}}$ and $\overline{w} = e^{\frac{a_0(1-\sigma)}{\sigma(1-a_0)}}$.

Moreover, as $w \to \underline{w}^+$, $\frac{1}{a_0} - \frac{1-\sigma}{\sigma \ln w} \to 0$ and as $w \to \overline{w}^-$, $\frac{1}{a_0} - \frac{1-\sigma}{\sigma \ln w} \to 1$. So $q^*(w)$ is continuous in

w.

Finally, for $\underline{w} < w < \overline{w}$, $q^{*\prime}(w) = \frac{1-\sigma}{\sigma}\frac{1}{(\ln w)^2}\frac{1}{w} > 0$ so that $q^*(w)$ is strictly increasing in w. □

Proof of Lemma 7:

Proof. Under the functional forms assumed, from (27), we have:

$$w = \left(\frac{\phi_1}{\bar{\ell}\sigma}\right)^{\frac{1}{1-\sigma q^*}} (A(1-a_0 q^*))^{\frac{1-\sigma}{1-\sigma q^*}} \tag{93}$$

From (92), consider the inverse of $q^*(w)$ for $0 < q^* < 1$:

$$w = e^{\frac{a_0(1-\sigma)}{\sigma(1-a_0 q^*)}}$$

substituting this into (93) we get:

$$e^{\frac{a_0(1-\sigma)}{\sigma(1-a_0 q^*)}} = \left(\frac{\phi_1}{\bar{\ell}\sigma}\right)^{\frac{1}{1-\sigma q^*}} (A(1-a_0 q^*))^{\frac{1-\sigma}{1-\sigma q^*}},$$

$$A = \frac{1}{1-a_0 q^*}\left[e^{\frac{a_0(1-\sigma q^*)}{\sigma(1-a_0 q^*)}}\left(\frac{\bar{\ell}\sigma}{\phi_1}\right)^{\frac{1}{1-\sigma}}\right]$$

As $q^* \to 0^+$

$$A \to e^{\frac{a_0}{\sigma}}\left(\frac{\bar{\ell}\sigma}{\phi_1}\right)^{\frac{1}{1-\sigma}} \equiv \underline{A}$$

As $q^* \to 1^-$:

$$A \to \frac{1}{1-a_0}\left[e^{\frac{a_0(1-\sigma)}{\sigma(1-a_0)}}\left(\frac{\bar{\ell}\sigma}{\phi_1}\right)^{\frac{1}{1-\sigma}}\right] \equiv \overline{A}.$$

We have $0 < \underline{A} < \overline{A} < \infty$.

Define:

$$h(q) = \frac{1}{1-a_0 q}\left[e^{\frac{a_0(1-\sigma q)}{\sigma(1-a_0 q)}}\left(\frac{\bar{\ell}\sigma}{\phi_1}\right)^{\frac{1}{1-\sigma}}\right]$$

First, $h(q)$ is continuous in q for $q \in [0,1]$ with $h(0) = \underline{A}$ and $h(1) = \overline{A}$.

Let $C = \left(\frac{\bar{\ell}\sigma}{\phi_1}\right)^{\frac{1}{1-\sigma}}$ and let $f(q) = \frac{a_0(1-\sigma q)}{\sigma(1-a_0 q)}$. The derivative of $h(q)$ is:

$$h'(q) = C \cdot \frac{e^{f(q)}}{1-a_0 q} \cdot \left[\frac{a_0}{1-a_0 q} + f'(q)\right],$$

where:

$$f'(q) = \frac{a_0(a_0 - \sigma)}{\sigma(1 - a_0 q)^2}.$$

Then:

$$h'(q) = C \cdot e^{f(q)} \cdot \frac{a_0^2(1 - \sigma q)}{\sigma(1 - a_0 q)^3} > 0$$

Therefore $h(q)$ is strictly increasing in q for $q \in (0, 1)$.

Since $h(q)$ is continuous, strictly increasing, it is a bijection. By the Inverse Function Theorem, there exists an inverse $h^{-1}(A) \equiv q^*(A)$ that is continuous and strictly increasing on $(\underline{A}, \overline{A})$, mapping onto $(0, 1)$. From the inverse of (92), define

$$w(q^*(A)) = e^{\frac{a_0(1-\sigma)}{\sigma(1-a_0 q^*(A))}}$$

Then $w(q^*(A))$ is continuous and strictly increasing in A for all $A \in (\underline{A}, \overline{A})$.

For $q^* = 0$, from (92) we have $w \leqslant \underline{w}$. From (93), we have:

$$w = \left(\frac{\phi_1}{\bar{\ell}^\sigma}\right)(A)^{1-\sigma}$$

Since $w \leqslant \underline{w} = e^{\frac{a_0(1-\sigma)}{\sigma}}$. Then:

$$\left(\frac{\phi_1}{\bar{\ell}^\sigma}\right)(A)^{1-\sigma} \leqslant e^{\frac{a_0(1-\sigma)}{\sigma}},$$

$$A \leqslant \left(\frac{\bar{\ell}^\sigma}{\phi_1}\right)^{\frac{1}{1-\sigma}} e^{\frac{a_0}{\sigma}} = \underline{A}$$

Then for all $A \leqslant \underline{A}$, $q^*(A) = 0$ and $w(q^*(A)) = \left(\frac{\phi_1}{\bar{\ell}^\sigma}\right)(A)^{1-\sigma}$ where $w(q^*(A))$ is continuous and strictly increasing in A.

Similarly, for $q^* = 1$, from (92) we have $w \geqslant \overline{w}$. From (93), we have:

$$w = \left(\frac{\phi_1}{\bar{\ell}^\sigma}\right)^{\frac{1}{1-\sigma}} (A(1 - a_0))$$

Since $w \geqslant \overline{w} = e^{\frac{a_0(1-\sigma)}{\sigma(1-a_0)}}$. Then:

$$\left(\frac{\phi_1}{\bar{\ell}^\sigma}\right)^{\frac{1}{1-\sigma}} (A(1 - a_0)) \geqslant e^{\frac{a_0(1-\sigma)}{\sigma(1-a_0)}},$$

$$A \geqslant \frac{1}{1 - a_0}\left[e^{\frac{a_0(1-\sigma)}{\sigma(1-a_0)}} \left(\frac{\bar{\ell}^\sigma}{\phi_1}\right)^{\frac{1}{1-\sigma}}\right] = \overline{A}$$

Then for all $A \geqslant \overline{A}$, $q^*(A) = 1$ and $w(q^*(A)) = \left(\frac{\phi_1}{\bar{\ell}^\sigma}\right)^{\frac{1}{1-\sigma}} (A(1 - a_0))$ where $w(q^*(A))$ is continuous and strictly increasing in A.

Therefore, for all $A > 0$, $q^*(A)$ is a continuous function and strictly increasing in A for $A \in (\underline{A}, \overline{A})$ and $w(q^*(A))$ is continuous, strictly increasing in A for all $A > 0$. □

Proof of Lemma 8:

Proof. For $A < \underline{A}$ we have $q^*(A) = 0$ and using (31) we have:

$$\frac{w(q^*(A))}{A} = \left(\frac{\phi_1}{\bar{\ell}^\sigma}\right)(A)^{-\sigma}$$

which is strictly decreasing in A as $\sigma < 1$.

For $A > \overline{A}$ we have $q^*(A) = 1$ and using (31) we have:

$$\frac{w(q^*(A))}{A} = \left(\frac{\phi_1}{\bar{\ell}^\sigma}\right)^{\frac{1}{1-\sigma}}(1 - a_0)$$

which is constant in A.

For $\underline{A} < A < \overline{A}$, we have $0 < q^*(A) < 1$ and from (31) we have:

$$w(q^*(A)) = e^{\frac{a_0(1-\sigma)}{\sigma(1-a_0 q^*(A))}}$$

where $q^*(A)$ solves

$$A = \frac{1}{1 - a_0 q^*(A)} e^{\frac{a_0(1-\sigma q^*(A))}{\sigma(1-a_0 q^*(A))}} \left(\frac{\bar{\ell}^\sigma}{\phi_1}\right)^{\frac{1}{1-\sigma}}$$

From Lemma 7, we know that $\frac{dq^*(A)}{dA} > 0$ for $\underline{A} < A < \overline{A}$. Combining the above we have that:

$$\frac{A}{w(q^*(A))} \equiv g(q^*(A)) = \frac{1}{1 - a_0 q^*(A)} e^{\frac{a_0(1-q^*(A))}{1-a_0 q^*(A)}} \left(\frac{\bar{\ell}^\sigma}{\phi_1}\right)^{\frac{1}{1-\sigma}}$$

Taking logs and differentiating:

$$\frac{d}{dq}\log g(q) = \frac{a_0}{1 - a_0 q} + \frac{d}{dq}\left(\frac{a_0(1-q)}{1 - a_0 q}\right) = \frac{a_0^2(1-q)}{(1 - a_0 q)^2} > 0 \quad \text{for } q \in (0, 1)$$

Then:

$$\frac{d}{dA}\log g(q^*(A)) = \frac{d}{dq}\log g(q) \cdot \frac{dq^*}{dA} > 0 \quad \text{for } q^*(A) \in (0, 1).$$

Therefore, $\frac{w(q^*(A))}{A} = \frac{1}{g(q^*(A))}$ is strictly decreasing in A for $\underline{A} < A < \overline{A}$.

□

Proof of Lemma 9:

Proof. Under the functional forms assumed, from (86), we have:

$$p_H(q^*(A), A) = \frac{w(q^*(A))}{A(1 - a_0 q^*(A))}$$

From Lemma 7, for $A \in (\underline{A}, \overline{A})$, we have:

$$w(q^*(A)) = e^{\frac{a_0(1-\sigma)}{\sigma(1-a_0 q^*(A))}}$$

and $q^*(A)$ is such that

$$A(1 - a_0 q^*(A)) = e^{\frac{a_0(1-\sigma q^*(A))}{\sigma(1-a_0 q^*(A))}} \left(\frac{\overline{\ell}^\sigma}{\phi_1}\right)^{\frac{1}{1-\sigma}}$$

Combining these, we get:

$$p_H(q^*(A), A) = \left(\frac{\phi_1}{\overline{\ell}^\sigma}\right)^{\frac{1}{1-\sigma}} e^{\frac{a_0(q^*(A)-1)}{(1-a_0 q^*(A))}} \tag{94}$$

Then:

$$\frac{dp_H(q^*(A), A)}{dA} = \left(\frac{\phi_1}{\overline{\ell}^\sigma}\right)^{\frac{1}{1-\sigma}} e^{\frac{a_0(q^*(A)-1)}{(1-a_0 q^*(A))}} \cdot \frac{a_0(1 - a_0)}{\sigma(1 - a_0 q^*(A))^2} \cdot \frac{dq^*(A)}{dA} > 0$$

since $0 < a_0 < 1$ and $\frac{dq^*(A)}{dA} > 0$ from Lemma 7, for $\underline{A} < A < \overline{A}$. Therefore, $p_H(q^*(A))$ is continuous and strictly increasing in A for $A \in (\underline{A}, \overline{A})$.

For $A \leqslant \underline{A}$, $q^*(A) = 0$ and $w(q^*(A)) = \left(\frac{\phi_1}{\overline{\ell}^\sigma}\right)(A)^{1-\sigma}$. Then:

$$p_H(0, A) = \left(\frac{\phi_1}{\overline{\ell}^\sigma}\right)(A)^{-\sigma}$$

so that $\frac{dp_H(0,A)}{dA} < 0$.

Similarly, for $A \geqslant \overline{A}$, $q^*(A) = 1$ and $w(q^*(A)) = \left(\frac{\phi_1}{\overline{\ell}^\sigma}\right)^{\frac{1}{1-\sigma}}(A(1 - a_0))$. Then:

$$p_H(1, A) = \left(\frac{\phi_1}{\overline{\ell}^\sigma}\right)^{\frac{1}{1-\sigma}}$$

so that $p_H(1, A)$ is constant. □