Import Price Shocks, FX-Monetary Policies, and Real Income Stabilization

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Abstract

We characterize jointly optimal exchange rate and monetary policies under commitment in a heterogeneous-agent, import-dependent small open economy facing essential import price spikes. When some households are borrowing constrained, exchange rate management becomes a tool for real income stabilization through the intertemporal margin, requiring temporary interest parity deviations, while optimal monetary policy targets the labor wedge. In an open economy, these roles are not substitutable. A Ramsey planner internalizes how pecuniary general equilibrium effects from real exchange rate movements redistribute real incomes: facing a spike, the planner leans against the depreciation, cutting exports and shifting resources toward constrained households through higher real wages. Calibrated to Japan's 2022 energy price path, optimal policy departs from representative-agent prescriptions and uses FX interventions to cut the contemporaneous co-movement of the real exchange rate with world energy prices while accepting costly interest parity deviations. Monetary policy complements FX policy by taking a disinflationary stance.

Keywords: Foreign exchange interventions; optimal monetary policy; heterogeneous agents; import price shocks; real income channel; energy shocks.

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1 Introduction

Recent years have repeatedly confronted import-dependent economies with large temporary spikes in the world prices of essential imports. The 2022 energy crisis, the renewed volatility in crude oil and LNG prices in 2026, and the rare-earth disruptions of 2025 all had the same economic structure: essential goods that were difficult to substitute away from in the short run became abruptly more expensive. For an import-dependent country, this is a terms-of-trade shock: the country must export more domestic tradeables to generate the purchasing power required to buy any given amount of imports. Because these imports are necessities, their short-run demand is highly price inelastic: demand falls little when the price rises, and the import bill goes up. This results in a real income loss for the country: more domestic output must be exported rather than consumed domestically to finance the higher import bill.

How much more of its output the country must hand over depends on the price at which foreigners buy it. When a country faces a downward-sloping demand for its goods, the world price of domestic tradeables must fall to induce foreigners to buy more and supply the requisite purchasing power. Consequently, an exogenous rise in the world price of an essential import can be accompanied by an endogenous decline in the world price of domestic output, which compounds the real-income losses: the country pays more for what it buys and receives less for what it sells. In a monetary economy, such real income losses inevitably take the form of domestic price inflation and nominal and real exchange rate depreciations: a real depreciation makes domestic output cheaper for foreigners, who buy more of it, and the export revenues thus generated enable the country to pay for its imports.

In this paper we address a policy-relevant question that directly relates to the mandate of monetary authorities in many countries: how should monetary and exchange rate policies be conducted by import-dependent economies facing temporary spikes in essential import prices[1]? Since real exchange rate depreciations can amplify real income losses, should policymakers intervene in foreign exchange markets to resist depreciations, or should they let the real exchange rate adjust freely in response to such shocks? And what path of domestic inflation should be chosen while the shock lasts?

In the canonical representative agent small-open-economy New Keynesian framework, the con-

[1] Exchange rate policy is a broad term. We define this policy as one where the monetary authority distorts cross-border financial flows to influence the path of the real exchange rate. This can be done through outright capital controls or foreign exchange interventions.

ventional policy prescription is to use monetary policy to target nominal rigidities while allowing the real exchange rate to move freely to clear the foreign exchange market. In this literature the relevant frictions are nominal rigidities and the associated labor market distortions, which give monetary policy its role; the depreciation-induced real income losses that accompany an import price spike create no separate role for exchange rate policy to distort the path of the real exchange rate. That prescription rests on the assumption that agents have perfect access to international asset markets. Asset markets determine how much of the increased import bill is paid at each date and, therefore, how much the real exchange rate depreciates at each date and how closely its path tracks the path of import prices. The real exchange rate must eventually depreciate, but it need not do so contemporaneously with the import price spike. By running current account deficits in high-price periods, the country can retain more of its output in these periods while exporting more in future when the shock has passed, thereby spreading the real income losses over time. A transitory import-price spike is then just a present-value real income loss. Policy cannot undo this loss, and has no separate reason to distort the timing of when the economy pays for it.

The recent heterogeneous-agent literature instead has departed from this assumption. When some households are liquidity constrained and cannot borrow against future incomes, the real income loss from an import price spike is no longer just a present value loss but also a current income loss: it falls on their real disposable income in the period the price rises. Restricted participation by households in asset markets then has general equilibrium consequences: constrained households would borrow against their future incomes during the spike if they could, and since they cannot, the economy borrows less in the aggregate than it would if every household were on its Euler path. More of the import bill must then be paid contemporaneously, more is exported in the high price periods, and the real exchange rate becomes overly correlated with import prices. The effective increase in the price of imports faced by the economy, in units of domestic output, is therefore amplified: not only does the exogenous world price go up, but the real exchange rate also depreciates simultaneously, and imports become dearer for both reasons. This feeds back and further erodes the real incomes of households, with the largest burden falling on those that are already borrowing constrained and consume out of current incomes.

The representative-agent prescription, monetary policy targeting nominal rigidities alongside an undistorted, freely floating exchange rate, need not remain optimal in this environment. When some households are borrowing constrained and cannot smooth income shocks intertemporally, how should exchange rate and monetary policies be conducted in response to temporary spikes in essential import prices? This paper studies this question in an import-dependent small open economy with heterogeneous households. Direct fiscal transfers are a natural way to redistribute real incomes; we study environments where that answer is unavailable: fiscal instruments capable of delivering lump-sum transfers to constrained households are assumed absent. Exchange rate and monetary policies cannot undo the real income loss in present-value terms; the question is whether they can influence how that loss is borne across households and across time, and to what extent they should.

Given this framework, this paper argues and illustrates that exchange rate policy takes on a prominent and distinctive role that it does not possess in a representative-agent world: stabilizing the real incomes of borrowing-constrained households. By altering the economy's net foreign asset position, directly through its own balance sheet or indirectly by changing the returns faced by participating households, the public agent can influence the path of the real exchange rate. Because the real exchange rate determines the purchasing power of domestic incomes over imports, this influence extends, through pecuniary general equilibrium effects, to the real incomes of all households, including those who are borrowing constrained. Facing a temporary import price spike, if additional borrowed foreign purchasing power is brought in while import prices are high and repaid after the shock has passed, less must be exported during the spike, the depreciation is smaller when the necessity is most expensive and larger after it has become cheap again, thereby reducing the excess co-movement of the real exchange rate with the shock. This lowers the effective price of imports in domestic units and raises the real incomes of households across the economy when the adverse shock hits. The gain matters most for borrowing-constrained households, who have high marginal utility in these periods and would like to consume more but cannot borrow to do so. The inflows reverse once the shock has passed: the exchange rate depreciates, exports expand, and real incomes are lowered as the economy pays for its imports. By tilting the real exchange rate path, policy shifts real incomes toward the high-price periods and away from the low-price periods, mitigates the depreciation-induced amplification, and spreads the real incomes and consumption of borrowing-constrained agents more evenly across the price cycle. We argue that a public agent who cares about the welfare of constrained households finds this policy desirable relative to the conventional prescription that leaves financial markets and the resulting exchange rate path undistorted. We identify this as the open economy real-income stabilization channel of exchange rate policy delivered through the policy's influence on the intertemporal margin.

Monetary policy's role also extends beyond targeting nominal rigidities. However, monetary policy cannot perform the intertemporal task. The terms on which the economy borrows abroad are set in world markets, and domestic interest rate policy does not move them; the re-timing described above requires an instrument that acts on the economy's foreign financial flows and that role can be performed by exchange rate policy. What monetary policy does move, where nominal rigidities give it traction, is current employment and output, and through them the earnings of constrained households. In the spirit of the heterogeneous-agent New Keynesian (HANK) monetary policy literature, this is the familiar real-income earnings channel of monetary policy, though here it operates through the intratemporal consumption-leisure margin rather than the intertemporal margin. In an import-dependent economy, we illustrate that the strength and direction of this channel depend on how complementary imports are to domestic employment. When the two move closely together, expanding employment in the high-price periods can erode, rather than support, the real incomes of constrained households, and optimal monetary policy turns out tighter, not looser, than conventional representative-agent prescriptions imply.

We formalize these arguments in a partially small open economy populated by two types of households: hand-to-mouth households, who consume their current income each period, and Ricardian households, who save and borrow freely in international asset markets. Firms produce the domestic tradeable good by combining labor with an essential imported input in fixed proportions, so that domestic activity and import demand move together, and the economy faces a downward-sloping, price-elastic export demand, so that export revenues in foreign units rise only with a real depreciation. Fiscal instruments capable of delivering lump-sum transfers to hand-to-mouth households are absent. In this environment we solve for jointly Ramsey-optimal monetary and exchange rate policies where monetary policy moves the labor wedge and exchange rate policy distorts the intertemporal margin. A two-period version of the model delivers the core results in closed form. We first study the model under inelastic labor supply, which shuts down the employment margin and isolates the role of exchange rate policy. With no hand-to-mouth households, optimal policy leaves the intertemporal margin undistorted for any path of import prices: this nests the conventional prescription as its representative-agent special case. With a positive mass of hand-to-mouth households and a temporary import price spike, optimal policy instead distorts the intertemporal margin and induces stronger capital inflows in high-price periods. The resulting real exchange rate is more appreciated in high-price periods and depreciates more once the shock has passed, and is therefore more volatile across periods, while the domestic units price of imported inputs, real incomes, and the consumption of hand-to-mouth households are all relatively stabilized.

The ability of policy to raise the real incomes of constrained households in the high-price periods, without transfers and without their participation in any asset market, rests on pecuniary general equilibrium effects. The intertemporal distortions, decentralized as a subsidy to Ricardian financial flows, induce them to borrow more and bring the borrowed foreign purchasing power to the foreign exchange market to bid for domestic output. Their bids crowd out foreigners and appreciate the real exchange rate, which lowers the domestic cost of the imported input and hence firms' marginal cost of production. Firms compete to expand imports and employment, and bid up real wages: the cost saving passes through into real wages. The wage gains accrue to all households, including hand-to-mouth households, whose consumption rises with their current labor income. Once the shock has passed and the borrowed resources are repaid, the same channel runs in reverse: the exchange rate depreciates, input costs rise, and real wages fall. Exchange rate policy thus uses the asset market access of Ricardian households to move the labor incomes of all households across periods, and it is this pass-through from the real exchange rate into real wages that delivers the real-income stabilization channel of exchange rate policy in our framework.

Allowing for an elastic labor supply introduces the employment margin and isolates the role of monetary policy. A familiar static terms-of-trade distortion gives monetary policy a role in this environment even without household heterogeneity: competitive firms do not internalize that their collective demand for imported inputs moves the real exchange rate against the economy, so employment and imports run higher than a price-taking economy would choose for itself, and optimal policy restrains them. This correction is already present in the representative-agent economy; what heterogeneity adds lies beyond this static correction. Because labor and the imported input are complementary, a marginal expansion of employment must be matched by additional imports; paying for the additional imports requires more export revenue, and hence a real depreciation. The depreciation creates a revaluation effect on Ricardian financial flows: in high import-price periods, when Ricardians are net borrowers, a depreciation revalues the real value of Ricardian financial flows so that the aggregate real income gains from higher employment are distributed disproportionately toward Ricardian households. The revaluation effect endogenously transfers purchasing power from the high-marginal-utility hand-to-mouth households towards the low-marginal-utility Ricardian households. A public agent who cares about the welfare of constrained households internalizes this endogenous redistribution and considers employment expansion in the high-price periods costlier than the representative-agent prescription does. Optimal monetary policy is therefore contractionary relative to the conventional stance.

We embed these arguments in an infinite-horizon two-agent quantitative model with nominal wage rigidities and financial intermediation frictions, which provides the exact decentralization of the intertemporal and labor wedges while preserving the economic intuition of the analytical framework. Monetary policy sets the nominal interest rate; exchange rate policy is implemented by a public agent that shifts its asset positions between domestic and foreign markets, so intertemporal distortions take the form of costly interest parity gaps. The quantitative model is calibrated to study optimal policies in response to energy price shocks; we calibrate the model to Japan, which is among the most energy-import-dependent countries in the world, and a clear case of exchange-rate-led amplification of domestic energy prices in the 2022 episode. We analyze perfect-foresight impulse responses and present a historical policy counterfactual on the realized 2022 path of world energy prices, which peaked 157 percent above its pre-shock level. We compare optimal policy with the conventional representative-agent prescription, which allows no intertemporal distortions and would be optimal were no household borrowing constrained. Followed in the heterogeneous economy, that prescription makes the real exchange rate co-move excessively with the world price: at the peak of the 2022 episode, 71 percent of the depreciation is attributable to borrowing constraints, measured against a counterfactual economy in which no household is constrained. Optimal policy instead accepts temporary, costly interest parity gaps and more than undoes the excess: it leans against the depreciation at the peak and turns the co-movement with world energy prices negative. The gains appear in domestic energy prices and real wages: at the peak, energy is 19.5 percent cheaper than under the conventional prescription and 29 percent of the peak real-wage loss is eliminated. Monetary policy complements the intervention: a disinflationary stance prevents an excessive expansion of employment and imports and holds the domestic price level at the peak 2.5 percent lower than under that prescription.

The rest of the paper proceeds as follows. Section 2 relates the paper to the literature. Section 3 presents the analytical model, characterizes the constrained-efficient allocation, and shows how monetary policy and foreign exchange intervention decentralize the two policy margins. Section 4 develops the quantitative model, calibrates it to imported energy shocks, and compares optimal policy with restricted policy regimes. Section 5 concludes.

2 Related Literature

How monetary and exchange rate policy should respond to import price shocks is a longstanding question in open economy macroeconomics. The conventional prescription grew out of the representative-agent small open economy New Keynesian framework of Galí and Monacelli (2005) and the classic case for flexible exchange rates (Friedman 1953): monetary policy addresses the distortions arising from nominal rigidities, international borrowing remains undistorted, and the exchange rate responds freely to shocks. With full participation in international asset markets, the depreciation that accompanies an import price spike is the expenditure switching required by a privately chosen current account path, and no separate role for exchange rate policy arises. The paper isolates the consequences of one departure from that prescription: unequal household access to intertemporal markets. When a positive mass of households is off its Euler equation and targeted transfers are unavailable, both the optimal external-financing policy and the optimal monetary stance change.

In heterogeneous-agent closed economies, monetary policy redistributes through unequal earnings exposure, the revaluation of nominal balance sheets, and unequal interest rate exposure, which move aggregate consumption when gains and losses fall on households with different marginal propensities to consume (Auclert 2019); more broadly, heterogeneous intertemporal responses alter the transmission of monetary policy to aggregate demand and employment (Kaplan, Moll and Violante 2018; Werning 2015; Auclert, Rognlie and Straub 2024). Dávila and Schaab (2023) characterize Ramsey-optimal policy when the planner internalizes these distributional consequences, and Acharya, Challe and Dogra (2023) derive optimal monetary policy in a tractable HANK economy that trades off consumption inequality, productive efficiency, and price stability. The earnings component of that transmission runs from the domestic real interest rate through aggregate demand to employment and real wages; the channel in this paper instead runs from external financial flows through foreign exchange market clearing to the real exchange rate, imported input costs, and the real wage, and it operates at a fixed level of employment. Nor can monetary policy perform this intertemporal task itself: the world return that governs external borrowing is beyond its reach, so reallocating the country's foreign purchasing power across dates requires an instrument that acts on external asset positions. On the earnings margin, imported-input complementarity adds a distributional cost to employment expansion, so the argument for supporting constrained households by raising employment need not carry over: in our environment the expansion's financing depreciates the exchange rate and shifts purchasing power away from constrained households.

A second literature establishes that exchange rates and external shocks have first-order distributional consequences in open economies with household heterogeneity. Cravino and Levchenko (2017) document that large devaluations fall hardest on low-income households. Auclert et al. (2021) show that the real income losses from depreciation weaken monetary transmission when short-run trade elasticities are low. de Ferra, Mitman and Romei (2020) study how household portfolio composition, leverage, and foreign-currency debt shape the transmission of a current-account reversal, and Oskolkov (2023) compares exchange rate regimes under household heterogeneity. Closest on the monetary side, Acharya and Challe (2025) characterize optimal monetary policy in a small open economy with uninsured income risk and unequal access to bond markets, where inefficient consumption inequality can call for more output and exchange rate stabilization than in the representative-agent economy, and Acharya, Challe and Corsetti (2026) study the optimal monetary stabilization of export shocks when incomplete insurance and imperfect worker mobility generate consumption inequality. These papers ask how heterogeneity changes what monetary policy alone should do, or how shocks transmit under given rules and regimes. Here the intertemporal task is assigned to a separate instrument: monetary policy controls employment, while foreign exchange intervention changes the timing of the foreign purchasing power that finances an essential import.

A third literature studies foreign exchange intervention when financial intermediation between home and foreign bond markets is imperfect, as in Gabaix and Maggiori (2015): Cavallino (2019) characterizes optimal intervention against capital flow shocks, Liu and Spiegel (2015) solve numerically for jointly optimal capital account restrictions and monetary policy, Amador et al. (2020) characterize the interventions required to sustain an exchange rate path at the zero lower bound, Bianchi and Coulibaly (2023) provide a theory of fear of floating, and Basu et al. (2020); International Monetary Fund (2023) integrate these instruments into unified policy frameworks. In these papers intervention addresses financial or aggregate distortions. Fanelli and Straub (2021), closest to this paper, derive a distributional motive: in their baseline, a world interest rate shock moves rich Ricardian spending, the relative price of tradables and nontradables turns against poor households with subsistence tradable needs, and optimal intervention moderates the resulting expenditure and exchange rate volatility. The two papers share a reduced-form portfolio-balance implementation with costly return spreads and the distributional use of the real exchange rate. The object of intervention differs: their planner restrains intertemporal movements in Ricardian spending after a capital flow shock; ours expands foreign borrowing during an import price spike, because the missing borrowing of constrained households leaves aggregate external finance too low. Here both household types consume the same good and the imported input enters production: borrowed foreign purchasing power replaces export revenue, the appreciation passes into real wages through firms' zero-profit condition, and monetary and exchange rate policy are chosen jointly, with the assignment of the labor and intertemporal margins itself the object of the characterization.

Itskhoki and Mukhin (2023), building on the exchange-rate disconnect of Itskhoki and Mukhin (2021), characterize jointly optimal exchange-rate and monetary policy in a representative-agent economy: monetary policy is assigned to domestic stabilization, and foreign exchange intervention uses the public balance sheet to remove the inefficient interest-parity deviations that intermediation creates; Egorov and Mukhin (2023) characterize optimal monetary policy under dollar pricing, also with a representative agent. Aktuğ and Rezghi (2026) study jointly optimal monetary and foreign exchange policy under oil shocks in a representative-agent economy with segmented financial markets, where intervention likewise closes the intermediation wedge. The representative-agent case of this paper shares this assignment: the public balance sheet undoes the financial friction and restores interest parity. Household heterogeneity reverses it: the planner here opens costly parity deviations deliberately, because they bring foreign purchasing power into the periods in which constrained households have high marginal utility. To our knowledge, this paper provides the first joint Ramsey characterization in which unequal household access to intertemporal markets itself generates the optimal intervention wedge.

The finite elasticity of export demand also connects the paper to optimal terms-of-trade and capital-control policy (Johnson 1953; Dixit and Norman 1980; Farhi and Werning 2014, 2016). Costinot, Lorenzoni and Werning (2014) derive capital controls from strategic manipulation of world intertemporal prices, and Dávila et al. (2025) study dynamic trade policy with heterogeneous households. In our economy, export-market power is corrected period by period through the labor wedge; the optimal intertemporal wedge is zero with a representative household for every import-price path and emerges only with unequal household access to intertemporal markets.

Finally, the macroeconomic effects of oil and energy price shocks and the monetary response to them have a long tradition (Bernanke, Gertler and Watson 1997; Blanchard and Galí 2007).

Drechsel et al. (2026) characterize welfare-optimal monetary and exchange rate frameworks for commodity-exposed economies with nominal rigidities. Chan, Diz and Kanngiesser (2024) study optimal monetary policy in a small open economy TANK model in which imported energy and labor are complementary production inputs: higher energy prices reduce constrained households' labor income and aggregate demand, leading optimal monetary policy to be less contractionary than in the representative-agent economy. Our monetary prescription differs once foreign exchange intervention is chosen jointly: employment expansion raises imported-input demand, induces depreciation, and changes the distribution of the resulting income gains. Auclert et al. (2023) study the fiscal and monetary response to the 2022 episode in an energy-importing heterogeneous-agent economy, where monetary tightening has limited influence over imported inflation and energy-price subsidies can insulate an individual importing economy while imposing adverse spillovers on other energy importers. In that policy debate, distributional protection is assigned to targeted fiscal transfers (Ari et al. 2022), and across OECD and European economies much of the support announced or implemented during the 2021–2022 energy crisis was broad-based rather than targeted (Sgaravatti, Tagliapietra and Zachmann 2023; Hemmerlé et al. 2023). When fiscal targeting is incomplete, the exchange rate itself shapes the incidence of the loss: the question here is the optimal timing of external finance when fiscal instruments cannot reach constrained households, a margin the conventional assignment leaves unused.

3 Analytical Framework

This section develops a two-period model of a small open economy that imports an essential intermediate input and is populated by households who are heterogeneous in their ability to participate in asset markets and smooth consumption intertemporally. A fraction of households are hand-to-mouth and consume their current income each period, while the remaining households can save and borrow freely in international asset markets. Given a temporary spike in import prices that erodes current household real incomes, we characterize optimal monetary and exchange rate management policies for this economy.

The framework presented in this section is tractable enough to allow for a closed-form characterization of optimal policies. We describe a utilitarian Ramsey planner's optimization problem with only two policy margins: the labor wedge and the intertemporal wedge. We interpret the labor wedge as the margin that monetary policy can move, and the intertemporal wedge as the margin that exchange rate policy can influence. While we leave the full decentralization with nominal rigidities and financial intermediation frictions to the quantitative model presented in Section 4, this section takes a simplified, reduced-form approach to policy implementation.

We proceed as follows. Section 3.1 describes the environment and defines the equilibrium given policy instruments. Section 3.2 sets up the Ramsey planner's problem. Section 3.3 characterizes optimal policy in the representative-agent special case as a benchmark. Section 3.4 turns to the heterogeneous-agent economy, first under inelastic labor supply to isolate the intertemporal dimension, then under elastic labor supply to characterize the jointly optimal intertemporal and labor wedges.

3.1 Environment

The economy operates over two periods, $t \in \{1, 2\}$. It produces a single final tradeable good and interacts with the rest of the world through three channels: it exports the domestic good, imports a productive input, and borrows or lends on international asset markets. We describe each component of the economy in turn.

3.1.1 Goods and Relative Prices

Three tradeable goods circulate in the world economy. The first is the domestically produced good, produced only in the small open economy. It is consumed by domestic households and exported to the rest of the world. Let C_t, X_t, and Y_t denote aggregate domestic consumption, exports, and output. The resource constraint is given by:

$$C_t + X_t = Y_t \tag{1}$$

The second good is a foreign consumption good. It represents the foreign consumption basket. Domestic households do not consume it, and domestic firms do not use it in production. Bonds traded in international asset markets are claims denominated in units of this good. Moreover, this good also serves as the world numeraire: we normalize its foreign currency price, $P_t^* = 1$ so that one unit of foreign currency equals one unit of this good, and all world prices are expressed in units of this good. The world gross real interest rate is assumed to be constant and denoted as R^*, i.e., one unit of the foreign good saved in period t returns R^* units in period $t + 1$. The economy is price-taking with respect to this interest rate.

The third good is an intermediate foreign good which the economy imports. The economy uses this good in the production of the domestic tradeable good. The world price of this input is denoted P_t^{T*} in units of the foreign good. The economy is also price-taking with respect to this relative price.

The domestically produced tradeable good is the only final good that the economy consumes. Let P_t denote the price of the domestic good in units of domestic currency and let e_t denote the nominal exchange rate, i.e., the price of the foreign currency in units of the domestic currency. With the world consumption good as the numeraire, e_t is also the price of the world consumption good in domestic currency units. Moreover, since the domestic economy only consumes the domestic good, P_t also denotes the domestic price level. Then let ϵ_t denote the real exchange rate, simply given by $\epsilon_t = \frac{e_t}{P_t}$. By the law of one price for tradeables, the inverse of the real exchange rate, $\frac{1}{\epsilon_t}$, is the world price of the domestic good in units of the world numeraire. The economy's terms of trade is then given by $\frac{P_t^{T*}}{\frac{1}{\epsilon_t}} = \epsilon_t P_t^{T*}$, which is the price of the imported input in units of the domestic good. Finally, let W_t denote the nominal wage and $w_t = \frac{W_t}{P_t}$ denote the real wage in units of the domestic good.

3.1.2 Production Technology

A unit mass of identical firms produces the domestic good, Y_t, using labor, N_t, and the imported intermediate input, M_t, according to a Leontief technology:

$$Y_t = A_t \cdot \min\left\{N_t, \frac{M_t}{\theta}\right\} \tag{2}$$

where $A_t > 0$ is total factor productivity and $\theta > 0$ is the input requirement per unit of labor employed. The Leontief specification captures an essential feature that motivates this paper: the imports are critical to the domestic economy, i.e., the imported and domestic inputs are strict complements in the short run, so that a rise in the input price cannot be offset by substituting toward domestic factors. With no substitution effects, relative price changes create pure income effects, which is helpful in isolating how real income effects affect the economy.

The firms' problem is to choose N_t and M_t to minimize the cost of producing Y_t subject to the production technology:

$$\min_{\{N_t, M_t\}} w_t N_t + \epsilon_t P_t^{T*} M_t \quad \text{s.t. (2)} \tag{3}$$

Given this Leontief technology, a cost-minimizing firm will always deploy the two inputs in fixed proportions to produce any given level of output:

$$M_t = \theta N_t \tag{4}$$

The output produced can be expressed as a linear function of labor input:

$$Y_t = A_t N_t \tag{5}$$

The firms' real marginal cost of production is therefore $\frac{w_t}{A_t} + \epsilon_t P_t^{T*} \frac{\theta}{A_t}$: each additional unit of output requires $\frac{1}{A_t}$ units of labor at wage w_t and $\frac{\theta}{A_t}$ units of the imported input at domestic cost $\epsilon_t P_t^{T*}$ per unit. Since firms are competitive, an equilibrium requires that firms make zero economic profits, i.e., the real marginal cost of production must equal one. This gives a linear expression for the real wage in terms of the price of the imported input and the real exchange rate:

$$w_t = A_t - \epsilon_t P_t^{T*} \theta \tag{6}$$

A_t denotes the total output produced per unit of labor employed, while $\epsilon_t P_t^{T*} \theta$ denotes the output paid by the firm to buy θ units of the imported input that together with a unit of labor produces A_t units of output. The residual, $A_t - \epsilon_t P_t^{T*} \theta$, is paid to labor. The real wage, therefore, has a linear one-for-one negative relationship with the price of the imported input and the real exchange rate. A rise in the price of the imported input or a real depreciation raises the marginal cost of production. At the prevailing wage, firms make losses and therefore decrease their demand for the input. Since inputs and labor are perfect complements, the demand for labor also falls. Competition for the released labor weakens until the wage has dropped one-for-one with the rise in input costs to restore zero profits. A fall in the price of the imported input or a real appreciation has the opposite effect: firms make positive profits and expand, bidding up wages until zero profits are restored.

Let Y_t^N denote the labor income that accrues to domestic workers in period t:

$$Y_t^N = w_t N_t = (A_t - \epsilon_t P_t^{T*} \theta) N_t \tag{7}$$

Since Y_t^N is the share of gross final output that accrues to the primary domestic factors of production, it also represents the economy's GDP. The remainder, $\epsilon_t P_t^{T*} \theta N_t$, is the import bill, i.e., the share of domestic output that firms sell in the foreign exchange market in return for buying θN_t units of the imported input. The real exchange rate determines the split: a depreciation raises the import bill and compresses GDP; an appreciation does the reverse.

3.1.3 Export Demand

As illustrated by the resource constraint (1), the domestic good is either consumed domestically or exported, where X_t denotes the share of output consumed externally in period t. Given the world price of the domestic good, $\frac{1}{\epsilon_t}$, we assume that the economy faces a downward-sloping, isoelastic export demand:

$$X_t = \zeta \, \epsilon_t^{1+\chi} \tag{8}$$

where $\zeta > 0$ is a scale parameter and $-(1+\chi)$ is the price elasticity of export demand. We assume that $\chi > 0$, i.e., the export demand is relatively price-elastic. Given this downward sloping demand structure, a real depreciation is required to induce the foreigners to buy more of the domestic good, and a real appreciation reduces the share of the domestic good sold abroad. Moreover, since the demand is relatively price elastic, the export revenue, $\frac{X_t}{\epsilon_t} = \zeta \, \epsilon_t^{\chi}$, in units of the foreign good is increasing in the real exchange rate. This is the Marshall-Lerner condition in this environment: a depreciation is required to increase the supply of the foreign purchasing power in the domestic economy. χ therefore measures the elasticity of supply of foreign purchasing power (in units of the foreign good) with respect to the real exchange rate.

3.1.4 Households

The economy is populated by a unit mass of households of two types. A fraction $\lambda \in [0, 1)$ are hand-to-mouth (HtM) households and the remaining $1 - \lambda$ are Ricardian households. Both types consume only the domestic good and supply labor. Let C_t^p and C_t^r denote the consumption of HtM and Ricardian households in period t.

All households supply N_t units of labor and take this quantity as given. In the analytical model presented in this section, we assume that N_t is a policy instrument. We discuss this further in subsection 3.1.5.

The period utility of household $i \in \{r, p\}$ is:

$$u(C_t^i) - \nu(N_t)$$

where $u : \mathbb{R}_{++} \to \mathbb{R}$ and $v : \mathbb{R}_{++} \to \mathbb{R}$ are twice continuously differentiable, with $u' > 0$, $u'' < 0$, $v' > 0$, and $v'' > 0$. They satisfy the standard Inada conditions: $\lim_{C \downarrow 0} u'(C) = \infty$, $\lim_{C \to \infty} u'(C) = 0$, $\lim_{N \downarrow 0} v'(N) = 0$, and $\lim_{N \to \infty} v'(N) = \infty$. All households share the same preferences. The common discount factor is $\beta \in (0, 1)$, and we impose $\beta R^* = 1$ throughout.

Hand-to-mouth households. HtM households cannot access asset markets. Given labor incomes, $w_t N_t$, they choose consumption, C_t^p, to maximize lifetime utility subject to period-by-period budget constraints:

$$C_t^p = w_t N_t \tag{9}$$

Since they do not participate in asset markets, the budget constraints imply that they consume their current labor income each period. Any income shocks pass through to their consumption one-for-one. There is no intertemporal smoothing.

Ricardian households. Ricardian households, in addition to consuming the domestic good and supplying labor, also participate in international asset markets. Let f^* denote a Ricardian household's net foreign asset position in period 1, denominated in units of the foreign good, with $f^* > 0$ denoting saving and $f^* < 0$ borrowing. Given labor income, the real exchange rate, the world interest rate, and government policy (τ, T), a Ricardian household's problem is to choose (C_t^r, f^*) to maximize lifetime utility subject to budget constraints:

$$\begin{aligned} &\max_{\{C_1^r, C_2^r, f^*\}} \; u(C_1^r) + \beta\, u(C_2^r) \qquad \text{s.t.} \\ &C_1^r + \epsilon_1 f^*(1 + \tau) = w_1 N_1 + T \\ &C_2^r = w_2 N_2 + \epsilon_2 R^* f^* \end{aligned} \tag{10}$$

where (τ, T) are policy instruments discussed in subsection 3.1.5. The optimal asset choice satisfies the Euler equation:

$$u'(C_1^r)\, \epsilon_1 (1 + \tau) = \beta R^*\, u'(C_2^r)\, \epsilon_2 \tag{11}$$

3.1.5 Fiscal-Monetary Policy

We assume that public policy has access to two margins: the intertemporal wedge and the labor wedge. The policy that influences the intertemporal wedge is interpreted as the exchange rate policy and is implemented through a capital control tax τ on the gross foreign asset positions of

Ricardian households. Tax revenue is rebated lump-sum to Ricardian households as a transfer T. The public agent's budget constraint is:

$$(1-\lambda)\,\tau\,\epsilon_1 f^* = (1-\lambda)\,T \tag{12}$$

The public agent's ability to influence the labor wedge is modeled in a reduced-form manner: the public agent directly sets employment, N_t, and we interpret this policy as monetary policy. This keeps the analytical framework clean while preserving the trade-offs that monetary policy faces. We defer the full decentralization to Section 4 with nominal wage rigidities and financial intermediation frictions. Monetary policy sets the inflation rate and exchange rate policy is implemented through FX interventions. The qualitative results from the analytical model carry over to the quantitative model; only the decentralization changes.

No direct fiscal transfers. The framework has no fiscal instrument capable of making lump-sum transfers to HtM households; the capital control revenue is rebated only to Ricardians. This modeling choice keeps the focus on redistribution through pecuniary general equilibrium effects created by exchange rate and monetary policies. Direct transfers to HtM households would trivially accomplish such redistribution. Without that fiscal channel, redistribution must operate through prices.

3.1.6 Balance of Payments

The final piece in the model is the balance-of-payments (BoP) accounting condition, which is already implicit in the specified equations. Combining the household budget constraints, the firms' zero-profit condition (6), goods market clearing (1), and the fiscal budget constraint (12) yields the balance-of-payments condition, which is interpreted as the economy's external budget constraint. Denote aggregate consumption as $C_t = \lambda C_t^p + (1-\lambda)C_t^r$ and let $F^* \equiv (1-\lambda)f^*$ be the aggregate net foreign asset position of the economy. Finally, substitute in the export demand function, (8), to express the period-by-period balance of payments in units of the foreign good:

$$\begin{aligned} P_1^{T*}\theta N_1 + F^* &= \zeta\,\epsilon_1^{\chi} \\ P_2^{T*}\theta N_2 &= \zeta\,\epsilon_2^{\chi} + R^* F^* \end{aligned} \tag{13}$$

The BoP constraints have a natural interpretation of equating demand and supply of foreign purchasing power[2] in the domestic economy's foreign exchange market. Firms are always net buyers of foreign purchasing power: they produce the domestic good and must pay for the imported input, so they are always exchanging domestic output for foreign purchasing power. Foreigners are always net buyers of the domestic good and suppliers of foreign purchasing power. Ricardian households switch sides depending on the sign of financial flows. When $F^* < 0$, Ricardians are net buyers of the domestic good in the foreign exchange market: they borrow foreign purchasing power and use it to buy domestic goods, joining foreigners on the supply side. When $F^* > 0$, they are net sellers of the domestic good in the foreign exchange market: they surrender domestic output out of their current labor income in exchange for future claims on foreign purchasing power, effectively joining firms on the demand side. The real exchange rate clears this market each period: it adjusts to ensure that the demand for foreign purchasing power equals the supply.

Combining the two period-by-period BoP constraints yields the intertemporal budget constraint for the economy:

$$P_1^{T*}\theta N_1 + \frac{1}{R^*} P_2^{T*}\theta N_2 = \zeta\, \epsilon_1^X + \frac{1}{R^*}\, \zeta\, \epsilon_2^X \tag{14}$$

The left-hand side of (14) is the present value of import expenditure in units of the foreign good, while the right-hand side is the present value of export revenue in foreign units. The intertemporal balance of payments requires that the present value of import expenditure equals the present value of export revenue. The economy cannot be a net borrower or lender in the long run; it cannot run a trade deficit or surplus in present value terms.

Define the trade balance in units of the foreign good:

$$NX_t \equiv \zeta\, \epsilon_t^X - P_t^{T*}\theta N_t \tag{15}$$

$NX_t > 0$ is a surplus and $NX_t < 0$ is a deficit. The trade balance is increasing in ϵ_t and decreasing in N_t: depreciation raises export revenue; higher employment raises the consumption of the intermediate input and hence increases the import bill. The intertemporal constraint requires

[2] We use the term foreign purchasing power to imply that it does not matter whether the agents actually exchange the foreign good or the intermediate input against the domestic good, as the foreign good is convertible into the intermediate input in the world markets at the price P_t^{T*}. In other words, the foreign good is not a medium of exchange in the model. However, for the sake of intuitive interpretation, henceforth, we adopt a convention: foreign exchange transactions involve sellers supplying the foreign good in exchange for the domestic good. Firms, who are always sellers of the domestic good, buy the foreign good and then use it to buy the intermediate input in the world market at the given exogenous price. In this sense, the model is equivalent to firms having access to a linear production technology with productivity $\frac{1}{P_t^{T*}}$ which converts the foreign good to the intermediate input.

$NX_1 + R^{*-1}NX_2 = 0$.

3.1.7 Equilibrium Definition

Given fiscal-monetary policy, (τ, T, N_t), an equilibrium is a set of allocations and prices, $\{C^p_t, C^r_t, F^*, M_t, Y_t, X_t, w_t, \epsilon_t\}$ such that:

i. HtM households satisfy their budget constraints, (9).

ii. Ricardian households satisfy the Euler equation, (11) and the budget constraints in (10).

iii. Firms satisfy the cost minimization conditions, (4), (5); and wage determination condition, (6).

iv. Export demand satisfies (8).

v. The fiscal budget constraint, (12), holds.

vi. The domestic goods market clears, i.e., (1) holds.

vii. The labor market clears, i.e., the labor demand by the firms equals the labor supply, N_t.

viii. The balance of payments conditions, (13), hold.

3.2 Optimal Policy Problem

The equilibrium definition in Section 3.1.7 describes the set of all feasible decentralized equilibria, indexed by the government's choice of fiscal-monetary policy (τ, T, N_t). We now describe the optimal policy problem of a utilitarian government that chooses a sequence of policies, $\{N_t, \tau, T\}$, to maximize social welfare subject to the equilibrium conditions listed in subsection 3.1.7.

We take the primal approach and state the optimization problem of a Ramsey planner who directly chooses $\{N_t, \epsilon_t\}$ subject to the implementability constraints implied by household and firm optimization. We then illustrate that the Ramsey planner's solution can be implemented as an equilibrium with an appropriate choice of capital control policy, $\{\tau, T\}$.

Consider the following constrained optimization problem:

$$\begin{gathered} \max_{\{N_t, \epsilon_t\}} \sum_{t=1}^{2} \beta^{t-1} \left[\lambda u\left(C^p_t(N_t, \epsilon_t)\right) + (1-\lambda) u\left(C^r_t(N_t, \epsilon_t)\right) - \nu(N_t)\right] \qquad \text{s.t.} \\ NX_1(N_1, \epsilon_1) + \frac{1}{R^*} NX_2(N_2, \epsilon_2) = 0 \end{gathered} \tag{16}$$

where:

$$C_t^p(N_t, \epsilon_t) = Y_t^N(N_t, \epsilon_t) \tag{17}$$

$$C_t^r(N_t, \epsilon_t) = Y_t^N(N_t, \epsilon_t) - \frac{\epsilon_t NX_t(N_t, \epsilon_t)}{1-\lambda} \tag{18}$$

and $Y_t^N(N_t, \epsilon_t)$ and $NX_t(N_t, \epsilon_t)$ are given by labor income equation (7) and the trade balance equation (15), respectively. Equation (17) is obtained by combining the HtM households' budget constraint, (9), with wage determination equation (6). Equation (18) is obtained by combining the Ricardian households' budget constraints in (10) with the wage determination equation, (7), and the balance of payments conditions, (13). This equation implies that if $NX_t < 0$, then the Ricardian households are net borrowers in period t and consume more than their period labor income. Conversely, if $NX_t > 0$, then the Ricardian households are net savers in period t and consume less than their period labor income. Moreover, since Ricardians save and borrow in units of the foreign good, the actual consumption in domestic units that such saving/borrowing can buy is a product of the real exchange rate, ϵ_t, and the trade balance in foreign units, NX_t. Finally, the constraint in (16) is the intertemporal budget constraint, (14): the present discounted value of net export revenue must equal zero.

We denote the solution to the optimization problem, (16), as the constrained efficient allocation. In Appendix A.1, we derive the first-order conditions for this problem and obtain the system of equations that characterize the solution. The solution in $\{N_t, \epsilon_t\}$ is characterized by an Euler-like intertemporal condition, (73), an intratemporal consumption-labor condition, (77) for $t = 1, 2$, and the intertemporal budget constraint in (16). In Appendix A.2 we discuss sufficient conditions that ensure that the solution is a unique maximum.

The constrained planning problem, (16), internalizes all equilibrium constraints except the Ricardian Euler equation, (11), and the goods-market-clearing condition, (1). The Ricardian Euler equation is not an implementability constraint for the planner because the planner can distort the intertemporal margin: the intertemporal wedge chosen by the Ramsey planner can be supported in the decentralized economy through an appropriate choice of τ. The goods-market-clearing condition is implied by Walras' identity: if all other equilibrium conditions are satisfied, then goods market clearing must hold. Lemma 1 formally shows that the solution to (16) can be supported as a decentralized equilibrium with an appropriate choice of policy, $\{\tau, T\}$. We define this decentralized equilibrium as the Ramsey equilibrium.

Lemma 1. *Let $\{N_t, \epsilon_t\}$ be a solution to the constrained optimization problem in* (16)*. Then there exists a policy, $\{\tau, T\}$, such that the solution to* (16) *is implementable as an equilibrium defined in subsection 3.1.7.*

Proof. See Appendix A.3. □

Before we proceed to characterize the constrained efficient allocation under different assumptions on λ and import price sequences, $\{P_t^{T*}\}$, it is useful to state a few definitions. Define the average marginal utility, $\bar{u}'_t$, as the population-weighted average of the marginal utilities of the two types of households:

$$\bar{u}'_t \equiv \lambda\, u'(C_t^p) + (1-\lambda)\, u'(C_t^r) \tag{19}$$

Define the labor wedge, τ_t^N, as the deviation of the average marginal rate of substitution between consumption and leisure from the real wage:

$$1 - \tau_t^N \equiv \frac{\nu'(N_t)}{w_t \bar{u}'_t} \tag{20}$$

Using the Ricardian Euler equation, (11), the intertemporal wedge corresponds to the capital control tax, τ:

$$1 + \tau \equiv \frac{\beta R^*\, u'(C_2^r)\, \epsilon_2}{u'(C_1^r)\, \epsilon_1} \tag{21}$$

In Appendix A.1, we use the first-order conditions of the Ramsey planner's problem to obtain closed-form expressions for the optimal labor and intertemporal wedges in terms of the optimal allocation, $\{N_t, \epsilon_t\}$. The optimal labor wedge is given by equation (79), and the optimal intertemporal wedge is given by equation (80).

Policy is said to be operative on a margin if it selects a non-zero wedge on that margin. On the other hand, an equilibrium is defined as laissez-faire if the policy chooses no distortion of the intertemporal and labor margins, i.e., sets the wedges to zero: $\tau = 0$ and $\tau_t^N = 0$. In what follows, we characterize the constrained efficient allocation under different assumptions on λ and import prices, P_t^{T*}, and compare this solution with laissez-faire equilibria and equilibria with sub-optimally chosen wedges.

3.3 Optimal Policy: Representative-Agent Benchmark

The representative-agent case is nested in the heterogeneous-agent problem by setting $\lambda = 0$. It provides a useful benchmark in which no household is borrowing constrained and the household's

Euler equation always holds. Proposition 1 characterizes its constrained-efficient allocation.

Proposition 1. *If $\lambda = 0$, then given any sequence of import prices and productivity, $\{P_t^{T*}, A_t\} > 0$, the constrained efficient allocation involves:*

- *No intertemporal distortions: $\tau = 0$.*
- *A strictly positive labor wedge that implements the optimal terms-of-trade:*

$$\tau_t^N = \frac{1}{\chi}\frac{\epsilon_t P_t^{T*}\theta}{w_t} > 0$$

Proof. See Appendix A.4. □

With a single household type that participates freely in international asset markets, there is no role for policy to distort the intertemporal margin: the representative agent can optimally smooth consumption over time by borrowing and lending in international asset markets. The proposition establishes that the undistorted Ricardian Euler equation coincides with the planner's optimality condition. The representative agent's borrowing and saving decisions are optimal for any path of import prices and productivities, and the planner has no incentive to distort the intertemporal price. Therefore, in the decentralized economy, the optimal capital control tax is zero.

Second, given that the economy faces an upward-sloping supply of foreign purchasing power in the form of export revenues, the model features a classic terms-of-trade distortion: competitive firms do not internalize the effect of their import demand on the real exchange rate. Each additional unit of import demand must depreciate the real exchange rate in order to induce foreigners to supply additional foreign purchasing power. This depreciation increases the firms' costs in the form of domestic output paid to foreigners: not only for the marginal unit of foreign purchasing power supplied by them but also for all inframarginal units. Noting that exports, X_t, can be written as a product of the real exchange rate, ϵ_t, and the foreign good supplied by foreigners, $\zeta\epsilon_t^{\chi}$, we have:

$$dX_t = \epsilon_t \chi\zeta\epsilon_t^{\chi-1}d\epsilon_t + \zeta\epsilon_t^{\chi}d\epsilon_t$$

where the first term is the direct quantity effect: a real depreciation increases the quantity of the foreign good supplied at the margin; and the second term is the inframarginal price effect: a real depreciation increases the price of the foreign good for all inframarginal units supplied by foreigners. Holding NX_t constant, taking the total differential of (15) yields:

$$P_t^{T*}\theta dN_t = \chi\zeta\epsilon_t^{\chi-1}d\epsilon_t$$

Together this gives us:

$$dX_t = \epsilon_t\, P_t^{T*}\theta dN_t + \frac{1}{\chi}\epsilon_t P_t^{T*}\theta dN_t$$

where the first and second terms have the same interpretation as above. The second term is the terms-of-trade effect, which constitutes a resource loss for the economy that price-taking firms do not internalize. Such terms-of-trade effects have been discussed in the classic optimal trade policy literature at least since Johnson (1953). They are also closely related to the general-equilibrium argument for optimal tariffs in Dixit and Norman (1980). A Ramsey planner internalizes such terms-of-trade effects and considers the marginal cost of expanding employment and imports higher than what private agents do. Define $\hat{w}_t$ as the effective marginal product of labor, i.e., the net resources retained by the domestic economy when employment is increased by a marginal unit after accounting for the terms-of-trade effect. $\hat{w}_t$ is defined as:

$$\hat{w}_t \equiv A_t - \frac{dX_t}{dN_t} = w_t - \frac{1}{\chi}\epsilon_t P_t^{T*}\theta$$

The Ramsey planner internalizes the terms-of-trade effect and values the marginal unit of labor at the effective output that the domestic economy retains, $\hat{w}_t$. As a result, the constrained efficient allocation restricts employment and imports below the laissez-faire level, which reduces output and export supply and results in a more appreciated real exchange rate in equilibrium relative to the competitive outcome[3]. The optimal labor wedge is proportional to the inframarginal price effect term, $\frac{1}{\chi}\epsilon_t P_t^{T*}\theta$, that we derived above.

This classic terms-of-trade distortion is neither a focus nor a requirement for the main results of the paper. It naturally arises in our framework due to the upward-sloping export supply assumption. Moreover, to isolate the impact of heterogeneity on optimal policy, it is useful to identify and separate out the effect of this terms-of-trade distortion in the overall policy response. The representative-agent framework is therefore a useful benchmark for this purpose.

[3]In our framework, import-export tariffs are unavailable, however due to the Leontief nature of the production technology, the labor wedge internalizes the terms-of-trade distortion to implement the same allocation which an optimal tariff would implement. Moreover, with isoelastic export demand, this distortion is purely static. The labor wedge corrects the terms-of-trade distortion period by period independently, and the intertemporal wedge provides no additional leverage for this purpose. It follows that if labor supply is perfectly inelastic, the optimal intertemporal wedge is still zero. The planner has no instrument with which to internalize the terms-of-trade distortion in that case and the constrained efficient allocation coincides with the competitive laissez-faire allocation.

Before proceeding to the heterogeneous agent model, it is useful to do a comparative statics exercise with respect to the import price, P^{T*}. Lemma 2 characterizes the constrained efficient allocation for the case where import prices are constant over time: $P_1^{T*} = P_2^{T*} = P^{T*}$ and then discusses how an increase in the period 1 price induces households to borrow in international asset markets to smooth this income shock.

Lemma 2. *Let $\lambda = 0$, $A_t = A > 0$ for $t \in \{1, 2\}$, and $\beta R^* = 1$. Let all other parameters satisfy the conditions of Lemma 5.*

i. ***Symmetric equilibrium.*** *If $P_1^{T*} = P_2^{T*} = P^{T*}$, then the constrained efficient allocation satisfies*

$$N_1 = N_2, \qquad \epsilon_1 = \epsilon_2, \qquad NX_1 = NX_2 = 0$$

ii. ***Comparative statics.*** *In addition, assume that utility over consumption satisfies $u'(C) = C^{-\sigma}$ with $\sigma \geqslant \frac{1}{1+\chi}$. Fix $P_2^{T*} = P^{T*}$ and let $P_1^{T*} > P^{T*}$. Then the constrained efficient allocation satisfies*

$$NX_1 < 0 < NX_2$$

Proof. See Appendix A.5. □

When import prices are equal across periods, part (i) establishes that the constrained efficient allocation is symmetric: employment, the real exchange rate, and the trade balance are identical in the two periods, and trade is intratemporally balanced. With $\beta R^* = 1$ and no asymmetry between the two periods, the representative agent has no incentive to shift resources intertemporally, and equal consumption across periods is optimal. The labor wedge is active in each period, statically correcting the terms-of-trade distortion, but the intertemporal wedge is undistorted.

Part (ii) considers a temporary increase in the import price in period 1, with $P_1^{T*} > P^{T*}$. The high input price reduces real wages and compresses real income in period 1. The representative agent responds by borrowing in international asset markets to smooth consumption across periods, running a trade deficit in period 1 and a surplus in period 2. Since the agent can borrow freely at the world interest rate, the transitory income shortfall in period 1 does not prevent consumption smoothing. The planner sets $\tau = 0$: the privately optimal current account path is also socially optimal, and there is no role for intertemporal policy. The terms-of-trade wedge continues to operate in each period independently.

The representative-agent benchmark therefore delineates the role of each policy instrument. Given the absence of import-export tariffs, the labor wedge through monetary policy internalizes the terms-of-trade distortion period by period, and the optimal capital control tax is zero for any sequence of import prices. This outcome is the close analog of conventional policy prescriptions in representative-agent small open economy models: since there are no frictions in asset markets, real income shocks do not call for interventions in foreign exchange markets. In response to import price spikes, the exchange rate adjusts freely as households optimally smooth consumption. However, when a fraction of households cannot borrow against future income to smooth consumption, asset markets need not be left undistorted. The following subsections discuss optimal policies in a heterogeneous-agent economy with a non-zero mass of hand-to-mouth households.

3.4 Heterogeneous Agents ($\lambda \in (0, 1)$)

We now turn to the case $\lambda \in (0, 1)$. A fraction λ of households cannot borrow against future income. When import prices are temporarily high, Ricardian households respond by borrowing in international asset markets, insulating their consumption from the income shock. HtM households have no such recourse: they consume their current labor income each period and bear the full income loss contemporaneously. This asymmetry in asset market access creates a possible role for policy.

To isolate the intertemporal dimension of optimal policy, we begin with the case of inelastic labor supply, $N_t = \bar{N}$. With employment fixed exogenously, the only instrument available to the government is the capital control tax, τ, and the Ramsey planner's problem reduces to choosing a point on the feasible frontier defined by the intertemporal budget constraint. This simplification allows us to isolate the role of exchange rate policy. The labor wedge and its interaction with the intertemporal wedge are taken up in Section 3.4.2.

3.4.1 Inelastic Labor Supply ($N_t = \bar{N}$)

Equal import prices. We begin with the case $P_1^{T*} = P_2^{T*} = P^{T*}$. With $\beta R^* = 1$ and no asymmetry between the two periods, neither household type has an incentive to shift resources intertemporally. The privately optimal asset position for Ricardian households is $f^* = 0$, implying balanced trade in each period. With $NX_1 = NX_2 = 0$, both household types consume their current labor income in each period. Since the income path is identical across periods, consumption is equal over time for both types; and since trade is balanced in each period, Ricardian and HtM households also consume equally within each period: $C_t^p = C_t^r$ for $t \in \{1, 2\}$.

This allocation is optimal for both Ricardian and HtM households. The HtM households would choose this allocation even if they had access to asset markets: in this special case, the inability of HtM households to smooth consumption is inconsequential. The planner therefore has no incentive to distort the intertemporal margin, and the constrained efficient allocation coincides with laissez-faire: $\tau = 0$ and the exchange rate path is undistorted. This result is the direct counterpart of part (i) of Lemma 2: with equal prices and $\beta R^* = 1$, the intertemporal wedge is zero both in the representative-agent economy and in the heterogeneous-agent economy. Lemma 3 formally summarizes this result.

Lemma 3. *Assume inelastic labor supply:* $N_t = \bar{N} > 0$. *Let* $\lambda \in (0, 1)$, $A_t = A > 0$ *for* $t \in \{1, 2\}$, *and* $\beta R^* = 1$. *Let* $P_1^{T*} = P_2^{T*} = P^{T*} > 0$. *Then the constrained efficient allocation involves no intertemporal distortions:* $\tau = 0$, *i.e., it coincides with the laissez-faire equilibrium, and satisfies:*

$$\epsilon_1 = \epsilon_2, \qquad NX_1 = NX_2 = 0, \qquad C_1^r = C_1^p = C_2^r = C_2^p$$

Proof. Follows as a Corollary to Proposition 5. □

The laissez-faire equilibrium under $P_1^{T*} > P_2^{T*}$. Now consider the case where $P_1^{T*} > P_2^{T*} = P^{T*}$. Before turning to optimal policy we characterize the laissez-faire equilibrium. This provides the reference point against which the planner's intervention is measured. The response of the economy to this price path is analogous to the representative-agent case studied in Lemma 2. The high input price in period 1 compresses real labor incomes, and Ricardian households borrow in international asset markets to smooth consumption over their lifetime. As in the representative-agent case, the economy runs a trade deficit in period 1 and a surplus in period 2: $NX_1 < 0 < NX_2$.

The heterogeneity in asset market access, however, introduces a dimension that is absent in the representative-agent economy. Ricardian households supplement their compressed period-1 labor income with the resources borrowed from abroad, consuming strictly more than their current incomes. HtM households, by contrast, consume their current labor income each period. In period 1, Ricardians consume strictly more than HtM households; in period 2, the pattern reverses as Ricardians service their debt while HtM households enjoy the full recovery of labor incomes. The laissez-faire equilibrium therefore features a consumption differential between the two household types that varies over the income cycle: $C_1^r > C_1^p$ and $C_2^r < C_2^p$. Lemma 4 formally establishes these properties of the laissez-faire equilibrium.

Lemma 4. *Assume inelastic labor supply:* $N_t = \bar{N} > 0$. *Let* $\lambda \in (0,1)$, $A_t = A > 0$, *for* $t \in \{1,2\}$, *and* $\beta R^* = 1$. *Let* $P_2^{T*} = P^{T*} > 0$ *and* $P_1^{T*} > P^{T*}$. *Consider the laissez-faire equilibrium, which imposes* $\tau = 0$. *The equilibrium satisfies:*

$$NX_1 < 0 < NX_2, \qquad C_1^r > C_1^p, \qquad C_2^r < C_2^p$$

Proof. See Appendix A.6. □

A perturbation along the feasible frontier. Before characterizing optimal policy, it is useful to examine how a capital control tax affects equilibrium outcomes. The intertemporal budget constraint, $NX_1 + R^{*-1}NX_2 = 0$, defines the set of feasible decentralized equilibria that can be reached for some choice of τ. We take the laissez-faire equilibrium characterized in Lemma 4 as the reference point and consider the impact of a capital control subsidy[4]. By distorting Ricardian households' saving decisions, this policy affects the intertemporal path of the real exchange rate in equilibrium. This influences resource sharing between the domestic economy and the rest of the world, as well as the distribution of domestically retained resources between the two household types, thereby affecting the consumption path of each type.

A capital subsidy, $d\tau < 0$, lowers the private cost of foreign borrowing for Ricardian households, inducing additional period-1 borrowing. Let $B \equiv -NX_1 > 0$ denote period-1 borrowing; the subsidy generates an additional $dB > 0$. By the intertemporal budget constraint, this additional borrowing must be repaid: $dNX_2 = R^* dB > 0$. In what follows, we trace the equilibrium consequences of a perturbation, $dB > 0$.

The period-1 balance of payments condition can be written as:

$$P_1^{T*}\theta N = \zeta \epsilon_1^X + B$$

Written this way, the BoP condition has the natural interpretation of equating demand and supply of foreign purchasing power in the domestic economy, i.e., an equilibrium condition in the domestic

[4] While we use the laissez-faire allocation as the starting point, the economic intuition discussed in this subsection holds for any point on the intertemporal budget constraint satisfying $NX_1 < 0 < NX_2$, given any import price sequence $\{P_1^{T*}, P_2^{T*}\}$.

economy's foreign exchange market. The left-hand side is firms' demand for foreign purchasing power to pay for the imported input. Given a perfectly inelastic labor supply, the firms' demand for foreign purchasing power is also perfectly inelastic in equilibrium. The right-hand side is the supply of foreign purchasing power: foreigners supply $\zeta\epsilon_1^{\chi}$ by purchasing the domestic good, and Ricardians supply B through their borrowing in international asset markets. The purchasing power supplied by foreigners is increasing in ϵ_1: a real depreciation induces them to buy more of the domestic good and consequently supply more in return, and a real appreciation does the reverse. Since the net demand by firms is perfectly inelastic, a rise in B crowds out export revenues one-for-one. In other words, Ricardians bring additional foreign purchasing power and seek to exchange it for the domestic good, creating an excess supply. Consequently, they bid down the real exchange rate and crowd out export revenues until the foreign exchange market clears. Taking the total differential of the BoP condition shows that the equilibrium requires a real appreciation:

$$d\epsilon_1 = -\frac{dB}{\chi\zeta\epsilon_1^{\chi-1}} < 0 \tag{22}$$

By inducing Ricardians to borrow more, the public agent substitutes export revenues with borrowed funds as a source of foreign purchasing power for firms. This reduces the need to sell domestic good to foreigners in the current period. To trace how this changes the flow of domestic output in the foreign exchange market, we multiply the period-1 BoP condition by ϵ_1 to express it in domestic units:

$$\epsilon_1 P_1^{T*}\theta N = \zeta\epsilon_1^{1+\chi} + \epsilon_1 B$$

The left-hand side is the share of domestic output sold by firms in the foreign exchange market in return for buying the foreign input, which represents a share of firms' production costs. The first term on the right-hand side represents exports, X_1, and the second term, $\epsilon_1 B$, is the domestic output purchased by Ricardians with borrowed foreign resources. Taking the total differential gives:

$$P_1^{T*}\theta N \, d\epsilon_1 = (1+\chi)\zeta\epsilon_1^{\chi} \, d\epsilon_1 + (\epsilon_1 dB + B d\epsilon_1) \tag{23}$$

Since $d\epsilon_1 < 0$, the left-hand side of (23) measures the total cost savings for firms. As Ricardians bid down the real exchange rate, the domestic-good cost of the imported input, $\epsilon_1 P_1^{T*}$, and therefore firms' marginal cost of production, falls. As the real exchange rate appreciates, given the initial real wage, hiring a unit of labor and buying θ units of input to produce A units of output now creates a positive profit opportunity for firms. Firms attempt to expand production and demand more inputs, and since inputs and labor are perfect complements, the demand for labor also expands. With the labor endowment fixed at N in equilibrium, competition for workers instead bids up the real wage until positive profit opportunities have been eliminated. The firms' zero-profit condition, (6), implies that the real wage must rise one-for-one with a real appreciation. In other words, cost savings by firms are passed through to workers in the form of higher real labor incomes. The increase in labor income is therefore:

$$dY_1^N = -P_1^{T*}\theta N \, d\epsilon_1 > 0 \tag{24}$$

While workers gain resources, the right-hand side of (23) shows who loses access to domestic resources. The net cost reduction for firms can be decomposed into two components. The first expression is $dX_1 = (1+\chi)\zeta\epsilon_1^{\chi} \, d\epsilon_1$, the decrease in exports. Using (22) we have:

$$dX_1 = -\left(\epsilon_1 dB + \frac{1}{\chi}\epsilon_1 dB\right) < 0 \tag{25}$$

This equation shows that the decrease in exports is itself decomposed into a quantity effect, $\epsilon_1 dB$, as well as an inframarginal price effect, $\frac{1}{\chi}\epsilon_1 dB$. The quantity effect is the domestic output no longer sold to foreigners because Ricardian borrowing replaces foreign-supplied purchasing power at the margin. The price effect arises due to an upward-sloping export supply curve: a real appreciation decreases the domestic output paid to foreigners at the margin and against all inframarginal units of foreign purchasing power supplied by them.

The second expression on the right-hand side of (23) represents the net change in the quantity of domestic good bought by Ricardians by exchanging the borrowed foreign good. This is also decomposed into a quantity effect and a price effect. The quantity effect, $\epsilon_1 dB$, is positive: an increase in borrowing allows Ricardians to buy more of the domestic good at any given price. The price effect, $B d\epsilon_1$, is negative: a real appreciation lowers the domestic-good value of all foreign resources exchanged by Ricardians, not only the marginal unit. The sign of the net change depends on whether the quantity effect dominates the price effect or vice versa.

Substituting (24) and (25) into (23), the net increase in labor income can be expressed as the sum of two inframarginal price effects associated with a real exchange rate appreciation:

$$dY_1^N = \left(\frac{\epsilon_1}{\chi} - B\frac{d\epsilon_1}{dB}\right) dB > 0 \tag{26}$$

The first term is the inframarginal price effect on foreigners: an appreciation lowers the domestic output paid against all inframarginal units of foreign purchasing power supplied through exports. The second term is the inframarginal price effect on Ricardians: an appreciation lowers the domestic-good value of all inframarginal foreign resources borrowed by Ricardians[5] Equation (26) therefore summarizes the net redistribution of resources: both household types gain through higher labor income which is financed by foreigners and Ricardians giving up their claims on domestic resources.

Thus, a subsidy to Ricardian borrowing induces a real exchange rate appreciation, which, through cost reduction for firms, results in an endogenous transfer of real resources toward labor incomes and away from foreign exchange market transactions. Since labor incomes are gained by both Ricardians and HtM households, this policy redistributes through pecuniary general equilibrium effects. Exports fall, while HtM consumption rises one-for-one with labor income. The change in Ricardian consumption is ambiguous: the labor-income gain and the positive quantity effect of additional borrowing are offset partly or fully by the inframarginal price effect:

$$dC_1^p = dY_1^N > 0$$

$$dC_1^r = dY_1^N + \frac{1}{1-\lambda}\left[\epsilon_1 dB + B\, d\epsilon_1\right]$$

However, in period 2 Ricardians face a larger debt burden. The intertemporal resource constraint requires that an appreciation in period 1 must be followed by a depreciation in period 2. The economy must export more in period 2 to pay back the larger period-1 borrowing. A real depreciation in period 2 does an analogous endogenous resource transfer but in the opposite direction: it reduces labor incomes and increases the share of output sold to foreigners. We have $dNX_2 = R^* dB > 0$. Using the period-2 BoP condition, we have:

$$d\epsilon_2 = \frac{R^* dB}{\chi \zeta \epsilon_2^{\chi-1}} > 0 \tag{27}$$

A larger repayment obligation depreciates the real exchange rate in period 2. This raises the domestic-good price of the imported input and hence firms' marginal cost, inducing them to reduce production. At the prevailing wage, firms make losses. Labor demand falls, and the real wage declines until zero profits are restored. Hence labor income falls:

$$dY_2^N = -P_2^{T*} \theta N\, d\epsilon_2 < 0 \tag{28}$$

[5]The positive quantity effect on Ricardians cancels against the negative quantity effect on foreigners. The net gain in labor income is the sum of the two inframarginal price effects.

Since HtM households consume current labor income, their consumption falls in period 2. Ricardians are also worse off in period 2. Not only does the labor income fall, they also face a higher repayment burden in domestic units. Both the quantity and price effects are unambiguously negative in period 2:

$$dC_2^p = dY_2^N < 0$$
$$dC_2^r = dY_2^N - \frac{1}{1-\lambda}\left[\epsilon_2 R^* dB + R^* B\, d\epsilon_2\right] < 0$$

The net fall in labor income can be decomposed in a way that parallels the period-1 expression:

$$dY_2^N = -\left[\frac{\epsilon_2}{\chi} - B\frac{d\epsilon_2}{dB}\right] R^* dB \tag{29}$$

The first term is the inframarginal export-price effect: the depreciation increases domestic output sold to foreigners against all inframarginal units of foreign purchasing power supplied by them. The second is the inframarginal price effect on Ricardian repayment obligations. A depreciation increases the domestic-good value of their existing debt burden. Since $B > 0$, this effect places part of the burden directly on Ricardians and dampens the fall in labor income.

Thus, a marginal increase in period-1 borrowing redistributes resources toward labor income in period 1 and away from labor income in period 2. Exports fall in period 1 and rise in period 2. HtM households gain in period 1 and lose in period 2. Ricardians face an ambiguous period-1 effect and an unambiguous period-2 loss. This intertemporal endogenous redistribution of resources is driven by the path of the real exchange rate, which policy can choose by distorting the intertemporal margin.

The planner's first-order condition. To characterize optimal policy, we return to the primal problem, (16), where a utilitarian Ramsey planner directly chooses the optimal exchange rate path by internalizing the pecuniary general equilibrium effects of exchange rate movements. In Appendix A we derive the Ramsey planner's first-order condition with respect to ϵ_t, which we restate below to make explicit the net marginal benefits of an appreciation at time t:

$$(\beta R^*)^{t-1}\left[\bar{u}_t'\left(\frac{\epsilon_t}{\chi} - NX_t\frac{d\epsilon_t}{dNX_t}\right) + u'(C_t^r)\epsilon_t + u'(C_t^r)NX_t\frac{d\epsilon_t}{dNX_t}\right] = \mu \tag{30}$$

where $\bar{u}_t'$ is average marginal utility, defined in (19), and μ is the Lagrange multiplier on the intertemporal budget constraint. The left-hand side captures the net benefit of a marginal appreciation in period t. The first term is the labor-income gain valued at average marginal utility. The term in parenthesis is identical to the net change in labor income decomposed into two inframarginal price effects, as illustrated in equations (24) and (29) above. The second term is the direct quantity effect on Ricardian foreign asset position. The third is the price effect: an appreciation lowers the domestic-good value of all inframarginal foreign resources exchanged by Ricardians. While the first and second terms have positive signs, the third term's sign depends on the sign of NX_t: if the Ricardians are net borrowers ($NX_t < 0$), the price effect is negative, and if they are net savers ($NX_t > 0$), the price effect is positive.

Equation (30) can be rearranged to group appreciation-induced resource transfers among agents into two net transfers. First, a net transfer between foreigners and the domestic economy, and, second, a net transfer between the two household types within the domestic economy:

$$(\beta R^*)^{t-1}\left[u'(C_t^r)\epsilon_t\left(1+\frac{1}{\chi}\right)+\lambda\big(u'(C_t^p)-u'(C_t^r)\big)\left(\frac{\epsilon_t}{\chi}-NX_t\frac{d\epsilon_t}{dNX_t}\right)\right]=\mu \tag{31}$$

The first term is interpreted as a net resource transfer from foreigners to domestic agents valued at Ricardian marginal utility: an appreciation reduces exports at the margin and lowers the domestic output paid against inframarginal foreign purchasing power. The second term captures the net resource transfer between domestic households. It represents the pecuniary general equilibrium effect of a real appreciation on HtM households through their labor incomes. Since a share λ of the overall increase in the labor incomes is accrued to HtM households, it is effectively equivalent to a net resource transfer from Ricardians to HtMs. The planner values this transfer at the marginal utility difference between the two types: how much more do HtM households value these resources relative to Ricardians.

The Lagrange multiplier μ captures the marginal cost of a real exchange rate appreciation in period t through a tightening of the intertemporal budget constraint. Feasibility dictates that an appreciation in period t must be followed by a depreciation in period t + 1. A marginal depreciation at time t + 1 relaxes the intertemporal budget constraint, the marginal utility benefit of which is also μ. An optimizing planner equates the benefits of inducing a marginal appreciation at time t to the costs of inducing a marginal depreciation at time t + 1. This yields the planner's intertemporal optimality condition:

$$\epsilon_1\, u'(C_1^r)+\Omega_1=\beta R^*\Big[\epsilon_2\, u'(C_2^r)+\Omega_2\Big] \tag{32}$$

where

$$\Omega_t \equiv \frac{\lambda\left[u'(C_t^p) - u'(C_t^r)\right]\left(\frac{\epsilon_t}{\chi} - NX_t \frac{d\epsilon_t}{dNX_t}\right)}{\frac{1+\chi}{\chi}} \tag{33}$$

Ω_t is the redistributive wedge that captures the social value of a net resource transfer between domestic households per unit of the resource transfer between foreigners and the domestic economy induced by a marginal real exchange rate change at time t.

Optimal policy under $P_1^{T*} > P_2^{T*}$. The intertemporal optimality condition (32) and the intertemporal budget constraint characterize the constrained efficient allocation for any import price sequence. From Lemma 4, the laissez-faire allocation satisfies $NX_1^{LF} < 0 < NX_2^{LF}$, which implies $C_1^{r,LF} > C_1^{p,LF}$ and $C_2^{r,LF} < C_2^{p,LF}$. HtM households therefore have strictly higher marginal utility than Ricardians in period 1 and strictly lower marginal utility in period 2, giving $\Omega_1^{LF} > 0$ and $\Omega_2^{LF} < 0$. Moreover, the laissez-faire allocation satisfies the Euler equation: $\epsilon_1^{LF} u'(C_1^{r,LF}) = \beta R^* \epsilon_2^{LF} u'(C_2^{r,LF})$. Evaluating the left- and right-hand sides of (32) at the laissez-faire allocation, we get:

$$\epsilon_1^{LF}\, u'(C_1^{r,LF}) + \Omega_1^{LF} \;>\; \epsilon_2^{LF}\, u'(C_2^{r,LF}) + \Omega_2^{LF}$$

Thus the marginal benefit of a real exchange rate appreciation in period 1 exceeds the marginal cost of the required depreciation in period 2 at the laissez-faire allocation whenever $P_1^{T*} > P_2^{T*}$. The inability of HtM households to intertemporally smooth consumption in response to a transitory fall in real incomes makes their income and consumption excessively volatile from the perspective of a utilitarian planner. By inducing a real exchange rate appreciation in period 1, and a depreciation in period 2, the planner can tilt the economy's access to resources toward the high-input price period and away from the low-input price period. In a representative agent economy there is no motive to do this. In a heterogeneous-agent economy, this policy induces an endogenous resource transfer to HtM households in the high-input price period and away from them in the low-input price period.

The period-1 appreciation reduces the domestic-good cost of the imported input, raises firms' residual profits at the prevailing wage, and through the no-arbitrage condition (6) bids wages up until zero profits are restored. Labor income rises in period 1, and since HtM households consume current labor income one-for-one, their consumption rises. In period 2, the depreciation raises input costs, compresses labor incomes, and reduces HtM consumption. By tilting the exchange rate path, the planner makes HtM consumption intertemporally smoother than under laissez-faire: the compression in period 1 is partially offset, and the recovery in period 2 is partially dampened. For

Ricardian households, the change in consumption in period 1 is ambiguous: the labor income gain and the positive quantity effect are partially or fully offset by the appreciation-induced reduction in the domestic-good value of their foreign borrowing. In period 2 their consumption unambiguously falls, as they face both lower labor income and a higher domestic-good cost of repayment. The net effect on social welfare is strictly positive, however: the FOC inequality establishes that a marginal appreciation at $t = 1$, with the required depreciation at $t = 2$, always raises social welfare at utilitarian Pareto weights starting from the laissez-faire allocation. We formally prove this in Proposition 2.

Therefore, optimal policy involves a more appreciated real exchange rate in period 1 and a more depreciated real exchange rate in period 2 relative to laissez-faire: $\epsilon_1^O < \epsilon_1^{LF}$ and $\epsilon_2^O > \epsilon_2^{LF}$. The optimal policy makes the intertemporal path of the real exchange rate more volatile than laissez-faire. Moreover, the trade deficit must be larger in period 1 and the trade surplus must be larger in period 2 relative to laissez-faire. At the same time, this appreciation partially offsets the exogenous spike in the world input price by compressing its domestic-good equivalent in the high-price period at the cost of a higher domestic equivalent in the low-price period, thereby stabilizing the country's terms of trade. Since real labor income, Y_t^N, i.e., the country's GDP, moves one-for-one against the terms of trade, it is also relatively stabilized: higher in the high-input price period and lower in the low-input price period under the constrained efficient allocation. Proposition 2 summarizes these results.

Finally, the constrained efficient allocation involves a strictly positive intertemporal wedge, i.e., in a decentralized equilibrium, this allocation is implementable through a strictly positive capital control tax on Ricardian asset holdings, or equivalently a subsidy on Ricardian borrowing. The tax on Ricardian saving raises their private cost of accumulating foreign assets, inducing them to borrow more in period 1 than they would choose under laissez-faire. The public agent in effect uses the Ricardian households' asset market participation to redistribute resources, operating through the pecuniary general equilibrium effects of real exchange rate changes. In Proposition 2 we establish that whenever $P_1^{T*} > P_2^{T*}$, the constrained efficient allocation is decentralized with a strictly positive capital control tax at $t = 1$, i.e., $\tau > 0$:

$$\tau = \frac{\Omega_1^O - \beta R^* \Omega_2^O}{\epsilon_1^O \, u'(C_1^{r,O})} > 0 \tag{34}$$

Proposition 2. *Assume inelastic labor supply:* $N_t = \bar{N} > 0$. *Let* $\lambda \in (0,1)$, $A_t = A > 0$ *for* $t \in \{1,2\}$,

$\beta R^* = 1$, $0 < \chi \leqslant 1$, $P_2^{T*} = P^{T*} > 0$, *and* $P_1^{T*} > P^{T*}$. *Let* $(\epsilon_t^O, C_t^{r,O}, C_t^{p,O}, NX_t^O, Y_t^{N,O})$ *denote the constrained efficient allocation and* $(\epsilon_t^{LF}, C_t^{r,LF}, C_t^{p,LF}, NX_t^{LF}, Y_t^{N,LF})$ *denote the laissez-faire allocation. The constrained efficient allocation involves:*

i. A strictly positive intertemporal wedge: $\tau > 0$.

ii. A relatively volatile real exchange rate: Exchange rate is strictly more appreciated in period 1 *and strictly more depreciated in period* 2*:*

$$\epsilon_1^O < \epsilon_1^{LF}, \qquad \epsilon_2^O > \epsilon_2^{LF}$$

iii. A relatively volatile trade balance: The trade deficit is strictly larger in period 1 *and the trade surplus is strictly larger in period* 2*:*

$$NX_1^O < NX_1^{LF} < 0 < NX_2^{LF} < NX_2^O$$

iv. Input price in domestic tradeable units is strictly lower in period 1 *and strictly higher in period* 2*:*

$$\epsilon_1^O P_1^{T*} < \epsilon_1^{LF} P_1^{T*}, \qquad \epsilon_2^O P_2^{T*} > \epsilon_2^{LF} P_2^{T*}$$

v. Real labor income is strictly higher in period 1 *and strictly lower in period* 2*:*

$$Y_1^{N,O} > Y_1^{N,LF}, \qquad Y_2^{N,O} < Y_2^{N,LF}$$

vi. Under sufficiently weak conditions established in Lemma 6, input prices and labor incomes are relatively stabilized:

$$|\epsilon_1^O P_1^{T*} - \epsilon_2^O P_2^{T*}| < |\epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}|$$
$$\left|Y_2^{N,O} - Y_1^{N,O}\right| < \left|Y_2^{N,LF} - Y_1^{N,LF}\right|$$

Proof. See Appendix A.7. □

In the high input price period, exchange rate policy suppresses exports below their laissez-faire level and retains a larger share of domestic output within the economy. Due to inframarginal price effects on foreigners and Ricardians, the resources thus retained flow disproportionately toward labor incomes, and since HtM households consume their current labor income without any recourse to asset markets, this channel raises their consumption in states when the import prices are high. In future states when the shock has subsided, the exchange rate depreciates to service the accumulated external obligation, reversing the resource transfer: the export share rises, the domestic share contracts, and real wages fall. The net effect is a partial but meaningful smoothing of the income and consumption path of HtM households relative to laissez-faire: exchange rate policy provides a degree of implicit insurance against transitory import price shocks even in the absence of direct fiscal transfers or explicit insurance markets. Proposition 3 summarizes the corresponding changes in output shares.

Proposition 3. *Assume that conditions of Proposition 2 hold. Relative to the laissez-faire equilibrium, the Ramsey equilibrium involves:*

- *a lower export share of output, a higher domestic share, a higher HtM share of output, and an ambiguous change in the Ricardian share of output in the high input price period, i.e., at* $t = 1$.
- *a higher export share of output and a lower domestic share, a lower HtM and Ricardian share of output in the low input price period, i.e., at* $t = 2$.

Proof. See Appendix A.8. □

We interpret the intertemporal policy discussed in this subsection as the real income redistribution channel of exchange rate policy, which in some ways is the open economy analogue of the intertemporal substitution real income channel of monetary policy in closed-economy HANK models, albeit with important differences. In closed-economy models, monetary policy typically works through the domestic real interest rate: when the real rate changes, unconstrained households move spending across time, and changes in aggregate demand then influence employment, affecting the real incomes of borrowing-constrained households. By contrast, intertemporal distortions in our model influence the path of the real exchange rate by changing market clearing conditions in foreign exchange markets. By distorting the intertemporal margin, the public agent can change the net supply of foreign purchasing power in the domestic economy's foreign exchange market, which in turn changes the real exchange rate and imported-input costs. Firms pass these cost changes into wages, thereby affecting the real incomes of constrained households. Therefore, the real income redistribution in our framework is delivered through the real exchange rate, even when employment is exogenously fixed. Moreover, in a small open economy, the laissez-faire intertemporal price is disciplined by international asset markets, and monetary policy cannot distort this margin. This necessitates a separate policy instrument that directly distorts the intertemporal margin, which in our analytical model is the capital control tax. In the quantitative model, we show that the constrained efficient allocation is implementable through FX interventions. Consequently, neither a freely floating nor a fixed real exchange rate implements the constrained efficient allocation; the optimal policy endogenously requires a managed float.

However, monetary policy in an open economy plays a role in distorting the labor wedge. In the next subsection, we relax the inelastic labor supply assumption to isolate this role and show that household heterogeneity modifies the optimal labor wedge beyond the static terms-of-trade correction identified in the representative-agent framework.

3.4.2 Elastic Labor Supply

The preceding subsection fixes employment at $\bar{N}$ and characterizes the optimal intertemporal wedge in isolation. This subsection relaxes that assumption: we endogenize the level of employment and input utilization to characterize the optimal labor wedge. We interpret this labor wedge as implementable through monetary policy in a decentralized economy with nominal rigidities. Our analytical framework maintains a reduced-form specification wherein the public agent directly chooses employment. We characterize the jointly optimal exchange rate and monetary policies in our heterogeneous-agent economy facing a transitory import price shock, $P_1^{T*} > P_2^{T*}$.

The representative-agent framework of Section 3.3 establishes that monetary policy already has a role: the labor wedge corrects for the terms-of-trade distortion that arises because competitive firms do not internalize the effect of their employment and import decisions on the real exchange rate. Facing an upward-sloping export-supply rule, a change in their collective import demand depreciates the real exchange rate and the economy must give up more domestic output to foreigners against all inframarginal units of foreign purchasing power supplied by them. The public agent internalizes this and restricts the economy's employment and import levels below laissez-faire through a strictly positive labor wedge, $\tau_t^N > 0$. In Section 3.3 we showed that a Ramsey planner values a marginal unit of employment at the effective marginal product, i.e., the net output retained by the economy for a marginal increase in employment, defined as $\hat{w}$:

$$\hat{w}_t = w_t - \frac{\epsilon_t P_t^{T*} \theta}{\chi}$$

where the second term is the terms-of-trade correction. The planner's first-order condition with respect to employment in the representative-agent economy is given by $v'(N_t) = \hat{w}_t u'(C_t)$. To isolate any distortion to the labor wedge beyond the static terms-of-trade correction, we define the terms-of-trade-adjusted labor wedge as:

$$1 - \hat{\tau}_t^N \equiv \frac{\nu'(N_t)}{\hat{w}_t u'(C_t)} \tag{35}$$

Proposition 1 establishes that the representative-agent constrained efficient allocation satisfies $\hat{\tau}_t^N = 0$ and $\tau = 0$ for any sequence of import prices.

Define the RA-equivalent policy as the one that imposes these same restrictions in the heterogeneous-agent economy: $\hat{\tau}_t^N = 0$ for $t \in \{1, 2\}$ and $\tau = 0$. In other words, the RA-equivalent policy satisfies the undistorted Ricardian Euler equation and implements a labor wedge that just corrects for the static terms-of-trade distortion with no distortions beyond that. The RA-equivalent policy together with the intertemporal budget constraint, satisfies:

$$\epsilon_1 u'(C_1^r) = \beta R^* \epsilon_2 u'(C_2^r) \tag{36}$$

$$\nu'(N_t) = \hat{w}_t \bar{u}'_t \qquad t \in \{1, 2\} \tag{37}$$

In what follows, we derive the jointly optimal intertemporal and labor wedges in the heterogeneous-agent economy and compare them to the RA-equivalent policy wedges. While the sign of the optimal intertemporal wedge is the same as in the previous subsection, we show that the optimal terms-of-trade-adjusted labor wedge is strictly positive, $\hat{\tau}_t^N > 0$ whenever $P_1^{T*} \neq P_2^{T*}$. In other words, the optimal monetary policy is strictly contractionary in proportionate terms relative to the RA-equivalent policy.

A perturbation to N_t. Before characterizing the planner's optimality condition with respect to employment, it is useful to consider the partial effect of a perturbation to N_t, $dN_t > 0$, on equilibrium outcomes, holding the trade balance, NX_t, fixed. Since the trade balance is fixed, the perturbation is purely intratemporal. An increase in the net labor supply, through labor-market clearing, requires that firms hire all additional labor, which raises domestic output in equilibrium. The domestic goods resource constraint, (1), together with the domestic output expression, (5), implies:

$$A_t \, dN_t = (1 - \lambda) dC_t^r + \lambda dC_t^p + dX_t \tag{38}$$

This equation shows that the additional output must be divided between domestic consumption and exports. Moreover, since labor and imported inputs are perfect complements, the labor market and the foreign exchange market must adjust simultaneously. An increased labor supply creates downward pressure on the real wage, lowering the marginal cost and creating positive profit opportunities for firms. Firms attempt to expand employment and simultaneously increase their demand for the imported input.

Holding NX_t fixed, the additional foreign purchasing power needed to pay for the imported input must come from higher export revenue. Because the supply of foreign purchasing power is increasing in the real exchange rate, the real exchange rate depreciates. Taking the total differential of the trade balance, $NX_t = \zeta\epsilon_t^{\chi} - P_t^{T*}\theta N_t$ and imposing $dNX_t = 0$ gives:

$$d\epsilon_t = \frac{P_t^{T*}\theta}{\chi\zeta\epsilon_t^{\chi-1}}\, dN_t > 0 \tag{39}$$

In equilibrium, zero profits must be restored: as the real exchange rate depreciates, the real wage must fall one-for-one until positive profit opportunities have been eliminated:

$$dw_t = -P_t^{T*}\theta\, d\epsilon_t = -\frac{(P_t^{T*}\theta)^2}{\chi\zeta\epsilon_t^{\chi-1}}\, dN_t < 0 \tag{40}$$

Thus, holding the trade balance fixed, higher employment raises output but depreciates the real exchange rate and lowers the real wage. The net effect on labor income is therefore ambiguous. While the direct effect of higher employment is to raise labor income, the indirect general-equilibrium effect through the real exchange rate and real wage is to lower labor income:

$$dY_t^N = w_t\, dN_t + N_t\, dw_t = \left(A_t - \epsilon_t P_t^{T*}\theta - \frac{(P_t^{T*}\theta)^2}{\chi\zeta\epsilon_t^{\chi-1}} N_t\right) dN_t \gtreqless 0 \tag{41}$$

where the first term, $w_t\, dN_t$, is the direct effect of higher employment on labor income: higher employment raises labor income at the prevailing wage, while the second term, $N_t\, dw_t$, is the negative general-equilibrium effect through the real exchange rate and real wage: higher employment depreciates the real exchange rate and lowers the real wage, reducing labor income. After rearranging, the net effect on labor income can be expressed in terms of the effective marginal product of labor, $\hat{w}_t$:

$$dY_t^N = \hat{w}_t dN_t + NX_t d\epsilon_t \tag{42}$$

where $\hat{w}_t$ is the net marginal output retained domestically, after accounting for the depreciation-led increase in exports: $\hat{w}_t = A_t - dX_t/dN_t$.

Since HtM households consume their current labor income, we have $dC_t^p = dY_t^N$. The Ricardian households are also affected by the change in labor income, but they also face a revaluation effect on their foreign financial flows:

$$dC_t^r = dY_t^N - \frac{1}{1-\lambda} NX_t d\epsilon_t$$

where the second term captures the revaluation effect, the sign of which depends on the sign of NX_t: if they are net borrowers ($NX_t < 0$), the revaluation effect is negative, and it is positive if they are net savers ($NX_t > 0$). The net changes in consumption of the two household types can therefore be expressed as:

$$dC_t^p = \hat{w}_t dN_t + NX_t d\epsilon_t \tag{43}$$

$$dC_t^r = \hat{w}_t dN_t - \frac{\lambda}{1-\lambda} NX_t d\epsilon_t \tag{44}$$

The above equations imply that the revaluation effect on Ricardians is equivalent to a net transfer of resources between the two household types. For a marginal increase in employment, the domestic economy gains $\hat{w}_t$ units of output, but the distribution of these resources between the two household types is determined endogenously through the real exchange rate and the sign of the country's trade balance. If the country is a net importer/net borrower ($NX_t < 0$), a real depreciation implies an endogenous transfer from HtM households to Ricardians, and if the country is a net exporter/net saver ($NX_t > 0$), the direction of the transfer is reversed.

To summarize, a marginal increase in employment, holding the trade balance fixed, induces a real depreciation and a fall in real wages. This depreciation raises exports and allows firms to import and produce more, and the domestic economy potentially gains resources from this increased production. However, the real depreciation creates a general-equilibrium revaluation effect on Ricardian financial flows, due to which such gains are unequally distributed between the two household types. In a period where Ricardians are net borrowers, the revaluation effect is equivalent to a net transfer of resources from HtM households to Ricardians, and in a period where Ricardians are net savers, the transfer is from Ricardians to HtM households.

The planner's first-order condition. The perturbation exercise above shows that changing employment creates general-equilibrium revaluation effects through real exchange rate movements, which affect the consumption of the two household types differently. A Ramsey planner internalizes these general-equilibrium effects. In Appendix A we derive the planner's first-order condition with respect to employment, which we restate below:

$$\nu'(N_t) = \hat{w}_t \bar{u}'_t + \lambda\left[u'(C^p_t) - u'(C^r_t)\right] NX_t \frac{d\epsilon_t}{dN_t}\bigg|_{NX_t} \tag{45}$$

The left-hand side is the social marginal disutility of labor. The first term on the right-hand side captures the effective gain to the domestic economy from a marginal increase in employment: after accounting for the terms-of-trade distortion, the economy retains $\hat{w}_t = A_t - \frac{1+\chi}{\chi}\epsilon_t P^{T*}_t \theta$ units of output from a marginal increase in employment, which the planner values at the average marginal utility. The second term captures how the endogenous real-exchange-rate-induced revaluation effect redistributes resources between the two household types. A marginal increase in employment requires the economy to import more inputs since labor and imported inputs are perfect complements. Higher imports require the economy to export more, which requires a real depreciation. If Ricardians are net borrowers ($NX_t < 0$), the revaluation effect is equivalent to a net transfer of resources from HtM households to Ricardians, and if they are net savers ($NX_t > 0$), the direction of the transfer is reversed. The planner values this transfer at the marginal utility difference between the two types: how much more HtM households value these resources relative to Ricardians. This revaluation effect disappears in a representative-agent economy, where $\lambda = 0$; when trade is intratemporally balanced, $NX_t = 0$; or when the two household types consume the same amount of the good, $C^p_t = C^r_t$.

Contractionary terms-of-trade-adjusted labor wedge under $NX_t \neq 0$. Given any sequence of import prices and productivity, $\{P^{T*}_t, A_t\}$, the constrained efficient allocation satisfies the intertemporal budget constraint, $NX_1 + R^{*-1}NX_2 = 0$, together with the optimal intertemporal condition, (32), derived in the previous subsection, and the intratemporal optimality conditions, (45), for each t. The optimal terms-of-trade-adjusted labor wedge can be expressed as:

$$\hat{\tau}^N_t = -\frac{\lambda\left[u'(C^p_t) - u'(C^r_t)\right] NX_t \frac{d\epsilon_t}{dN_t}\big|_{NX_t}}{\hat{w}_t \bar{u}'_t} \tag{46}$$

It follows that if this solution satisfies $NX_t = 0$ then employment choice just accounts for the terms-of-trade distortion, and the optimal terms-of-trade-adjusted labor wedge is zero: $\hat{\tau}^N_t = 0$. However, if the optimal solution involves $NX_t \neq 0$, then the optimal terms-of-trade-adjusted labor wedge is always strictly positive. This is because, with strictly concave utility, $u(.)$, the sign of $[u'(C^p_t) - u'(C^r_t)]NX_t$ is always negative whenever $NX_t \neq 0$. This follows from $C^r_t - C^p_t =$

$-\epsilon_t NX_t/(1-\lambda)$. If Ricardians are net borrowers in a period ($NX_t < 0$), then they consume more than HtM households, and consequently have a lower marginal utility. Similarly, if Ricardians are net savers in a period ($NX_t > 0$), then they consume less than HtM households, and have a higher marginal utility. Due to exchange-rate-induced revaluation effects that redistribute resources between the two household types, the planner always considers expanding employment to be more costly at the margin than the disutility from labor. This is because this endogenous redistribution always transfers resources from the higher marginal utility household type to the lower marginal utility household type, which goes against the direction of redistribution that a utilitarian planner would prefer. Consequently, whenever $NX_t \neq 0$, the optimal terms-of-trade-adjusted labor wedge is always strictly positive: $\hat{\tau}_t^N > 0$. Redistributive revaluation effects therefore introduce a role for monetary policy in the heterogeneous-agent economy that is absent in the representative-agent economy. In proportionate terms, the optimal monetary policy is always weakly more contractionary than in a representative-agent economy facing the same exogenous variables. Proposition 4 formally summarizes this result.

Proposition 4. *Given any* $\{P_t^{T*}, A_t\}$, *and* $\lambda \in (0,1)$, *consider the constrained efficient allocation that solves* (16). *Assume that it satisfies* $\hat{w}_t > 0$ *for* $t \in \{1,2\}$. *Then this allocation involves a strictly positive terms-of-trade-adjusted labor wedge,* $\hat{\tau}_t^N = 1 - \frac{\nu'(N_t)}{\hat{w}_t \bar{u}'_t} > 0$, *whenever* $NX_t \neq 0$.

Proof. See Appendix A.9. □

Optimal policy under $P_1^{T*} = P_2^{T*}$. The preceding discussion allows us to characterize optimal policy in this case almost immediately because it is analogous to the representative-agent case discussed in Proposition 1 and Lemma 2 in the previous subsection. When import prices are equal across periods and $\beta R^* = 1$, the primitives of the planner's problem are symmetric across the two dates. Given this symmetry, a date-asymmetric allocation would only create dispersion in consumption and labor across otherwise identical periods. Averaging such an allocation across the two dates preserves feasibility and, by concavity of consumption utility and convexity of labor disutility, weakly raises welfare. Hence the constrained efficient allocation is symmetric: $N_1 = N_2$ and $\epsilon_1 = \epsilon_2$. Therefore, the two periods must then have the same trade balance; the intertemporal budget constraint implies that $NX_1 = NX_2 = 0$. Ricardians neither borrow nor save, so both household types consume their current labor income in each period and $C_t^p = C_t^r$ for $t \in \{1,2\}$.

With balanced trade and equal consumption across household types, the revaluation effect term in the planner's FOC with respect to employment, (45), vanishes. The employment optimality condition reduces to $\nu'(N_t) = \hat{w}_t \bar{u}'_t$, which is the same as RA-equivalent policy condition (37), i.e., the solution involves a strictly positive labor wedge that accounts for the static terms-of-trade distortion: $\tau^N_t = \frac{1}{\chi}\frac{\epsilon_t P^{T*}\theta}{w_t} > 0$, and the terms-of-trade-adjusted labor wedge is zero: $\hat{\tau}^N_t = 0$. Since the planner chooses a symmetric allocation, it satisfies the undistorted Ricardian Euler equation, so $\tau = 0$. The constrained efficient allocation therefore coincides with the RA-equivalent policy in this case. Proposition 5 formally summarizes optimal policy with symmetric primitives.

Proposition 5. *Let* $\lambda \in (0,1)$, $A_t = A > 0$, $P^{T*}_t = P^{T*} > 0$ *for* $t \in \{1,2\}$, *and* $\beta R^* = 1$. *Assume the conditions of Lemma 5. Consider the constrained efficient allocation that solves* (16). *This solution involves:*

- *No intertemporal distortion:* $\tau = 0$.
- *A strictly positive labor wedge implementing the optimal terms-of-trade:* $\tau^N_t = \frac{1}{\chi}\frac{\epsilon_t P^{T*}\theta}{w_t} > 0$, *i.e., the terms-of-trade-adjusted labor wedge is zero:* $\hat{\tau}^N_t = 0$.
- *Symmetric equilibrium across periods:* $N_1 = N_2$, $\epsilon_1 = \epsilon_2$, $NX_1 = NX_2 = 0$, *and* $C^p_1 = C^r_1 = C^p_2 = C^r_2$.

Proof. See Appendix A.10. □

Optimal policy under $P^{T*}_1 > P^{T*}_2$. When the import price is temporarily higher in period 1, the RA-equivalent policy no longer implements the constrained efficient allocation. In the representative-agent benchmark, the planner leaves the intertemporal margin undistorted and uses the labor wedge only to correct the static terms-of-trade distortion. With borrowing-constrained households, the utilitarian planner instead chooses both a positive intertemporal wedge and a positive terms-of-trade-adjusted labor wedge. The intertemporal wedge influences the real exchange rate path and redistributes resources toward labor incomes and away from exports in the high-price period and reverses this pattern in the low-price period. The labor wedge influences employment and imported-input demand within each period, accounting for terms-of-trade effects and for exchange-rate-induced revaluation effects on Ricardian financial flows that redistribute resources between the two household types. Proposition 6 formally summarizes that implementing the constrained efficient allocation requires both intertemporal and intratemporal distortions. The optimal policy prescription in a heterogeneous-agent economy facing a transitory import price shock therefore differs from the representative-agent benchmark: it requires a combination of exchange rate policy that distorts the intertemporal margin and monetary policy that distorts the consumption-labor margin, both performing independent roles in optimally redistributing resources across time and between household types.

Proposition 6. *Let* $\lambda \in (0,1)$, $A_t = A > 0$, $P_1^{T*} > P_2^{T*} = P^{T*} > 0$, *and* $\beta R^* = 1$. *Assume the conditions of Lemma 5. Consider the constrained efficient allocation that solves (16). Assume that this solution satisfies* $\hat{w}_t > 0$ *for* $t \in \{1,2\}$. *Assume further that* $0 < \chi \leqslant 1$ *and preferences satisfy* $u'(C) = C^{-\sigma}$ *with* $\sigma > 1/(1+\chi)$. *Then the constrained efficient allocation involves:*

- *A strictly positive intertemporal wedge,* $\tau > 0$.
- *A strictly positive terms-of-trade-adjusted labor wedge,* $\hat{\tau}_t^N > 0$ *for* $t \in \{1,2\}$.

Proof. See Appendix A.11. □

Proposition 6 only allows comparison between optimal policy and the RA-equivalent policy in proportionate terms: signs and magnitudes of the intertemporal and labor wedges. These wedge comparisons do not imply an unambiguous ranking of employment levels across policy regimes. The revaluation term in (45) tends to lower employment in period 1. At the same time, a positive intertemporal wedge appreciates the period-1 real exchange rate relative to the RA-equivalent allocation. This appreciation raises $\hat{w}_1$ and tends to raise period-1 employment. The ranking of N_1 and similarly N_2 across regimes is therefore ambiguous. While not globally provable, using numerical simulations in the next section, we show that under reasonable parameterizations, the statements of Propositions 2 and 3 hold true even under elastic labor supply: optimal policy appreciates the real exchange rate and worsens the trade balance in the high-price period and depreciates the exchange rate and improves the trade balance in the low-price period, making them intertemporally more volatile when compared to the RA-equivalent allocation. At the same time, optimal policy stabilizes the domestic-units price of the imported input, i.e., the terms of trade, $\epsilon_t P_t^{T*}$, and consequently stabilizes domestic labor income, Y_t^N, making them less volatile across periods relative to the RA-equivalent allocation. Finally, optimal policy lowers the export share of output and raises the HtM share of output in the high-price period, and reverses this pattern in the low-price period, relative to the RA-equivalent allocation.

This subsection has established a real income redistribution role for monetary policy that parallels in many ways the earnings channel of monetary policy emphasized in closed-economy HANK models, albeit with important differences. By influencing the level of inflation, monetary policy distorts the labor wedge and directly affects employment levels and therefore the real incomes of households. In these models, monetary expansions after adverse productivity shocks can raise employment and current earnings of constrained households. The implication is that monetary policy is relatively more expansionary than in a representative-agent world. In our import-dependent open economy studied here, employment is complementary to imported inputs. Given states with high import prices, higher employment raises the import bill; with an upward-sloping supply of foreign purchasing power, the required increase in imports depreciates the real exchange rate. The depreciation lowers real wages and revalues Ricardian financial flows. This revaluation effect distributes the aggregate real income gains from higher employment disproportionately toward borrowers, i.e., Ricardian households, and away from HtM households who are borrowing constrained. A utilitarian planner internalizes this endogenous redistribution and considers employment expansion in high-price states to be costly. Thus the earnings channel calls for a more contractionary labor wedge and monetary policy than in the representative-agent world.

The next section embeds the analytical framework developed in this section in an infinite-horizon quantitative model with nominal wage rigidities and financial intermediation frictions. In this extended environment, FX interventions decentralize the intertemporal policy, while monetary policy sets the inflation rate and controls the labor wedge. The quantitative model carries over the economic intuition and compares optimal policy to alternative policies under MIT shocks to import prices.

4 Quantitative Model

This section extends the analytical framework to an infinite-horizon quantitative economy. The real side of the model is unchanged: domestic output is produced with labor and an imported input, households differ in their access to asset markets, the economy faces downward-sloping export demand, and direct transfers to HtM households remain unavailable. What changes is the decentralization. Ricardian households now trade domestic nominal bonds, international financial intermediaries link domestic returns to external financing, unions set wages under nominal rigidity, monetary policy sets the nominal interest rate, and fiscal policy conducts foreign exchange intervention through its foreign asset position. The Ramsey problem in this environment selects the constrained efficient allocation, with the labor and intertemporal wedges of the analytical model now implemented by monetary policy and by the public agent's foreign asset positions. We calibrate the model to an energy-importing economy and compare optimal policy with restricted policy regimes along two imported-energy price paths, a transitory spike and the realized 2021–2023 sequence.

4.1 Environment

Time is discrete and indexed by $t \geqslant 1$. The economy is deterministic and faces an exogenously specified path of imported input prices and total factor productivity, $\{P_t^{T*}, A_t\}_{t \geqslant 1}$ where $P_t^{T*} > 0$ and $A_t > 0$ for all t and bounded. The preferences, household types, final domestic good production technology, export demand, and relative-price definitions are otherwise as in Section 3. The quantitative environment differs from the analytical model in the asset market structure and in the decentralization of public policy. The intertemporal policy is implemented through foreign exchange interventions in the presence of financial intermediation frictions, while the labor wedge is implemented through monetary policy in the presence of nominal wage rigidities. We describe each component of the quantitative model in the following subsections.

4.1.1 Households

As before, a fraction $(1 - \lambda)$ of households are Ricardian and a fraction λ are HtM. Ricardian households now trade one-period domestic nominal bonds. Let $B_{t+1}^{M,r}$ denote nominal bond holdings chosen in period t, and let R_{t+1}^{M} denote the gross nominal return paid on bonds carried into period $t + 1$. Ricardian households solve

$$\max_{\{C_t^r, B_{t+1}^{M,r}\}_{t \geqslant 1}} \sum_{t=1}^{\infty} \beta^{t-1} \left[u(C_t^r) - \nu(N_t) \right] \quad \text{s.t.} \tag{47}$$
$$P_t C_t^r + B_{t+1}^{M,r} = W_t N_t + R_t^M B_t^{M,r} + T_t$$

Let $\Pi_{t+1} \equiv P_{t+1}/P_t$ and $R_{t+1} \equiv R_{t+1}^{M}/\Pi_{t+1}$ denote gross inflation and the gross real interest rate in domestic units. The solution to the Ricardian household problem is characterized by an Euler equation:

$$\frac{u'(C_t^r)}{\beta u'(C_{t+1}^r)} = R_{t+1} \tag{48}$$

and a no-Ponzi-transversality condition:

$$\lim_{T\to\infty} \frac{B_{T+1}^r}{\prod_{s=2}^{T} R_s} = 0 \tag{49}$$

where $B_{t+1}^r \equiv B_{t+1}^{M,r}/P_t$ denotes the Ricardian household's bond holdings in real domestic units. While Ricardian households can participate in domestic asset markets, they do not have the ability to directly participate in international asset markets. HtM households remain excluded from all asset markets and direct fiscal transfers. They consume their current labor incomes each period:

$$C_t^p = w_t N_t \tag{50}$$

As before, both household types work the same number of hours in equilibrium, N_t. The hours are not chosen by households. We describe the labor market structure below.

4.1.2 Labor Aggregator

The final good production technology remains unchanged from the analytical model. Firms now use a final labor service together with imported inputs to produce the final good. The final labor service, N_t, is produced by a competitive labor packer who aggregates over $k \in [0, 1]$ tasks performed by task-specific labor, $N_{k,t}$, using a CES technology:

$$N_t = \left(\int_0^1 N_{k,t}^{\frac{\varepsilon_w - 1}{\varepsilon_w}}\, dk\right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \qquad \varepsilon_w > 1$$

Let $W_{k,t}$ denote the nominal wage for task k. Cost minimization gives task demand and the aggregate wage index:

$$N_{k,t} = \left(\frac{W_{k,t}}{W_t}\right)^{-\varepsilon_w} N_t, \qquad W_t = \left(\int_0^1 W_{k,t}^{1-\varepsilon_w}\, dk\right)^{\frac{1}{1-\varepsilon_w}} \tag{51}$$

4.1.3 Unions

We now describe the nominal wage setting and rigidity structure adapted from Erceg, Henderson and Levin (2000) and Auclert, Rognlie and Straub (2024). For each task, $k \in [0, 1]$, workers are represented by a union. Taking the aggregator's labor demand function for the task k as given, the union sets the nominal wage, $W_{k,t}$, and calls on all members to work the same number of hours, $N_{k,t}$. For each task, k, workers are representative of the population: a mass λ are HtM and $(1-\lambda)$ are Ricardian. The union seeks to maximize the population-weighted utility of its members. The union also faces quadratic utility costs for changing nominal wages. The union's optimization problem is to maximize:

$$\sum_{s\geqslant 0} \beta^s \left(\lambda u\left(C^p_{t+s}\right) + (1-\lambda)u\left(C^r_{t+s}\right) - \nu\left(N_{t+s}\right) - \frac{\psi_{nr}}{2}\left(\frac{W_{k,t+s}}{W_{k,t+s-1}} - 1\right)^2\right) \tag{52}$$

where $\psi_{nr} > 0$ parametrizes the degree of nominal wage rigidity. Each union is infinitesimal and only takes into account its marginal effect on every household's consumption and labor supply. Since each union's optimization problem is symmetric, in equilibrium all unions will have the same optimal wage and labor supply decisions: $W_{k,t} = W_t$ and $N_{k,t} = N_t$ for all k. In Appendix B.1 we derive the unions' optimal wage and labor supply decisions that yield the aggregate wage Phillips curve of the form:

$$\nu'(N_t) = (1-\tau^N_t) w_t \bar{u}'_t \tag{53}$$

where $(1-\tau^N_t)$ represents the labor wedge and is given by:

$$1-\tau^N_t = \frac{1}{\mu_w} + \frac{\left(\frac{w_t}{w_{t-1}}\Pi_t - 1\right)\frac{w_t}{w_{t-1}}\Pi_t - \beta\left(\frac{w_{t+1}}{w_t}\Pi_{t+1} - 1\right)\frac{w_{t+1}}{w_t}\Pi_{t+1}}{\frac{\varepsilon_w}{\psi_{nr}} w_t N_t \bar{u}'_t} \tag{54}$$

where $\mu_w \equiv \varepsilon_w/(\varepsilon_w - 1)$ is the wage markup and $\bar{u}'_t$ is the average marginal utility as defined in equation (19).

4.1.4 Financial Intermediaries

The financial intermediation structure in our model is adapted from Gabaix and Maggiori (2015). The world is populated by a unit mass of identical financial intermediaries who intermediate borrowing and lending transactions between domestic and world financial markets. These intermediaries are assumed to be owned outside the small open economy and take offsetting asset positions in domestic and world markets:

$$\frac{F_{t+1}}{e_t} + F^*_{t+1} = 0 \tag{55}$$

where F_{t+1} is the domestic asset position in units of domestic currency and F^*_{t+1} is the asset position in the international asset markets, in units of the world numeraire, i.e., the foreign good. A positive position denotes a saving and a negative position denotes a borrowing. The intermediaries' gross returns in foreign good units, ν_{t+1}, are given by:

$$\nu_{t+1} = R^{M}_{t+1}\frac{F_{t+1}}{e_{t+1}} + R^{*}F^{*}_{t+1} \tag{56}$$

There is a limited repayment commitment on the part of intermediaries: each period, immediately after taking their asset positions, an intermediary can divert a portion $\Gamma\left|F^{*}_{t+1}\right|$, with $\Gamma > 0$, of the position it intermediates: $\left|F^{*}_{t+1}\right|$. If the intermediary diverts the funds, the position is unwound and the lenders recover a portion $\left(1-\Gamma\left|F^{*}_{t+1}\right|\right)$ of their claims $\left|F^{*}_{t+1}\right|$. Since the lenders anticipate the incentives of the intermediary to divert funds, the intermediary is subject to a credit constraint such that discounted returns from intermediation are weakly higher than the returns from diverting the funds:

$$\frac{1}{R^{*}}\nu_{t+1} \geqslant \Gamma\left|F^{*}_{t+1}\right| \times \left|F^{*}_{t+1}\right| \tag{57}$$

The intermediary's constrained optimization problem is then given by:

$$\max_{\{F^{*}_{t+1}, F_{t+1}\}} \frac{1}{R^{*}}\nu_{t+1} \quad \text{s.t. to (55), (57)}$$

The solution to this optimization problem yields the intermediary's asset demand rule in the world asset markets:

$$F^{*}_{t+1} = \frac{1}{\Gamma}\left[1 - \frac{R^{M}_{t+1}}{R^{*}}\frac{e_t}{e_{t+1}}\right] = \frac{1}{\Gamma}\left[1 - \frac{R_{t+1}}{R^{*}}\frac{\epsilon_t}{\epsilon_{t+1}}\right] \tag{58}$$

Equation (58) shows that the intermediary's asset demand is linearly related to the interest parity gap $R_{t+1}\frac{\epsilon_t}{\epsilon_{t+1}} - R^{*}$. Define the domestic real interest rate in foreign units as $\tilde{R}_{t+1}$:

$$\tilde{R}_{t+1} \equiv R_{t+1}\frac{\epsilon_t}{\epsilon_{t+1}} \tag{59}$$

If $\tilde{R}_{t+1} = R^{*}$, intermediary asset demand is zero. When $\tilde{R}_{t+1} < R^{*}$, the intermediary saves in foreign markets and borrows in domestic markets; when $\tilde{R}_{t+1} > R^{*}$, the intermediary borrows in foreign markets and saves in domestic markets.

4.1.5 Monetary Policy

Monetary policy sets the path of domestic nominal interest rates, $\{R^{M}_{t+1}\}_{t\geqslant 1}$ where the nominal rate satisfies the Fisher equation:

$$R^{M}_{t+1} = R_{t+1}\Pi_{t+1} \tag{60}$$

4.1.6 Fiscal Policy

The public agent can directly participate in domestic as well as foreign asset markets, bypassing any private intermediaries. Let $B_{t+1}^{M,G}$ denote government domestic bond holdings in domestic currency units and let B_{t+1}^{G*} denote government's foreign asset position in units of the foreign good. The period-by-period government budget constraint in domestic currency units is given by:

$$B_{t+1}^{M,G} + e_t B_{t+1}^{G*} + (1-\lambda)T_t = R_t^M B_t^{M,G} + e_t R^* B_t^{G*}$$

where T_t denotes lump-sum taxes/transfers to Ricardian household. As in the analytical model, HtM households remain excluded from direct fiscal transfers by assumption. Define $B_{t+1}^G \equiv \frac{B_{t+1}^{M,G}}{P_t}$ as the government's domestic bond holdings in real domestic units. The government budget constraint in real domestic units is then given by:

$$B_{t+1}^G + \epsilon_t B_{t+1}^{G*} + (1-\lambda)\frac{T_t}{P_t} = R_t B_t^G + \epsilon_t R^* B_t^{G*} \tag{61}$$

The government is subject to no-Ponzi conditions in both markets:

$$\lim_{T\to\infty} \frac{B_{T+1}^G}{\prod_{s=2}^T R_s} = 0, \qquad \lim_{T\to\infty} \frac{B_{T+1}^{G*}}{R^{*\,T-1}} = 0 \tag{62}$$

together with a consistency condition on interest rates which ensures that government asset positions satisfy no-Ponzi conditions in domestic as well as foreign units:

$$0 < \inf_{T\geqslant 2} \prod_{s=2}^{T} \frac{\tilde{R}_s}{R^*} \leqslant \sup_{T\geqslant 2} \prod_{s=2}^{T} \frac{\tilde{R}_s}{R^*} < \infty \tag{63}$$

4.1.7 Domestic Bonds Market Clearing

Domestic bonds market clearing requires that the asset positions by Ricardians, government, and financial intermediaries must sum to zero: $(1-\lambda)B_{t+1}^{M,r} + B_{t+1}^{M,G} + F_{t+1} = 0$. Define the aggregate Ricardian position in real domestic units as $B_{t+1} \equiv (1-\lambda)B_{t+1}^r$. Using the intermediary balance sheet condition, (55), and expressing the bonds market clearing condition in real domestic units, we have:

$$B_{t+1} + B_{t+1}^G = \epsilon_t F_{t+1}^* \tag{64}$$

4.1.8 Balance of Payments and Intertemporal Budget Constraint

The balance of payments condition is implicitly satisfied, given household, firm, government budget constraints, and goods and domestic bonds market clearing conditions are satisfied. In Appendix B.2, we derive the period-by-period balance of payments condition which is given by:

$$P_t^{T*}\theta N_t + F_{t+1}^* + B_{t+1}^{G*} = \zeta\epsilon_t^X + R_t\frac{\epsilon_{t-1}}{\epsilon_t}F_t^* + R^*B_t^{G*} \tag{65}$$

The left-hand side represents the demand for foreign purchasing power by firms, private intermediaries, and the government at time t. The right-hand side represents the supply of foreign purchasing power from export revenues and inherited foreign positions. The BoP conditions represent the effective external budget constraint of the economy. The period-by-period constraints can be combined to obtain the intertemporal budget constraint:

$$\sum_{t=1}^{\infty}\frac{\frac{1}{\Gamma}\left(1-\frac{\tilde{R}_{t+1}}{R^*}\right)^2}{{R^*}^{t-1}} + \sum_{t=1}^{\infty}\frac{P_t^{T*}\theta N_t}{{R^*}^{t-1}} = \sum_{t=1}^{\infty}\frac{\zeta\epsilon_t^X}{{R^*}^{t-1}} + R^*B_1^{G*} + R_1\frac{\epsilon_0F_1^*}{\epsilon_1} \tag{66}$$

See Appendix B.2 for the derivation of this condition. The first term on the left-hand side represents the present discounted value of the carry cost associated with allowing interest parity deviations, i.e., allowing intermediaries to take non-zero asset positions in the domestic bonds market. This cost arises due to the assumption that intermediaries are owned outside the small open economy and must be compensated over and above the world interest rate for participating in the domestic bonds market. The second term represents the present discounted value of the import bill and first term on the right-hand side represents the present discounted value of the export revenues. The second and third terms represent inherited government and intermediary claims at $t = 1$.

4.1.9 Decentralized Equilibrium

Given the initial states: $\{R_1^M, B_1, B_1^{G*}, \epsilon_0, F_1^*, w_0\}$, exogenous sequence of import prices and productivity, $\{P_t^{T*}, A_t\}_{t=1}^{\infty}$, and fiscal-monetary policy: $\{B_{t+1}^G, B_{t+1}^{G*}, R_{t+1}^M, \frac{T_t}{P_t}\}_{t=1}^{\infty}, R_1$, a decentralized equilibrium consists of an allocation: $\{C_t^r, C_t^p, B_{t+1}, N_t, M_t, Y_t, F_{t+1}^*, X_t\}_{t=1}^{\infty}$ and prices $\{\epsilon_t, w_t, \Pi_t, R_{t+1}\}_{t=1}^{\infty}$ that satisfy all optimality and market clearing conditions. We define the decentralized equilibrium and list all equations that characterize the equilibrium in Appendix B.3.

4.2 Optimal Policy Problem

Analogous to the approach in the analytical section, we consider the primal problem of a utilitarian Ramsey planner who directly chooses $\{N_t, \epsilon_t, \tilde{R}_{t+1}, C_t^r, C_t^p\}_{t=1}^{\infty}$ to maximize welfare subject to implementability constraints implied by the decentralized equilibrium. The general Ramsey problem features a time-inconsistency problem which arises from the financial intermediation structure in the model: at $t = 1$, the claim due to the intermediaries in foreign units is given by $\frac{R_1^M}{\Pi_1}\frac{\epsilon_0 F_1^*}{\epsilon_1}$. Since the intermediaries lent in domestic currency units, Π_1 and ϵ_1 can be chosen to influence the ex-post foreign-units value of the inherited claim. For the quantitative exercise, we initialize the economy at the Ramsey steady state, in which A_t and P_t^{T*} are constant over time. In that steady state, $R = R^*$ and hence $F_1^* = 0$. This initialization removes the planner's incentive at $t = 1$ to revalue an inherited intermediary position. It does not, however, eliminate time inconsistency more generally: following a shock, the commitment allocation can generate nonzero intermediary positions that a future planner would inherit. We therefore interpret the solution to (99), stated in Appendix B.4, as a full-commitment Ramsey allocation initialized at the Ramsey steady state[6].

Lemmas 7, 8, 9 stated in Appendix B.4 together establish conditions under which the constrained efficient allocation, i.e., the solution to (99), is implementable as a decentralized equilibrium. Under these restrictions, the planner can choose N_t directly, and the wage Phillips curve specified by equations (53), (54) enters only in the decentralization. As in the analytical framework, choosing N_t directly chooses the optimal labor wedge. Inflation and the nominal interest rate path, $\{\Pi_t, R_{t+1}^M\}$ can be constructed ex-post to implement the constrained efficient allocation in a decentralized equilibrium. The planner chooses the intertemporal wedge through $\tilde{R}_{t+1}$. The quantitative model implements this wedge through public foreign exchange interventions. By choosing domestic and foreign asset positions, $\{B_{t+1}^G, B_{t+1}^{G*}\}$, the public agent implements the constrained efficient allocation with optimal intertemporal distortions; in the analytical framework the same wedge is represented by a direct capital control tax. One difference from the analytical framework is that intertemporal distortions are costly: the carry cost term in the intertemporal budget constraint, (66), that arises due to the financial intermediation friction represents a cost paid by the economy for departing

[6]Given $F_1^* = 0$ and $\beta R^* = 1$, if $A_t = A > 0$ and $P_t^{T*} = P^{T*} > 0$ for all t, the solution to (99) is a steady state: $C_t^r = C^r$, $C_t^p = C^p$, $N_t = N$, $\epsilon_t = \epsilon$, and $\tilde{R}_{t+1} = R^*$. We verify this property numerically. The resulting allocation has $F_{t+1}^* = 0$ and $B_{t+1}^{G*} = B_1^{G*}$ in every period, so reoptimization reproduces the same allocation when the economy remains in this no-shock steady state. This stationarity property does not imply time consistency along a transition after a shock.

from interest parity[7].

4.3 Quantitative Application: Energy Price Shocks and Optimal Policy

The theory has studied optimal policy in response to essential import price shocks in general terms; its leading application is fossil fuel price spikes in an energy-importing economy. Fossil fuels fit the theory's definition of an essential import. They are necessary to household and business energy needs, substitution away from them is limited in the short run, and their prices are set in US dollars in global markets that no single importer can influence, so a price spike raises an import-dependent country's energy expenditure almost in proportion to its energy intensity. The 2022 energy crisis brought Brent crude and European natural gas to multiples of their pre-pandemic levels (International Energy Agency 2022; U.S. Energy Information Administration 2026; European Securities and Markets Authority 2023), and the 2026 disruption to crude and LNG flows through the Strait of Hormuz reproduced the pattern within months (International Energy Agency 2026).

The quantitative exercise calibrates the model to Japan. Japan is among the world's largest energy importers: it imports nearly 90 percent of its primary energy, was the largest importer of liquefied natural gas in 2022, and ranks fifth in global crude oil consumption with imports accounting for 97 percent of demand (Agency for Natural Resources and Energy 2024). In 2022, fossil fuel import bills rose to roughly 4 percent of GDP and contributed to the largest annual trade deficit on record at 20 trillion yen (Nippon.com 2023). Japan also fits the particular sense in which the model economy is small and open. It is small with respect to world asset markets and world energy markets: it takes world interest rates and dollar energy prices as given. It is not small in the market for its own exports: Japanese output is differentiated, and foreign demand for it is downward sloping, so earning additional foreign purchasing power within a period requires selling more cheaply.

Japan's experience during the 2022 spike illustrates the amplification the theory studies. The dashed line in the left panel of Figure 1 plots an index of world energy prices in US dollars, which peaked 172 percent above its January 2021 level. Over the same period the yen depreciated by 42 percent against the dollar, and Japan experienced unusually high inflation, with the CPI peaking 5.4 percent above its January 2021 level. The right panel combines the two: the CPI-deflated yen-dollar exchange rate, a measure of the real exchange rate, peaked 36 percent above its initial level. The

[7] Therefore, apart from instruments that implement the constrained efficient allocation, the quantitative model of this section retains the structure of the analytical framework, and only the implementation of the wedges differs.

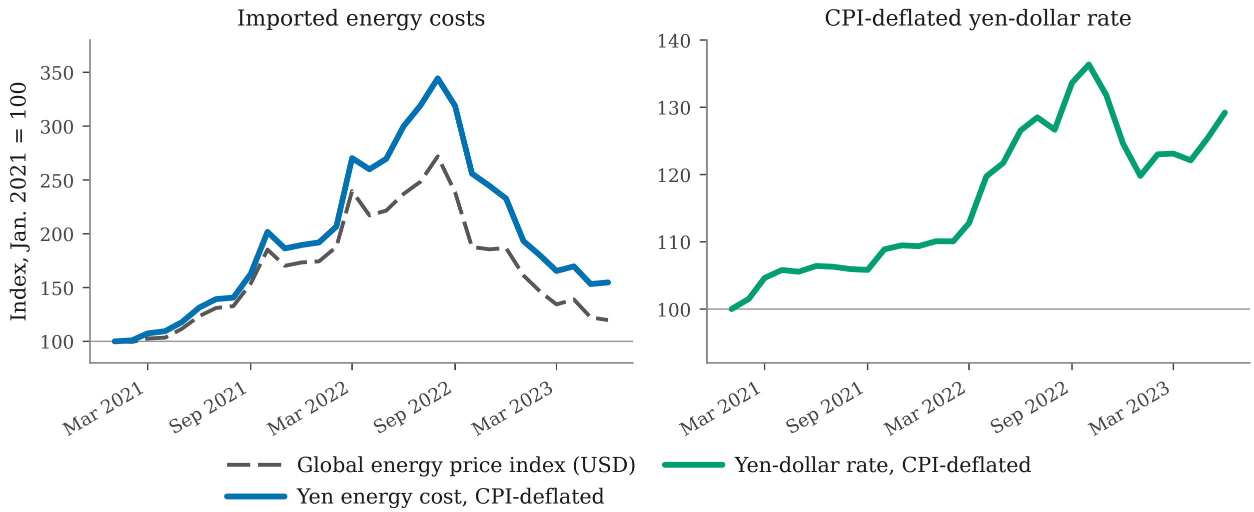

Figure 1. Energy prices and the yen, 2021–2023

Note: The left panel plots the IMF global energy price index in U.S. dollars and the same index converted into CPI-deflated yen. The right panel plots the CPI-deflated yen-dollar exchange rate. Monthly series are normalized to 100 in January 2021 and run through June 2023. Sources: IMF global energy price index and yen-dollar exchange rate from FRED; Japan CPI from the Statistics Bureau of Japan.

solid line in the left panel plots the price of imported energy in units of the Japanese consumption basket: at the peak, imported energy cost 1.27 times what it would have cost had only the world price risen with no real depreciation[8]. The remainder of this section characterizes jointly optimal exchange rate and monetary policies for the calibrated economy facing shocks of this kind, and asks what these policies would have prescribed over an episode like 2021–2023.

The quantitative exercise uses the calibrated economy to characterize the assignment of the two instruments: what exchange rate policy should do and through which margin, what monetary policy should do conditional on the exchange rate policy, and whether either instrument can reproduce the other's role in its absence. Two imported-energy price paths serve these questions. The first is an MIT shock, a perfect-foresight impulse response to a doubling of the energy price that decays at its estimated AR(1) persistence, which isolates the two roles. The second is a historical simulation along the realized path of world energy prices through the 2022 crisis, a deterministic hump-shaped sequence over 2021–2023 that peaks 157 percent above the pre-shock level, which gauges their quantitative importance. The exercise does not target a broad set of Japanese moments; it asks whether restricted asset market participation materially amplifies the domestic consequences of an energy price shock and how each instrument should respond. The model calibration is

[8]The figure is descriptive, and we do not attribute the real depreciation, in whole or in part, to the energy price spike. Under a downward-sloping export demand, however, a real depreciation accompanying an essential import price spike is what the theory predicts.

described in Appendix B.5.

We compare four policy regimes, each defined by the question it answers. The representative-agent-equivalent benchmark sets both the intertemporal wedge and the terms-of-trade-adjusted labor wedge to zero: it applies the policy corrections that are optimal without household heterogeneity. The interest-parity Ramsey policy chooses the labor wedge freely while suppressing the intertemporal wedge; it asks what monetary policy can accomplish when it cannot change the timing of foreign purchasing power. The FX-only policy chooses the intertemporal wedge freely while suppressing the terms-of-trade-adjusted labor wedge; it asks what exchange rate policy can accomplish when employment is not independently managed. The optimal policy chooses both.[9]

The representative-agent economy of Appendix B.6, in which no household is borrowing constrained and every household is on its Euler equation, fixes the reference point. Figures 6 and 7 report its allocation and implementation paths under the MIT shock. The two instruments separate along the lines of Itskhoki and Mukhin (2023): exchange rate policy is assigned to the financial friction, and monetary policy to the frictions in domestic production. The public agent's foreign asset position takes on the financial flows that intermediaries would otherwise price, so interest parity holds, no carry losses are incurred, and the intertemporal margin is left undistorted; monetary policy targets nominal rigidities and, in this economy, additionally corrects the static terms-of-trade distortion. Heterogeneity reverses the role of the intervention. Optimal policy in the heterogeneous economy no longer uses the public balance sheet to undo the intermediation friction: it opens interest parity gaps deliberately, and pays the carry losses that the friction attaches to them, because the gap is what tilts the path of the real exchange rate and, through firms' costs and the zero-profit condition, the real wages and current incomes of constrained households.

Figure 2 reports allocations and relative prices under the MIT shock. Under the RA-equivalent benchmark the real exchange rate depreciates as the energy price rises, so the domestic real price of energy rises 1.46 percent for every percent rise in the world price. Real wages, employment, and labor income fall; since HtM households consume their current labor income, the income loss passes directly into their consumption. Constrained households cannot borrow against future income, so

[9] We do not present laissez-faire policies in the comparison. A laissez-faire intertemporal policy is dominated even without household heterogeneity: private financial flows intermediated under the friction open interest parity gaps and carry losses that the public balance sheet can undo at no cost, which is why optimal policy in the representative-agent economy discussed below restores interest parity; Itskhoki and Mukhin (2023) characterize optimal exchange rate and monetary policy in a representative-agent economy with this class of financial frictions. The comparison here therefore focuses on how heterogeneity changes the role of policy beyond what representative-agent models already establish.

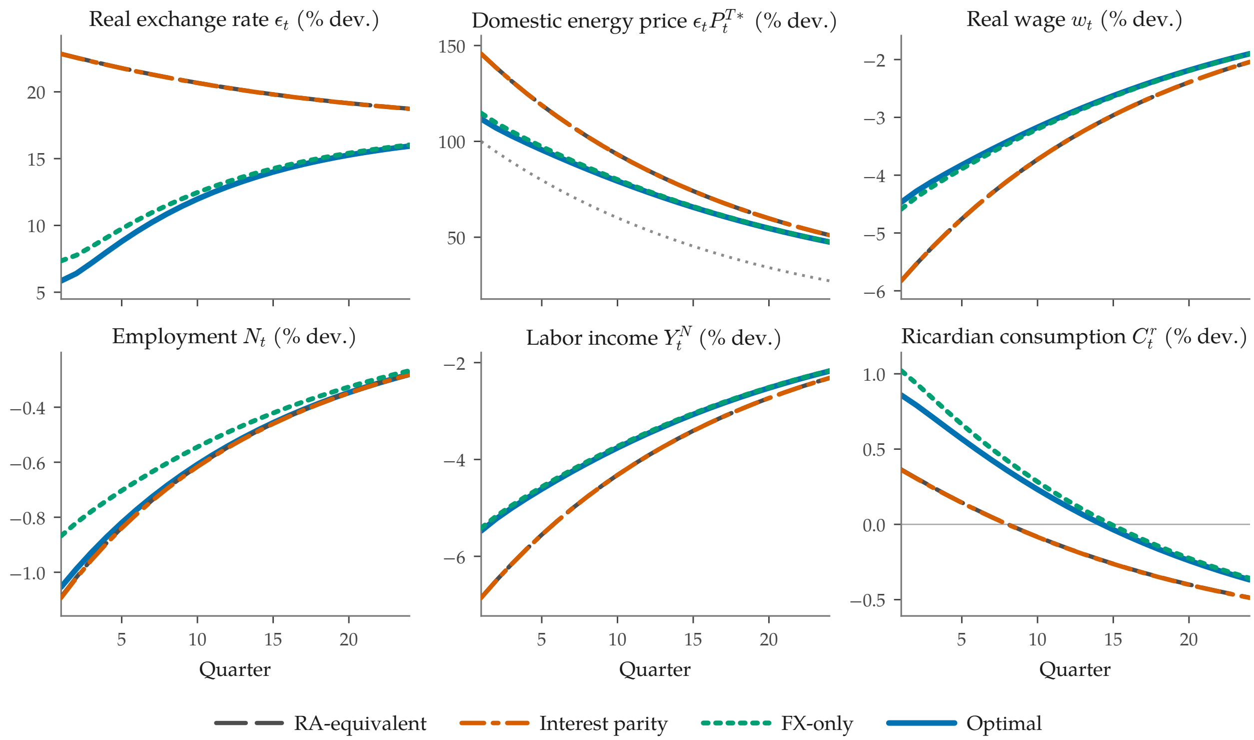

Figure 2. MIT shock: allocations and relative prices

Note: Percent deviations from the initial steady state. The dotted line is the world energy price.

the economy relies more heavily on contemporaneous export revenue to finance the import bill. Expanding exports requires a real depreciation, which further raises the domestic energy price and compresses real wages. In the representative-agent economy of Appendix B.6, the same shock produces a smaller depreciation, so 40 percent of the benchmark's impact depreciation reflects the borrowing that constrained households cannot undertake. Ricardian consumption instead rises on impact as households with access to asset markets borrow against the temporary loss, and then falls as the resulting foreign obligation is repaid. The interest-parity Ramsey policy barely departs from the benchmark. Monetary policy can choose employment, but it cannot move foreign purchasing power toward the high-price dates, and the employment response that is optimal without that ability is small; the remaining allocation paths are essentially unchanged.

Exchange rate policy operates on the margin monetary policy cannot reach. The FX-only policy borrows abroad in the high-price quarters, so borrowed foreign purchasing power replaces current export revenue; the real exchange rate appreciates relative to interest parity, the domestic cost of the imported input falls, and the real wage and labor income rise. This is most of the contemporaneous income stabilization that the joint optimum achieves. FX intervention alone, however, does not

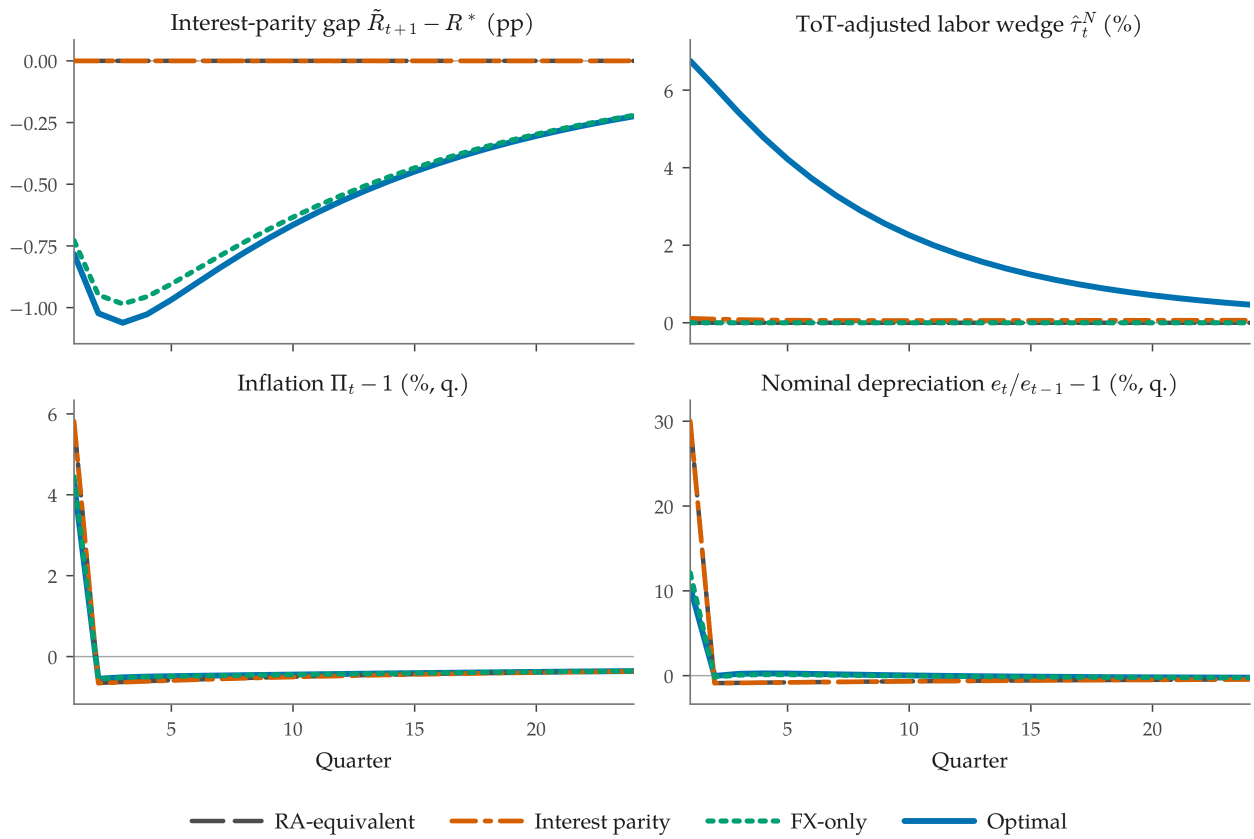

Figure 3. MIT shock: policy and implementation

determine employment or the division of the gain between wages and hours. With the labor wedge suppressed, the cheaper imported input induces firms to expand: employment and imports rise, financing the additional imports requires export revenue, and earning it gives back part of the appreciation. The FX-only policy therefore delivers slightly higher labor income than the joint optimum, through additional hours at a lower real wage. The resulting currency is weaker than under the joint optimum, which revalues the borrowed foreign resources in favor of Ricardian households; their consumption in Figure 2 is highest in this regime. These responses reverse as the foreign position is repaid. Exchange rate policy therefore changes the timing and incidence of the imported-resource loss; it does not remove that loss.

Monetary policy's role is defined conditional on the intervention. Figure 3 displays the two policy margins: the optimal policy opens a negative interest parity gap, the decentralized form of the positive intertemporal wedge, and sets a positive terms-of-trade-adjusted labor wedge. The wedge restrains employment relative to the FX-only allocation. Lower employment limits imported-input demand and the export revenue required to finance it, so less of the intervention-induced

Table 1: Analytical predictions under the MIT shock

Prediction	RA-equivalent	Optimal
Intertemporal response (Prop. 2)		
Real-exchange-rate volatility (%)	4.9	10.0
Real exchange rate, terminal steady state (% dev.)	17.09	17.61
Trade balance, high-price quarter (% of s.s. export revenue)	-106.5	-110.4
Trade balance, terminal steady state (% of s.s. export revenue)	4.06	4.18
Domestic price and labor income (Prop. 2)		
Volatility of domestic energy price (%)	109.8	80.0
Domestic energy price, terminal steady state (% dev.)	17.09	17.61
Volatility of labor income (%)	6.18	4.77
Labor income, terminal steady state (% dev.)	-0.72	-0.74
Volatility of the real wage (%)	5.18	3.79
Output shares (Prop. 3)		
Export share, high-price quarter (%)	4.40	3.65
Export share, terminal steady state (%)	4.10	4.12
HtM consumption share, high-price quarter (%)	31.69	32.15
HtM consumption share, terminal steady state (%)	33.424	33.417
Ricardian consumption share, terminal steady state (%)	62.48	62.46

appreciation is given back. Through the zero-profit condition, $w_t = A_t - \epsilon_t P_t^{T*}\theta$, the lower domestic imported-input cost then appears as a higher real wage rather than primarily as additional hours. The intervention thus determines the size of the current income gain, while the monetary stance determines its composition and its recipients: wages rather than hours, and labor income for all households rather than revaluation gains for the unconstrained. The nominal path that implements this allocation features lower inflation and substantially less front-loaded nominal depreciation than the RA-equivalent benchmark.

The output reallocation under the MIT shock is reported in Figure 10 of Appendix B.7. Relative to the RA-equivalent benchmark, borrowed foreign purchasing power replaces export revenue in the high-price quarters, so the export share falls and more output remains for domestic consumption. The appreciation lowers the import-bill share, while the higher real wage raises the HtM consumption share. The Ricardian share also rises slightly, a response that Proposition 3 leaves ambiguous in the high-price period. As the foreign position is repaid, the export and import-bill shares rise relative to the benchmark and both household consumption shares fall.

Table 1 sets the analytical predictions beside the calibrated responses under the MIT shock, and each of them holds. Volatility is measured as the maximum absolute deviation from the terminal steady state, so the permanent component that all regimes share does not enter. To stabilize the domestic price of energy and labor income, optimal policy makes the real exchange rate more volatile, while the domestic energy price, the real wage, and labor income all become less volatile:

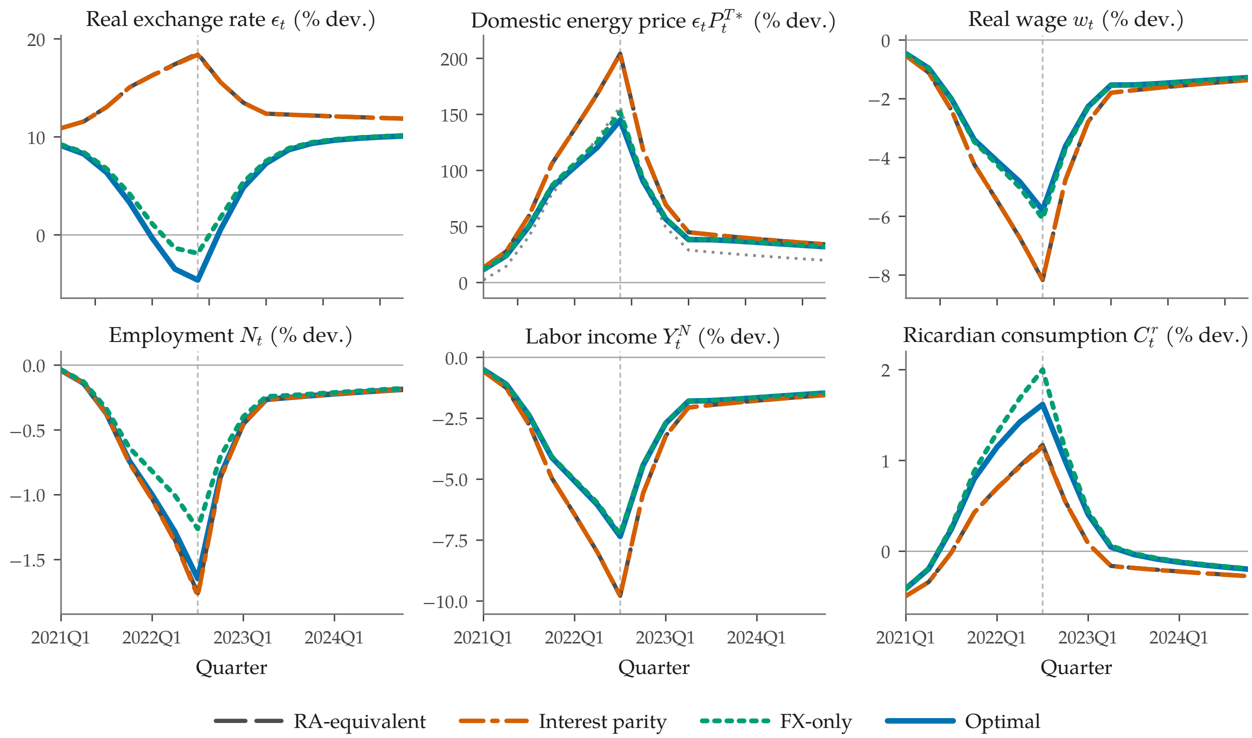

Figure 4. 2021–2023 shock: allocations and relative prices

Note: Percent deviations from the initial steady state. The dotted line is the world energy price; the vertical line marks its peak.

the exchange rate is the instrument of stabilization rather than its object. The high-price responses in Figure 2 and the terminal rows of the table show the same pattern in levels: the currency is stronger, the domestic energy price lower, and labor income higher while the import price is high, with each ranking reversed after the shock. The trade balance likewise moves from a larger deficit in the high-price quarter to a larger surplus later. The export and HtM shares move in the directions stated in Proposition 3, including the lower HtM and Ricardian shares after the shock. Finally, the negative interest parity gap and the positive terms-of-trade-adjusted labor wedge in Figure 3 verify the policy signs in Propositions 2, 4, and 6.

Figure 4 reports the historical simulation along the realized 2021–2023 path; the representative-agent reference paths for the same price sequence appear in Figures 8 and 9 of Appendix B.6. Relative to that reference, the borrowing that constrained households cannot undertake accounts for 71 percent of the RA-equivalent benchmark's peak depreciation. Under the benchmark, every percent rise in the world price of energy raises its domestic real price by 1.30 percent at the peak, close to the 1.27 in the Japanese data of Figure 1; under the optimal policy the factor is 0.92, so exchange rate policy more than eliminates the depreciation-driven amplification: at the peak, the domestic real

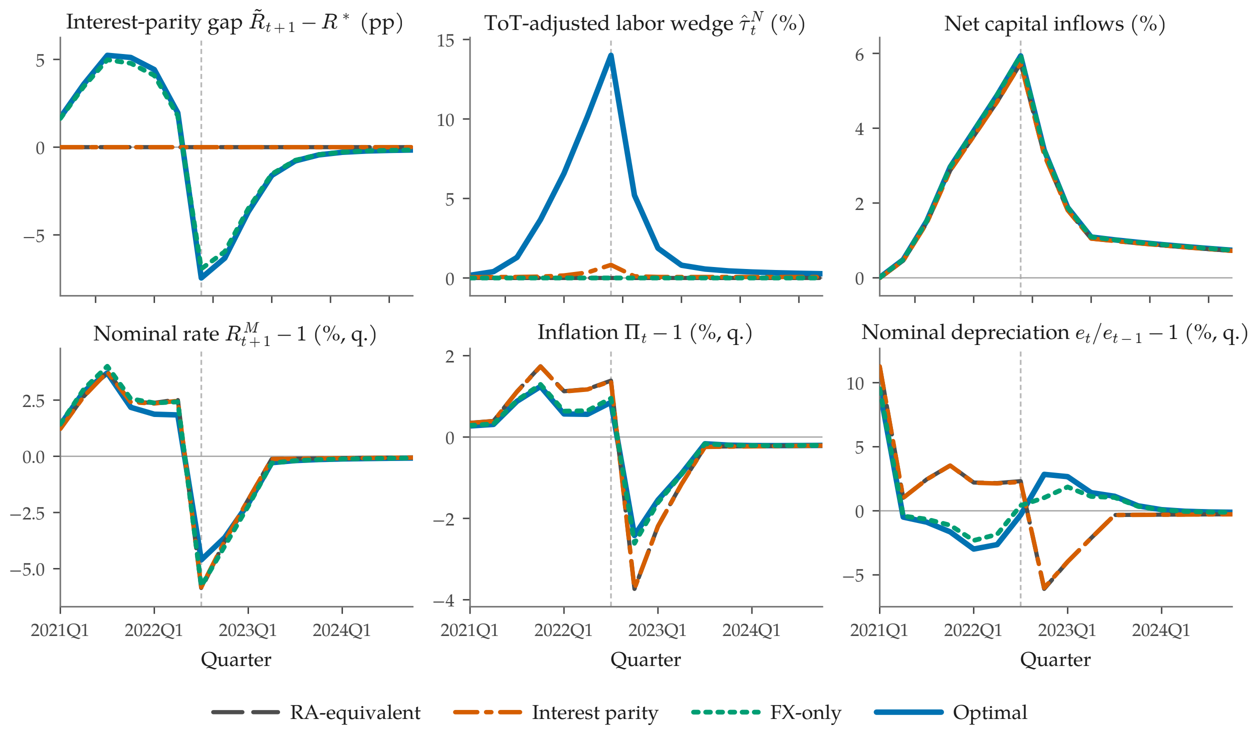

Figure 5. 2021–2023 shock: policy and implementation

Note: Net capital inflows are shown as a percent of initial steady-state gross output. The vertical line marks the energy-price peak.

price of energy is 19.5 percent lower than under the benchmark. The prescribed path appreciates the currency through the run-up, places the strongest currency, above its pre-shock level, at the peak of the world price, and depreciates it afterward as the foreign position is unwound. The optimal policy undoes 29 percent of the benchmark's peak real-wage loss and 25 percent of its peak labor-income loss. Employment remains below the FX-only path, so the income protection comes through a larger real wage rather than through additional hours. Ricardian consumption rises as foreign purchasing power is brought into the high-price quarters and falls as the position is repaid.

Figure 5 connects the two policy margins to their observable implementation. Net capital inflows build as the energy price rises and recede after its peak: foreign purchasing power replaces export revenue when the import bill is high and the position is unwound afterward. The interest parity gap is positive early in the run-up and turns negative as the price approaches its peak, tilting the intervention toward the quarters in which foreign purchasing power is most valuable. The terms-of-trade-adjusted labor wedge rises with the import price and restrains employment and imported-input demand exactly when the intervention holds the currency strong. Monetary policy is contractionary relative to the pre-shock steady state: the nominal rate rises to 3.7 percent during

Table 2: Policy effectiveness and implementation

	RA-equivalent	Interest parity	FX-only	Optimal
Panel A. MIT shock; impact quarter				
Domestic energy-price amplification factor	1.46	1.46	1.15	1.12
Real-wage loss undone relative to RA-equivalent (%)	0	0	21	23
Labor-income loss undone relative to RA-equivalent (%)	0	0	21	20
Inflation reduction relative to RA-equivalent (pp)	0.00	0.01	1.37	1.51
Interest parity gap (pp)	0.00	0.00	-0.73	-0.78
Terms-of-trade-adjusted labor wedge $\hat{\tau}^N$ (%)	0.00	0.11	0.00	6.76
Panel B. 2021–2023 energy-price path; peak quarter				
Domestic energy-price amplification factor	1.30	1.30	0.97	0.92
Real-wage loss undone relative to RA-equivalent (%)	0	0	26	29
Labor-income loss undone relative to RA-equivalent (%)	0	0	26	25
Inflation reduction relative to RA-equivalent (pp)	0.00	0.01	0.43	0.53
Interest parity gap (pp)	0.00	0.00	-6.93	-7.45
Terms-of-trade-adjusted labor wedge $\hat{\tau}^N$ (%)	0.00	0.82	0.00	14.00

the run-up and stands at 1.8 percent at the peak, above its pre-shock level throughout, and is cut steeply after the peak as inflation falls. Relative to the RA-equivalent benchmark, which sets the labor wedge to zero, the stance is tighter: inflation is lower through the run-up, peak-quarter inflation by 0.5 percentage points, and the domestic price level at the peak is 2.5 percent below the benchmark's. The peak nominal rate is nonetheless below the benchmark's 2.5 percent, because it implements the lower inflation path; the stance relative to the benchmark is read from the wedge and from inflation, not from the level of the rate. Following a large initial depreciation, the nominal depreciation rate turns negative through the early run-up and positive again around the peak as the foreign position starts to unwind.

Figure 11 of Appendix B.7 reports the corresponding output shares. Relative to the RA-equivalent benchmark, borrowed foreign purchasing power lowers the export share during the high-price quarters and leaves more output for domestic consumption. The appreciation lowers the import-bill share, while the higher real wage raises the HtM consumption share. The Ricardian share rises while foreign resources are brought forward. These differences recede as the energy price falls and the foreign position is unwound.

Table 2 collects the effectiveness measures and the policy wedges across regimes and both price paths. Under the MIT shock, the joint optimum lowers the amplification factor to 1.12 and undoes 23 percent of the benchmark's impact real-wage loss and 20 percent of its impact labor-income loss, with the domestic energy price 13.8 percent cheaper and the price level 1.4 percent lower than the benchmark at impact. The labor-income improvement is slightly smaller than under the FX-only policy because the positive labor wedge replaces part of the hours response with a larger real-wage response. Along the realized 2021–2023 path, these prescriptions constitute a full-commitment counterfactual: intervene to hold the currency stronger and restrain activity while the shock builds, then unwind as it fades.

5 Conclusion

This paper has studied how exchange rate and monetary policy should be jointly conducted by essential import-dependent economies facing temporary shocks to world prices of these imports. Conventional policy prescriptions, which rely on representative-agent models, assign monetary policy to target nominal distortions, and argue for letting the exchange rate float freely with no distortions to private financial flows. When some households are borrowing constrained and transfers cannot reach them, that assignment need no longer hold: the economy borrows too little during the spike, the real exchange rate depreciates contemporaneously with import prices, and the amplified real income loss falls hardest on households who consume out of current incomes.

We have argued that exchange rate policy then takes on a role it does not possess in representative-agent economies: stabilizing the real incomes of borrowing-constrained households. A public agent that distorts the intertemporal margin to support stronger capital inflows while import prices are high, and unwinds them afterward, cuts the excess co-movement of the real exchange rate with the shock: less must be exported while the necessity is expensive, so the depreciation is smaller at the peak and larger once the shock has passed. The gains reach constrained households through pecuniary general equilibrium effects of exchange rate movements: the stronger currency lowers the domestic price of imports and raises real incomes in the periods in which constrained households' marginal utility is highest. We call this the real-income stabilization channel of exchange rate policy delivered through its influence on the intertemporal margin in an open economy. Monetary policy cannot perform this role in an open economy, the margin it affects is the consumption-leisure wedge. When imports are complementary to domestic employment, a monetary expansion to support real incomes of constrained workers is costly as it requires a depreciation that shifts purchasing power towards unconstrained households through revaluation effects; optimal monetary policy complements FX interventions with a tighter rather than looser stance and lowers inflation below representative-agent prescriptions.

Calibrated to Japan and confronted with the realized 2022 path of world energy prices, our quantitative two-agent model compares jointly optimal policy with the representative-agent-equivalent policy and restricted optimal policies. The exercise isolates how heterogeneity in asset market access changes the policy prescription: optimal policy opens costly interest parity gaps to manage the real exchange rate and stabilize the incomes of the borrowing constrained, while monetary policy cuts inflation and restrains employment relative to the representative-agent-equivalent stance. Together the two instruments dampen the transmission of the world price shock to the domestic economy: around the 2022 episode, optimal policy more than mitigates the depreciation-driven amplification by turning the correlation between the real exchange rate and world energy prices negative, 29 percent of the peak real-wage loss under the conventional prescription is undone while energy prices are 19.5 percent lower at the peak of the crisis and the domestic price level is 2.5 percent lower. The present value of household consumption, meanwhile, is nearly unchanged across regimes: policy primarily re-times the loss and changes its incidence; it does not eliminate the underlying present-value resource loss.

We have deliberately excluded three margins, and they mark the natural extensions for future research: fiscal transfers that are partial or delayed rather than absent, with the division of the stabilization task between the fiscal and exchange rate instruments; policy without commitment, where the promised unwinding of the intervention must be credible; and shocks that arrive as risk rather than as realized paths, under which the public foreign asset position acquires a precautionary role alongside the re-timing role isolated here.

References

Acharya, Sushant, and Edouard Challe. 2025. "Inequality and Optimal Monetary Policy in the Open Economy." *Journal of International Economics*, 155: 104076.

Acharya, Sushant, Edouard Challe, and Giancarlo Corsetti. 2026. "Monetary Stabilization of Export Shocks, Revisited." Centre for Economic Policy Research CEPR Discussion Paper 21138.

Acharya, Sushant, Edouard Challe, and Keshav Dogra. 2023. "Optimal Monetary Policy According to HANK." *American Economic Review*, 113(7): 1741–1782.

Agency for Natural Resources and Energy. 2024. "Japan's Energy 2024." *Ministry of Economy, Trade and Industry, Japan*.

Aktuğ, Emrehan, and Abolfazl Rezghi. 2026. "Optimal Exchange Rate Policy with Oil Shocks." International Monetary Fund IMF Working Paper 2026/030.

Amador, Manuel, Javier Bianchi, Luigi Bocola, and Fabrizio Perri. 2020. "Exchange Rate Policies at the Zero Lower Bound." *Review of Economic Studies*, 87(4): 1605–1645.

Ari, Anil, Nicolas Arregui, Simon Black, Oya Celasun, Dora Iakova, Aiko Mineshima, Victor Mylonas, Ian Parry, Iulia Teodoru, and Karlygash Zhunussova. 2022. "Surging Energy Prices in Europe in the Aftermath of the War: How to Support the Vulnerable and Speed up the Transition Away from Fossil Fuels." International Monetary Fund IMF Working Paper 2022/152.

Auclert, Adrien. 2019. "Monetary Policy and the Redistribution Channel." *American Economic Review*, 109(6): 2333–2367.

Auclert, Adrien, Hugo Monnery, Matthew Rognlie, and Ludwig Straub. 2023. "Managing an Energy Shock: Fiscal and Monetary Policy." National Bureau of Economic Research NBER Working Paper 31543.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub. 2024. "The Intertemporal Keynesian Cross." *Journal of Political Economy*, 132(12): 4068–4121.

Auclert, Adrien, Matthew Rognlie, Martin Souchier, and Ludwig Straub. 2021. "Exchange Rates and Monetary Policy with Heterogeneous Agents: Sizing up the Real Income Channel." National Bureau of Economic Research NBER Working Paper 28872. Revised August 2024.

Bahmani-Oskooee, Mohsen, and Artatrana Ratha. 2004. "The J-Curve: A Literature Review." *Applied Economics*, 36(13): 1377–1398.

Basu, Suman S., Emine Boz, Gita Gopinath, Francisco Roch, and Filiz D. Unsal. 2020. "A Conceptual Model for the Integrated Policy Framework." International Monetary Fund IMF Working Paper 20/121.

Bernanke, Ben S., Mark Gertler, and Mark Watson. 1997. "Systematic Monetary Policy and the Effects of Oil Price Shocks." *Brookings Papers on Economic Activity*, 1997(1): 91–157.

Bianchi, Javier, and Louphou Coulibaly. 2023. "A Theory of Fear of Floating." National Bureau of Economic Research NBER Working Paper 30897.

Blanchard, Olivier J., and Jordi Galí. 2007. "The Macroeconomic Effects of Oil Price Shocks: Why Are the 2000s So Different from the 1970s?" National Bureau of Economic Research NBER Working Paper 13368.

Cavallino, Paolo. 2019. "Capital Flows and Foreign Exchange Intervention." *American Economic Journal: Macroeconomics*, 11(2): 127–170.

Chan, Jenny, Sebastian Diz, and Derrick Kanngiesser. 2024. "Energy Prices and Household Heterogeneity: Monetary Policy in a Gas-TANK." *Journal of Monetary Economics*, 147: 103620. Supplement.

Costinot, Arnaud, Guido Lorenzoni, and Iván Werning. 2014. "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation." *Journal of Political Economy*, 122(1): 77–128.

Cravino, Javier, and Andrei A. Levchenko. 2017. "The Distributional Consequences of Large Devaluations." *American Economic Review*, 107(11): 3477–3509.

Dávila, Eduardo, and Andreas Schaab. 2023. "Optimal Monetary Policy with Heterogeneous Agents: Discretion, Commitment, and Timeless Policy." National Bureau of Economic Research NBER Working Paper 30961.

Dávila, Eduardo, Andrés Rodríguez-Clare, Andreas Schaab, and Stacy Tan. 2025. "A Dynamic Theory of Optimal Tariffs." National Bureau of Economic Research NBER Working Paper 33898.

de Ferra, Sergio, Kurt Mitman, and Federica Romei. 2020. "Household Heterogeneity and the Transmission of Foreign Shocks." *Journal of International Economics*, 124: 103303.

Dixit, Avinash K., and Victor Norman. 1980. *Theory of International Trade: A Dual, General Equilibrium Approach.* Cambridge:Cambridge University Press.

Drechsel, Thomas, Michael McLeay, Silvana Tenreyro, and Enrico D. Turri. 2026. "How Should Central Banks Respond to Commodity Price Shocks? Optimal Monetary and Exchange Rate Frameworks for Commodity-Exposed Economies." National Bureau of Economic Research NBER Working Paper 35164.

Egorov, Konstantin, and Dmitry Mukhin. 2023. "Optimal Policy under Dollar Pricing." *American Economic Review*, 113(7): 1783–1824.

Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin. 2000. "Optimal Monetary Policy with Staggered Wage and Price Contracts." *Journal of Monetary Economics*, 46(2): 281–313.

European Securities and Markets Authority. 2023. "The August 2022 surge in the price of natural gas futures." ESMA TRV Risk Analysis.

Fanelli, Sebastián, and Ludwig Straub. 2021. "A Theory of Foreign Exchange Interventions." *The Review of Economic Studies*, 88(6): 2857–2885.

Farhi, Emmanuel, and Iván Werning. 2014. "Dilemma Not Trilemma? Capital Controls and Exchange Rates with Volatile Capital Flows." *IMF Economic Review*, 62(4): 569–605.

Farhi, Emmanuel, and Iván Werning. 2016. "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities." *Econometrica*, 84(5): 1645–1704.

Friedman, Milton. 1953. "The Case for Flexible Exchange Rates." In *Essays in Positive Economics*. 157–203. Chicago:University of Chicago Press.

Gabaix, Xavier, and Matteo Maggiori. 2015. "International liquidity and exchange rate dynamics." *The Quarterly Journal of Economics*, 130(3): 1369–1420.

Galí, Jordi, and Tommaso Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." *The Review of Economic Studies*, 72(3): 707–734.

Hemmerlé, Yannick, et al. 2023. "Aiming Better: Government Support for Households and Firms During the Energy Crisis." OECD Publishing OECD Economic Policy Paper 32.

International Energy Agency. 2022. "World Energy Outlook 2022." International Energy Agency.

International Energy Agency. 2026. "Sheltering from Oil Shocks." International Energy Agency.

International Monetary Fund. 2023. "Integrated Policy Framework: Principles for the Use of Foreign Exchange Intervention." IMF Policy Paper 2023/061.

International Monetary Fund. 2026. "Global Price of Energy Index [PNRGINDEXM]." *retrieved from FRED, Federal Reserve Bank of St. Louis*, Accessed June 26, 2026.

Itskhoki, Oleg, and Dmitry Mukhin. 2021. "Exchange rate disconnect in general equilibrium." *Journal of Political Economy*, 129(8): 2183–2232.

Itskhoki, Oleg, and Dmitry Mukhin. 2023. "Optimal Exchange Rate Policy." National Bureau of Economic Research NBER Working Paper 31933.

Johnson, Harry G. 1953. "Optimum Tariffs and Retaliation." *The Review of Economic Studies*, 21(2): 142–153.

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante. 2018. "Monetary Policy According to HANK." *American Economic Review*, 108(3): 697–743.

Kaplan, Greg, Giovanni L. Violante, and Justin Weidner. 2014. "The Wealthy Hand-to-Mouth." *Brookings Papers on Economic Activity*, 2014(1): 77–138.

Liu, Zheng, and Mark M. Spiegel. 2015. "Optimal Monetary Policy and Capital Account Restrictions in a Small Open Economy." *IMF Economic Review*, 63(2): 298–324.

Nippon.com. 2023. "Japan Posts Record ¥20 Trillion Trade Deficit in 2022." *Nippon.com*.

Oskolkov, Aleksei. 2023. "Exchange Rate Policy and Heterogeneity in Small Open Economies." *Journal of International Economics*, 142: 103750.

Rose, Andrew K. 1991. "The Role of Exchange Rates in a Popular Model of International Trade: Does the Marshall-Lerner Condition Hold?" *Journal of International Economics*, 30(3–4): 301–316.

Sgaravatti, Giovanni, Simone Tagliapietra, and Georg Zachmann. 2023. "National Fiscal Policy Responses to the Energy Crisis." *Bruegel Datasets*.

U.S. Energy Information Administration. 2026. "Europe Brent Spot Price FOB (Dollars per Barrel)." Accessed June 2026.

Werning, Iván. 2015. "Incomplete Markets and Aggregate Demand." National Bureau of Economic Research NBER Working Paper 21448.

Appendix

A Analytical Model Appendix

A.1 Optimality Conditions

Consider the two-period constrained planning problem, (16). Form the Lagrangian:

$$\mathcal{L} = \sum_{t=1}^{2} \beta^{t-1}\Big[\lambda\, u(C_t^p) + (1-\lambda)\, u(C_t^r) - \nu(N_t)\Big] + \mu\left[NX_1 + \frac{1}{R^*} NX_2\right] \tag{67}$$

where μ is the Lagrange multiplier on the intertemporal budget constraint.

FOC with respect to ϵ_t. Differentiating (67) with respect to ϵ_t, using $C_t^p = Y_t^N$ and $C_t^r = Y_t^N - \frac{\epsilon_t}{1-\lambda} NX_t$:

$$\beta^{t-1}\left[\bar{u}'_t\, \frac{\partial Y_t^N}{\partial \epsilon_t} - u'(C_t^r)\left(NX_t + \epsilon_t \frac{\partial NX_t}{\partial \epsilon_t}\right)\right] + \mu\, (R^*)^{-(t-1)}\, \frac{\partial NX_t}{\partial \epsilon_t} = 0$$

where $\bar{u}'_t \equiv \lambda\, u'(C_t^p) + (1-\lambda)\, u'(C_t^r)$. Solving for μ:

$$\mu = (\beta R^*)^{t-1} \cdot \frac{u'(C_t^r)\left(NX_t + \epsilon_t \dfrac{\partial NX_t}{\partial \epsilon_t}\right) - \bar{u}'_t\, \dfrac{\partial Y_t^N}{\partial \epsilon_t}}{\dfrac{\partial NX_t}{\partial \epsilon_t}} \tag{68}$$

Using $NX_t = \zeta \epsilon_t^{\chi} - P_t^{T*} \theta N_t$, rewrite Y_t^N as:

$$Y_t^N = A_t N_t - \zeta \epsilon_t^{1+\chi} + \epsilon_t NX_t$$

Differentiating with respect to ϵ_t and rearranging we get:

$$NX_t + \epsilon_t \frac{\partial NX_t}{\partial \epsilon_t} = \frac{\partial Y_t^N}{\partial \epsilon_t} + (1+\chi)\zeta \epsilon_t^{\chi} \tag{69}$$

Substitute (69) into the numerator of (68) and collect terms using $u'(C_t^r) - \bar{u}'_t = -\lambda[u'(C_t^p) - u'(C_t^r)]$:

$$\mu = (\beta R^*)^{t-1}\left[u'(C_t^r)\, \epsilon_t\left(1 + \frac{1}{\chi}\right) - \lambda\big[u'(C_t^p) - u'(C_t^r)\big]\frac{\dfrac{\partial Y_t^N}{\partial \epsilon_t}}{\dfrac{\partial NX_t}{\partial \epsilon_t}}\right] \tag{70}$$

Define:

$$\Omega_t \equiv \frac{\lambda\left[u'(C_t^p) - u'(C_t^r)\right]\left(-\frac{\partial Y_t^N/\partial \epsilon_t}{\partial NX_t/\partial \epsilon_t}\right)}{1 + \frac{1}{\chi}} \tag{71}$$

Setting $\tilde{\mu} \equiv \mu/(1 + 1/\chi)$:

$$(\beta R^*)^{t-1}\left[\epsilon_t\, u'(C_t^r) + \Omega_t\right] = \tilde{\mu} \tag{72}$$

Since $\tilde{\mu}$ is constant, equating (72) at $t = 1$ and $t = 2$ gives the intertemporal optimality condition:

$$\epsilon_1\, u'(C_1^r) + \Omega_1 = \beta R^*\left[\epsilon_2\, u'(C_2^r) + \Omega_2\right] \tag{73}$$

FOC with respect to N_t. Differentiating (67) with respect to N_t and substituting μ from (68):

$$v'(N_t) = \bar{u}'_t\left[\frac{\partial Y_t^N}{\partial N_t} - \frac{\partial Y_t^N}{\partial \epsilon_t}\cdot\frac{\frac{\partial NX_t}{\partial N_t}}{\frac{\partial NX_t}{\partial \epsilon_t}}\right] + u'(C_t^r)\, NX_t \cdot \frac{\frac{\partial NX_t}{\partial N_t}}{\frac{\partial NX_t}{\partial \epsilon_t}} \tag{74}$$

Define:

$$\hat{w}_t \equiv \frac{\partial Y_t^N}{\partial N_t} + \left(NX_t - \frac{\partial Y_t^N}{\partial \epsilon_t}\right)\frac{\frac{\partial NX_t}{\partial N_t}}{\frac{\partial NX_t}{\partial \epsilon_t}} = w_t - \tfrac{1}{\chi}\epsilon_t P_t^{T*}\theta \tag{75}$$

Define:

$$\Lambda_t \equiv \lambda\left[u'(C_t^p) - u'(C_t^r)\right] NX_t \cdot \frac{\frac{\partial NX_t}{\partial N_t}}{\frac{\partial NX_t}{\partial \epsilon_t}} \tag{76}$$

Using $u'(C_t^r) = \bar{u}'_t - \lambda[u'(C_t^p) - u'(C_t^r)]$ we get:

$$v'(N_t) = \bar{u}'_t\, \hat{w}_t - \Lambda_t \tag{77}$$

The solution to the constrained planning problem, (16), $\{\epsilon_t, N_t\}_{t=1}^2$, is characterized by the intertemporal optimality condition, (73), and the intratemporal optimality conditions, (77), for $t = 1, 2$, together with the intertemporal budget constraint:

$$NX_1 + \frac{1}{R^*} NX_2 = 0 \tag{78}$$

Optimal Wedges Using the optimality conditions and the definitions of the labor wedge, (20), and the intertemporal wedge, (21), we can express the optimal wedges as:

$$\tau_t^N = \frac{1}{\chi}\frac{\epsilon_t P_t^{T*}\theta}{w_t} + \frac{\Lambda_t}{\bar{u}_t' w_t} \tag{79}$$

$$\tau = \frac{\Omega_1 - \beta R^* \Omega_2}{\epsilon_1 u'(C_1^r)} \tag{80}$$

A.2 Uniqueness of the Solution

The following Lemma establishes sufficient conditions for the uniqueness of the solution to the constrained planning problem, (16).

Lemma 5. *Let $A_t > 0$ and $P_t^{T*} > 0$ for $t \in \{1,2\}$. Let $R^* > 0$, $\beta > 0$, $\zeta > 0$, $\theta > 0$, $\chi > 0$, and $\lambda \in [0,1)$. Let $u : \mathbb{R}_{++} \to \mathbb{R}$ be strictly increasing and strictly concave, and $\nu : \mathbb{R}_{++} \to \mathbb{R}$ be strictly increasing and strictly convex. Assume that there exists at least one solution, $\{N_t, \epsilon_t\}_{t=1}^2$, to equations (73), (77), and (78).*

For $\lambda \in (0,1)$, assume that for each $t \in \{1,2\}$ and each $\mu \in \mathbb{R}$, the function

$$\Phi_t(N_t, \epsilon_t; \mu) \equiv \beta^{t-1}\Big[\lambda\, u\big(C_t^p(N_t,\epsilon_t)\big) + (1-\lambda)\, u\big(C_t^r(N_t,\epsilon_t)\big) - \nu(N_t)\Big] + \mu\,(R^*)^{-(t-1)}\, NX_t(N_t,\epsilon_t)$$

admits a unique interior maximizer $\big(N_t^*(\mu),\, \epsilon_t^*(\mu)\big)$ on its domain.

Then the solution, $\{N_t, \epsilon_t\}_{t=1}^2$, is the unique maximizer of the constrained planning problem, (16).

Proof. Step 1: Uniqueness of the per-period maximizer.

For $\lambda \in (0,1)$, uniqueness of $\big(N_t^*(\mu),\, \epsilon_t^*(\mu)\big)$ is assumed directly.

For $\lambda = 0$, Φ_t simplifies to:

$$\Phi_t(N_t, \epsilon_t; \mu) = \beta^{t-1}\Big[u\big(A_t N_t - \zeta \epsilon_t^{1+\chi}\big) - \nu(N_t)\Big] + \mu\,(R^*)^{-(t-1)}\big(\zeta \epsilon_t^{\chi} - P_t^{T*}\theta N_t\big)$$

where $C_t^r = A_t N_t - \zeta \epsilon_t^{1+\chi}$ at $\lambda = 0$. Under the change of variables $z_t = \epsilon_t^{\chi}$, a bijection on $\mathbb{R}_{++}$ since $\chi > 0$, the exponent $(1+\chi)/\chi > 1$ implies that $z_t \mapsto z_t^{(1+\chi)/\chi}$ is strictly convex. Since u is strictly increasing and strictly concave, $u\Big(A_t N_t - \zeta z_t^{(1+\chi)/\chi}\Big)$ is strictly concave in (N_t, z_t). The term $-\nu(N_t)$ is strictly concave in N_t, and $\mu(R^*)^{-(t-1)}\big(\zeta z_t - P_t^{T*}\theta N_t\big)$ is linear. Therefore Φ_t is strictly concave in (N_t, z_t). Hence, whenever an interior maximizer exists, it is unique and maps back to a unique interior maximizer $\big(N_t^*(\mu),\, \epsilon_t^*(\mu)\big)$ in the original variables.

Step 2: Strict monotonicity of $h_t(\mu) \equiv NX_t\big(N_t^*(\mu), \epsilon_t^*(\mu)\big)$.

Let $\mu_2 > \mu_1$. We first show that

$$\big(N_t^*(\mu_2), \epsilon_t^*(\mu_2)\big) \neq \big(N_t^*(\mu_1), \epsilon_t^*(\mu_1)\big)$$

Suppose for contradiction that both equal the same interior point (N_t, ϵ_t). Since the maximizer is interior, it satisfies the first-order conditions of $\Phi_t(\,\cdot\,; \mu_i)$ for $i \in \{1, 2\}$:

$$\nabla_{(N_t,\epsilon_t)} U_t(N_t, \epsilon_t) + \mu_i\,(R^*)^{-(t-1)} \nabla_{(N_t,\epsilon_t)} NX_t(N_t, \epsilon_t) = 0, \qquad i \in \{1, 2\}$$

where

$$U_t(N_t, \epsilon_t) \equiv \beta^{t-1}\Big[\lambda u(C_t^p(N_t, \epsilon_t)) + (1-\lambda)u(C_t^r(N_t, \epsilon_t)) - \nu(N_t)\Big]$$

Subtracting the two conditions gives:

$$(\mu_2 - \mu_1)(R^*)^{-(t-1)} \nabla_{(N_t,\epsilon_t)} NX_t(N_t, \epsilon_t) = 0$$

Since $\mu_2 > \mu_1$ and $(R^*)^{-(t-1)} > 0$, this requires $\nabla_{(N_t,\epsilon_t)} NX_t(N_t, \epsilon_t) = 0$. But

$$\frac{\partial NX_t}{\partial N_t} = -P_t^{T*}\theta < 0$$

at every interior point, so $\nabla_{(N_t,\epsilon_t)} NX_t \neq 0$, a contradiction. Hence the maximizers at μ_1 and μ_2 are distinct.

Denote the unique interior maximizers by (N_t^i, ϵ_t^i) for $i \in \{1, 2\}$, where

$$(N_t^i, \epsilon_t^i) = \big(N_t^*(\mu_i), \epsilon_t^*(\mu_i)\big)$$

Set

$$U_t^i \equiv \beta^{t-1}\Big[\lambda u(C_t^p(N_t^i, \epsilon_t^i)) + (1-\lambda)u(C_t^r(N_t^i, \epsilon_t^i)) - \nu(N_t^i)\Big], \qquad H_t^i \equiv NX_t(N_t^i, \epsilon_t^i)$$

Since the maximizers are distinct, uniqueness at μ_2 and μ_1 respectively yields strict inequalities:

$$\begin{aligned} U_t^2 + \mu_2(R^*)^{-(t-1)}H_t^2 &> U_t^1 + \mu_2(R^*)^{-(t-1)}H_t^1 \\ U_t^1 + \mu_1(R^*)^{-(t-1)}H_t^1 &> U_t^2 + \mu_1(R^*)^{-(t-1)}H_t^2 \end{aligned}$$

Adding and cancelling $U_t^1 + U_t^2$ yields

$$(\mu_2 - \mu_1)(R^*)^{-(t-1)}(H_t^2 - H_t^1) > 0$$

Since $\mu_2 > \mu_1$ and $(R^*)^{-(t-1)} > 0$, it follows that

$$H_t^2 > H_t^1$$

Therefore $h_t(\mu)$ is strictly increasing in μ.

Step 3: Uniqueness of μ^* and of the maximizer.

Define

$$S(\mu) \equiv h_1(\mu) + R^{*-1} h_2(\mu)$$

Since h_1 and h_2 are strictly increasing and $R^* > 0$, S is strictly increasing. Hence there exists at most one μ^* satisfying

$$S(\mu^*) = 0$$

By assumption, there exists at least one solution $\{N_t, \epsilon_t\}_{t=1}^2$ to (73), (77), and (78). Any such solution defines a common multiplier μ through (68). Equation (73) implies that this multiplier is the same across the two periods, while (77) gives the corresponding first-order condition with respect to N_t. Hence the solution coincides with

$$\left\{(N_t^*(\mu), \epsilon_t^*(\mu))\right\}_{t=1}^2$$

Since (78) holds, this multiplier satisfies $S(\mu) = 0$. Therefore at least one μ^* satisfying $S(\mu^*) = 0$ exists. By strict monotonicity of S, this μ^* is unique. Given this unique μ^*, Step 1 yields a unique pair

$$\left(N_t^*(\mu^*), \epsilon_t^*(\mu^*)\right)$$

for each $t \in \{1, 2\}$.

It remains to verify that this allocation is the unique global maximizer of the constrained planning problem. Let

$$x_t = (N_t, \epsilon_t), \qquad x_t^* = \left(N_t^*(\mu^*), \epsilon_t^*(\mu^*)\right)$$

Since x_t^* uniquely maximizes

$$\Phi_t(x_t; \mu^*) = U_t(x_t) + \mu^*(R^*)^{-(t-1)} N X_t(x_t)$$

for each $t \in \{1, 2\}$, any allocation (x_1, x_2) satisfies

$$\sum_{t=1}^{2} \left[U_t(x_t) + \mu^*(R^*)^{-(t-1)} NX_t(x_t)\right] \leqslant \sum_{t=1}^{2} \left[U_t(x_t^*) + \mu^*(R^*)^{-(t-1)} NX_t(x_t^*)\right]$$

with strict inequality unless $x_t = x_t^*$ for both $t = 1, 2$.

For any feasible allocation,

$$NX_1(x_1) + R^{*-1} NX_2(x_2) = 0$$

The allocation $\{x_t^*\}_{t=1}^2$ also satisfies this condition because $S(\mu^*) = 0$. Therefore the multiplier terms cancel, and

$$U_1(x_1) + U_2(x_2) \leqslant U_1(x_1^*) + U_2(x_2^*)$$

with strict inequality unless $(x_1, x_2) = (x_1^*, x_2^*)$. Hence $\{x_t^*\}_{t=1}^2$ is the unique global maximizer of problem (16). □

Remark. Lemma 5 requires an additional assumption on the uniqueness of the static optimizer for any $\lambda \in (0, 1)$ since uniqueness of the solution cannot be derived from strict concavity of u alone. This is because $C_t^p = (A_t - \epsilon_t P_t^{T*}\theta)N_t$ and C_t^r are both bilinear in (N_t, ϵ_t), and a composition with a strictly concave u does not yield a globally concave Φ_t for $\lambda \in (0, 1)$. The additional assumption rules out multiple local optima arising from this non-concavity. In practice, this assumption is satisfied for a large range of parameters under CRRA preferences, and verified numerically.

A.3 Proof of Lemma 1

Proof. Let $\{N_t, \epsilon_t\}_{t=1}^2$ solve (16). The intertemporal resource constraint gives

$$NX_1 + \frac{1}{R^*} NX_2 = 0$$

Define prices and quantities by

$$w_t = A_t - \epsilon_t P_t^{T*}\theta, \qquad M_t = \theta N_t, \qquad Y_t = A_t N_t, \qquad X_t = \zeta \epsilon_t^{1+\chi}$$

and define household consumption by (17) and (18). Let aggregate net foreign assets be

$$F^* = NX_1$$

and set $f^* = F^*/(1-\lambda)$. The intertemporal resource constraint implies $NX_2 = -R^*F^*$, so the two balance-of-payments equations in (13) hold.

Choose τ to satisfy the Ricardian Euler equation,

$$1+\tau = \frac{\beta R^* u'(C_2^r)\epsilon_2}{u'(C_1^r)\epsilon_1}$$

and set $T = \tau\epsilon_1 f^*$. Then the fiscal budget constraint (12) holds. Moreover, using $F^* = NX_1$,

$$C_1^r = w_1 N_1 - \epsilon_1 f^*$$

so

$$C_1^r + \epsilon_1 f^*(1+\tau) = w_1 N_1 + T$$

Similarly, using $NX_2 = -R^*F^*$,

$$C_2^r = w_2 N_2 + \epsilon_2 R^* f^*$$

Thus the Ricardian budget constraints hold. The HtM budget constraints hold by (17). Firms' optimality conditions and zero profits hold by the definitions of M_t, Y_t, and w_t above. Export demand holds by construction. Goods-market clearing follows from Walras' identity, since household budgets, firms' zero-profit conditions, the fiscal budget constraint, and the balance of payments all hold. Hence the allocation is implementable as an equilibrium under the policy $\{\tau, T\}$. □

A.4 Proof of Proposition 1

Proof. Setting $\lambda = 0$ in (71) gives $\Omega_t = 0$. Equation (73) then coincides with the undistorted Ricardian Euler equation, so $\tau = 0$. Equation (77) becomes $v'(N_t) = u'(C_t^r)\hat{w}_t$. Therefore $1 - \tau_t^N = v'(N_t)/(u'(C_t^r)w_t) = \hat{w}_t/w_t$. Since $\hat{w}_t = w_t - \frac{1}{\chi}\epsilon_t P_t^{T*}\theta$, $\tau_t^N = \frac{1}{\chi}\epsilon_t P_t^{T*}\theta/w_t$. At an interior solution, $v'(N_t) > 0$ and $u'(C_t^r) > 0$ imply $\hat{w}_t > 0$, hence $w_t > 0$ and $\tau_t^N > 0$. □

A.5 Proof of Lemma 2

Proof. Let $z_t \equiv \epsilon_t^X$ and $q \equiv (1+\chi)/\chi > 1$, so that $C_t = AN_t - \zeta z_t^q$ and $NX_t = \zeta z_t - P_t^{T*}\theta N_t$.

Part 1. At $\lambda = 0$, the optimality system derived in the Appendix A reduces to

$$\epsilon_1 u'(C_1^r) = \beta R^* \epsilon_2 u'(C_2^r), \tag{81}$$

$$\nu'(N_t) = u'(C_t^r)\hat{w}_t, \qquad t \in \{1, 2\}, \tag{82}$$

$$NX_1 + \frac{1}{R^*} NX_2 = 0 \tag{83}$$

Suppose $P_1^{T*} = P_2^{T*} = P^{T*}$. Let (N^0, z^0) solve the one-period system

$$\zeta z^0 - P^{T*}\theta N^0 = 0$$

and

$$\nu'(N^0) = u'\left(AN^0 - \zeta(z^0)^q\right)\left[A - q\zeta(z^0)^{q-1}\frac{P^{T*}\theta}{\zeta}\right]$$

This system is the first-order condition of the strictly concave one-period problem obtained after imposing $NX = 0$, and hence has at most one interior solution. Setting

$$N_1 = N_2 = N^0, \qquad z_1 = z_2 = z^0$$

satisfies (82). It also satisfies (81) because $\beta R^* = 1$, and it satisfies (83) because $NX_1 = NX_2 = 0$. Hence it is a solution to the representative-agent optimality system. By Lemma 5, the constrained efficient allocation is unique. Therefore

$$N_1 = N_2, \qquad z_1 = z_2$$

It follows that $\epsilon_1 = \epsilon_2$ and $NX_1 = NX_2 = 0$.

Part 2. For $P > 0$ and $nx \in \mathbb{R}$, define the period value function:

$$V(P, nx) \equiv \max_{N,z} \{u(AN - \zeta z^q) - \nu(N)\} \quad \text{s.t.} \quad \zeta z - P\theta N = nx$$

Since the objective is concave in (N, z) and the constraint is affine, V is concave in nx. The two-period problem is equivalent to

$$\max_{nx_1} \; G(nx_1) \equiv V(P_1^{T*}, nx_1) + \beta V(P^{T*}, -R^* nx_1)$$

G is concave. Since $\beta R^* = 1$,

$$G'(0) = V_{nx}(P_1^{T*}, 0) - V_{nx}(P^{T*}, 0)$$

It suffices to show $V_{nx}(P, 0)$ is strictly decreasing in P.

At $nx = 0$ the constraint gives $\zeta z = P\theta N$. The envelope theorem yields $V_{nx}(P, 0) = -q\, C^{-\sigma} z^{q-1}$, so it suffices to show $C^{-\sigma} z^{q-1}$ is strictly increasing in P along the balanced-trade optimum.

Define $s \equiv \zeta z^q/(AN) \in (0, 1/q)$, so $C = AN(1 - s)$. The first-order condition with respect to N is:

$$A^{1-\sigma} N^{-\sigma} (1 - s)^{-\sigma} (1 - qs) = \nu'(N)$$

Implicit differentiation with respect to s gives:

$$\frac{d \log N}{ds} = \frac{\dfrac{\sigma}{1 - s} - \dfrac{q}{1 - qs}}{\sigma + \dfrac{N\nu''(N)}{\nu'(N)}}$$

Since ν is convex, $N\nu''(N)/\nu'(N) \geqslant 0$, so

$$\frac{d \log N}{ds} < \frac{\sigma}{\sigma(1 - s)} = \frac{1}{1 - s} \tag{84}$$

From $s = (\theta^q / A\zeta^{q-1}) P^q N^{q-1}$:

$$\frac{d \log P}{ds} = \frac{1}{q} \left[\frac{1}{s} - (q - 1) \frac{d \log N}{ds} \right]$$

Since $s < 1/q$ implies $(q - 1)/(1 - s) < 1/s$, combining with (84) gives $d \log P/ds > 0$: P is strictly increasing in s.

Up to a positive constant,

$$C^{-\sigma} z^{q-1} \propto s^{\frac{q-1}{q}} (1 - s)^{-\sigma} N^{\frac{q-1}{q} - \sigma}$$

Its log-derivative with respect to s is:

$$\frac{q - 1}{qs} + \frac{\sigma}{1 - s} + \left(\frac{q - 1}{q} - \sigma \right) \frac{d \log N}{ds}$$

Since $\sigma \geqslant (q - 1)/q = 1/(1 + \chi)$, the last coefficient is non-positive. Using the tighter bound $d \log N/ds < \sigma/[(\sigma + N\nu''/\nu')(1 - s)]$ from (84), the log-derivative is bounded below by:

$$\frac{q - 1}{qs} + \frac{\sigma}{1 - s} \left[1 - \frac{\sigma - (q - 1)/q}{\sigma + N\nu''(N)/\nu'(N)} \right] > 0$$

Hence $C^{-\sigma} z^{q-1}$ is strictly increasing in s, and since P is strictly increasing in s, it is strictly increasing in P. Therefore $V_{nx}(P, 0) = -q\, C^{-\sigma} z^{q-1}$ is strictly decreasing in P.

Since $P_1^{T*} > P^{T*}$, we have $G'(0) < 0$. Concavity of G then gives $NX_1 = nx_1^* < 0$, and (83) gives $NX_2 = -R^* NX_1 > 0$. □

A.6 Proof of Lemma 4

Proof. With $\tau = 0$ and $\beta R^* = 1$, the laissez-faire equilibrium satisfies:

$$\epsilon_1 u'(C_1^r) = \epsilon_2 u'(C_2^r), \qquad NX_1 + \frac{1}{R^*} NX_2 = 0$$

Suppose for contradiction $NX_1 \geqslant 0$, so $NX_2 \leqslant 0$. Since $P_1^{T*} > P^{T*}$:

$$\zeta \epsilon_1^{\chi} \geqslant P_1^{T*} \theta \bar{N} > P^{T*} \theta \bar{N} \geqslant \zeta \epsilon_2^{\chi}$$

giving $\epsilon_1 > \epsilon_2$. Since $NX_1 \geqslant 0$ and $NX_2 \leqslant 0$:

$$C_1^r \leqslant Y_1^N = (A - \epsilon_1 P_1^{T*} \theta) \bar{N} < (A - \epsilon_2 P^{T*} \theta) \bar{N} = Y_2^N \leqslant C_2^r$$

Therefore $u'(C_1^r) > u'(C_2^r)$, and with $\epsilon_1 > \epsilon_2$:

$$\epsilon_1 u'(C_1^r) > \epsilon_2 u'(C_2^r)$$

contradicting the Euler equation. Hence $NX_1 < 0$ and $NX_2 = -R^* NX_1 > 0$. Since $C_t^r - C_t^p = -\frac{\epsilon_t NX_t}{1-\lambda}$, it follows that $C_1^r > C_1^p$ and $C_2^r < C_2^p$. □

A.7 Proof of Proposition 2

Proof. Strict concavity and gradient formula. Since C_t^p is affine and non-constant in ϵ_t and u is strictly concave, $W(\epsilon_1, \epsilon_2) \equiv \sum_{t=1}^{2} \beta^{t-1} [\lambda u(C_t^p) + (1-\lambda) u(C_t^r)]$ is strictly concave. Direct differentiation of W, together with the definition of Ω_t, gives

$$\frac{\partial W}{\partial \epsilon_t} = -\beta^{t-1} \left(1 + \frac{1}{\chi}\right) \left[\epsilon_t u'(C_t^r) + \Omega_t\right] \frac{\partial NX_t}{\partial \epsilon_t} \tag{85}$$

From (71), (7), and (15):

$$|\Omega_t| = \frac{\lambda \left|u'(C_t^p) - u'(C_t^r)\right| P_t^{T*} \theta \bar{N}}{(1 + \chi^{-1}) \chi \zeta \epsilon_t^{\chi - 1}}$$

When $NX_t > 0$, $C_t^r < C_t^p$, so $\left|u'(C_t^p) - u'(C_t^r)\right| < u'(C_t^r)$, and $P_t^{T*} \theta \bar{N} < \zeta \epsilon_t^{\chi}$. Hence

$$|\Omega_t| < \frac{\lambda\, u'(C_t^r)\, \zeta \epsilon_t^{\chi}}{(1 + \chi^{-1}) \chi \zeta \epsilon_t^{\chi - 1}} = \frac{\lambda}{1 + \chi} \epsilon_t u'(C_t^r) < \epsilon_t u'(C_t^r) \tag{86}$$

where the last inequality uses $\lambda < 1 < 1 + \chi$. Hence $\epsilon_t u'(C_t^r) + \Omega_t > 0$ whenever $NX_t > 0$.

Proof of items ii and iii. By Lemma 4, $NX_1^{LF} < 0 < NX_2^{LF}$, so $\Omega_1^{LF} > 0 > \Omega_2^{LF}$. Setting $\tau = 0$ and

$\beta R^* = 1$ in (11) gives $\epsilon_1^{LF} u'(C_1^{r,LF}) = \epsilon_2^{LF} u'(C_2^{r,LF})$. Hence, using $\Omega_1^{LF} > 0$, $\Omega_2^{LF} < 0$, and (86) at $t = 2$:

$$H_1^{LF} \equiv \epsilon_1^{LF} u'(C_1^{r,LF}) + \Omega_1^{LF} \;>\; \epsilon_2^{LF} u'(C_2^{r,LF}) \;>\; \epsilon_2^{LF} u'(C_2^{r,LF}) + \Omega_2^{LF} \equiv H_2^{LF} > 0 \tag{87}$$

For any feasible $(\epsilon_1, \epsilon_2) \neq (\epsilon_1^{LF}, \epsilon_2^{LF})$, strict concavity of W, (85), and $\beta R^* = 1$ give:

$$W(\epsilon_1, \epsilon_2) < W^{LF} - \left(1 + \chi^{-1}\right) \underbrace{\left[H_1^{LF} \frac{\partial NX_1^{LF}}{\partial \epsilon_1} \Delta\epsilon_1 + R^{*-1} H_2^{LF} \frac{\partial NX_2^{LF}}{\partial \epsilon_2} \Delta\epsilon_2 \right]}_{\equiv \mathcal{B}}$$

where $\Delta\epsilon_t = \epsilon_t - \epsilon_t^{LF}$. Decompose $\mathcal{B}$ as:

$$\mathcal{B} = H_2^{LF} \left[\frac{\partial NX_1^{LF}}{\partial \epsilon_1} \Delta\epsilon_1 + R^{*-1} \frac{\partial NX_2^{LF}}{\partial \epsilon_2} \Delta\epsilon_2 \right] + \left(H_1^{LF} - H_2^{LF}\right) \frac{\partial NX_1^{LF}}{\partial \epsilon_1} \Delta\epsilon_1$$

By concavity of $NX_1 + R^{*-1} NX_2$ and the equality constraint, the first bracket is non-negative. When $\Delta\epsilon_1 \geqslant 0$, both terms are non-negative by (87) and $\partial NX_t / \partial \epsilon_t > 0$, and at least one is strictly positive for $(\epsilon_1, \epsilon_2) \neq (\epsilon_1^{LF}, \epsilon_2^{LF})$. Hence

$$W(\epsilon_1, \epsilon_2) < W^{LF} \quad \text{for all feasible } (\epsilon_1, \epsilon_2) \text{ with } \epsilon_1 \geqslant \epsilon_1^{LF}$$

Along the constraint, $R^{*-1} \partial NX_2 / \partial \epsilon_2 \, d\epsilon_2 = -\partial NX_1 / \partial \epsilon_1 \, d\epsilon_1$, so by (85) and $\beta R^* = 1$:

$$dW^{LF} = -\left(1 + \chi^{-1}\right) \left(H_1^{LF} - H_2^{LF}\right) \frac{\partial NX_1^{LF}}{\partial \epsilon_1} d\epsilon_1 > 0 \quad \text{for } d\epsilon_1 < 0$$

so LF is suboptimal and there exist feasible allocations with $W > W^{LF}$. Since every feasible allocation with $\epsilon_1 \geqslant \epsilon_1^{LF}$ satisfies $W \leqslant W^{LF}$ and LF is suboptimal, any optimum satisfies $\epsilon_1^O < \epsilon_1^{LF}$. Since both allocations satisfy the constraint and NX_t is strictly increasing in ϵ_t,

$$\epsilon_2^O > \epsilon_2^{LF}, \qquad NX_1^O < NX_1^{LF} < 0 < NX_2^{LF} < NX_2^O$$

Proof of items iv and v. Item iv follows immediately from item ii. Since $Y_t^N = (A - \epsilon_t P_t^{T*} \theta) \bar{N}$ is strictly decreasing in $\epsilon_t P_t^{T*}$, item v follows from item iv.

Proof of item i. Since $NX_1^O < 0 < NX_2^O$, we have $\Omega_1^O > 0 > \Omega_2^O$. With $\beta R^* = 1$, (73) gives $\epsilon_1^O u'(C_1^{r,O}) + \Omega_1^O = \epsilon_2^O u'(C_2^{r,O}) + \Omega_2^O$, and (11) gives $\epsilon_1^O (1 + \tau) u'(C_1^{r,O}) = \epsilon_2^O u'(C_2^{r,O})$. Substituting the latter into the former gives

$$\tau = \frac{\Omega_1^O - \Omega_2^O}{\epsilon_1^O u'(C_1^{r,O})} > 0$$

Proof of item vi. This follows directly from Lemma 6.

□

Lemma 6. *Assume the conditions of Proposition 2 hold. Let* $p \equiv \frac{P_1^{T*}}{P_2^{T*}} > 1$.

i. *Define the allocation that ensures equal input prices across periods,* $(\epsilon_1^E, \epsilon_2^E)$*:*

$$\epsilon_1^E P_1^{T*} = \epsilon_2^E P_2^{T*}$$

and satisfies the intertemporal budget constraint, (78). If such an allocation is infeasible or it is feasible and

$$\frac{\lambda}{1+\chi} \leqslant \frac{(p+\beta)}{p(1+\beta p^{\chi})}$$

then the constrained efficient allocation satisfies:

$$0 \leqslant \epsilon_1^O P_1^{T*} - \epsilon_2^O P_2^{T*} < \epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}$$

$$0 \leqslant Y_2^{N,O} - Y_1^{N,O} < Y_2^{N,LF} - Y_1^{N,LF}$$

ii. *Define the mirror allocation as one that ensures input prices are as volatile as the laissez-faire allocation,* $(\epsilon_1^M, \epsilon_2^M)$*:*

$$\epsilon_1^M P_1^{T*} - \epsilon_2^M P_2^{T*} = -\left(\epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}\right)$$

with $\epsilon_1^M < \epsilon_1^{LF}, \epsilon_2^M > \epsilon_2^{LF}$*, and satisfies the intertemporal budget constraint, (78). If such an allocation is infeasible or it is feasible and:*

$$-NX_1^M \leqslant \left(1 - \frac{\lambda}{1+\chi}\right) P_1^{T*} \theta \bar{N}$$

then the constrained efficient allocation satisfies:

$$|\epsilon_1^O P_1^{T*} - \epsilon_2^O P_2^{T*}| < |\epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}|$$

$$\left|Y_2^{N,O} - Y_1^{N,O}\right| < \left|Y_2^{N,LF} - Y_1^{N,LF}\right|$$

Proof. By Proposition 2,

$$\epsilon_1^O < \epsilon_1^{LF}, \qquad \epsilon_2^O > \epsilon_2^{LF}$$

and

$$NX_1^O < NX_1^{LF} < 0 < NX_2^{LF} < NX_2^O$$

Also, from the laissez-faire lemma,

$$\epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*} > 0$$

Since

$$Y_2^N - Y_1^N = \theta \bar{N} \left(\epsilon_1 P_1^{T*} - \epsilon_2 P_2^{T*} \right)$$

it is enough to prove the corresponding claims for domestic input prices.

We first record an ordering property of the feasible frontier. Let $D(\epsilon_1, \epsilon_2) \equiv \epsilon_1 P_1^{T*} - \epsilon_2 P_2^{T*}$ and let (ϵ_1, ϵ_2) and $(\tilde{\epsilon}_1, \tilde{\epsilon}_2)$ both satisfy (78). If $\tilde{\epsilon}_1 > \epsilon_1$, then $NX_1(\tilde{\epsilon}_1) > NX_1(\epsilon_1)$, so (78) implies $NX_2(\tilde{\epsilon}_2) < NX_2(\epsilon_2)$. Since NX_2 is strictly increasing in ϵ_2, $\tilde{\epsilon}_2 < \epsilon_2$. Hence

$$D(\tilde{\epsilon}_1, \tilde{\epsilon}_2) > D(\epsilon_1, \epsilon_2)$$

Along the feasible frontier, D is therefore strictly increasing in ϵ_1. Since NX_2 is continuous and strictly increasing, the feasible frontier is continuous wherever it is defined.

We first prove the equal-input-price cutoff result. Suppose first that no feasible allocation satisfying (78) and

$$\epsilon_1 P_1^{T*} = \epsilon_2 P_2^{T*}$$

exists in the direction $\epsilon_1 < \epsilon_1^{LF}$. Since $D(\epsilon_1^{LF}, \epsilon_2^{LF}) > 0$ and D is strictly increasing along the feasible frontier, the constrained efficient allocation, which satisfies $\epsilon_1^O < \epsilon_1^{LF}$ by Proposition 2, must satisfy

$$\epsilon_1^O P_1^{T*} - \epsilon_2^O P_2^{T*} \geqslant 0$$

Together with

$$\epsilon_1^O < \epsilon_1^{LF}, \qquad \epsilon_2^O > \epsilon_2^{LF}$$

this gives

$$0 \leqslant \epsilon_1^O P_1^{T*} - \epsilon_2^O P_2^{T*} < \epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}$$

Now suppose the equal-input-price allocation $(\epsilon_1^E, \epsilon_2^E)$ is feasible. Since

$$\epsilon_1^E P_1^{T*} = \epsilon_2^E P_2^{T*}$$

and $P_1^{T*} > P_2^{T*}$, we have

$$\epsilon_1^E < \epsilon_2^E$$

Using (78), this allocation satisfies

$$NX_1^E < 0 < NX_2^E$$

Thus

$$C_1^{r,E} > C_1^{p,E} = C_2^{p,E} > C_2^{r,E}$$

Let

$$\kappa_t^E \equiv \frac{P_t^{T*}\theta\bar{N}}{(1+\chi)\zeta(\epsilon_t^E)^{\chi-1}}$$

Expanding the two Ω terms at the equal-input-price allocation gives

$$\begin{aligned}
&\epsilon_2^E u'(C_2^{r,E}) + \Omega_2^E - \epsilon_1^E u'(C_1^{r,E}) - \Omega_1^E \\
&= (\epsilon_2^E - \epsilon_1^E)u'(C_1^{p,E}) + (\epsilon_1^E - \lambda\kappa_1^E)\left[u'(C_1^{p,E}) - u'(C_1^{r,E})\right] \\
&\quad + (\epsilon_2^E - \lambda\kappa_2^E)\left[u'(C_2^{r,E}) - u'(C_2^{p,E})\right]
\end{aligned}$$

The first term is strictly positive. Since $NX_2^E > 0$,

$$P_2^{T*}\theta\bar{N} < \zeta(\epsilon_2^E)^{\chi}$$

so

$$\lambda\kappa_2^E < \epsilon_2^E$$

Hence the third term is strictly positive. The condition

$$\frac{\lambda}{1+\chi} \leqslant \frac{p+\beta}{p(1+\beta p^{\chi})}$$

is equivalent, using (78) and $\epsilon_2^E = p\epsilon_1^E$, to

$$\lambda P_1^{T*}\theta\bar{N} \leqslant (1+\chi)\zeta(\epsilon_1^E)^{\chi}$$

and hence to

$$\lambda\kappa_1^E \leqslant \epsilon_1^E$$

Therefore the second term is non-negative, and so

$$\epsilon_2^E u'(C_2^{r,E}) + \Omega_2^E > \epsilon_1^E u'(C_1^{r,E}) + \Omega_1^E$$

We now use the strict concavity argument from Proposition 2. For any feasible allocation with $\epsilon_1 \leqslant \epsilon_1^E$, feasibility and concavity of the external resource constraint imply that the tangent feasibility term at $(\epsilon_1^E, \epsilon_2^E)$ is non-negative. Since the planner gradient at $(\epsilon_1^E, \epsilon_2^E)$ points strictly toward larger ϵ_1 along the feasible frontier, strict concavity implies that every feasible allocation with $\epsilon_1 \leqslant \epsilon_1^E$ is dominated. Hence the constrained efficient allocation must satisfy

$$\epsilon_1^O > \epsilon_1^E$$

Since both allocations satisfy (78) and D is strictly increasing along the feasible frontier, this implies

$$\epsilon_1^O P_1^{T*} - \epsilon_2^O P_2^{T*} \geqslant 0$$

Together with the directional comparison in Proposition 2, this gives

$$0 \leqslant \epsilon_1^O P_1^{T*} - \epsilon_2^O P_2^{T*} < \epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}$$

Multiplying by $\theta\bar{N}$ gives

$$0 \leqslant Y_2^{N,O} - Y_1^{N,O} < Y_2^{N,LF} - Y_1^{N,LF}$$

We next prove the mirror cutoff result. Suppose first that no feasible allocation satisfying (78) and

$$\epsilon_1 P_1^{T*} - \epsilon_2 P_2^{T*} = -\left(\epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}\right)$$

exists in the direction

$$\epsilon_1 < \epsilon_1^{LF}, \qquad \epsilon_2 > \epsilon_2^{LF}$$

Since $D(\epsilon_1^{LF}, \epsilon_2^{LF}) > 0$ and D is strictly increasing along the feasible frontier, the constrained efficient allocation, which satisfies $\epsilon_1^O < \epsilon_1^{LF}$ by Proposition 2, must satisfy

$$\epsilon_1^O P_1^{T*} - \epsilon_2^O P_2^{T*} > -\left(\epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}\right)$$

Using again

$$\epsilon_1^O P_1^{T*} - \epsilon_2^O P_2^{T*} < \epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}$$

we obtain

$$|\epsilon_1^O P_1^{T*} - \epsilon_2^O P_2^{T*}| < |\epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}|$$

Now suppose the mirror allocation $(\epsilon_1^M, \epsilon_2^M)$ is feasible. By construction,

$$\epsilon_1^M P_1^{T*} - \epsilon_2^M P_2^{T*} = -\left(\epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}\right)$$

with

$$\epsilon_1^M < \epsilon_1^{LF}, \qquad \epsilon_2^M > \epsilon_2^{LF}$$

Therefore

$$NX_1^M < NX_1^{LF} < 0 < NX_2^{LF} < NX_2^M$$

and

$$C_1^{r,M} > C_1^{p,M} > C_2^{p,M} > C_2^{r,M}$$

Let

$$\kappa_t^M \equiv \frac{P_t^{T*}\theta\bar{N}}{(1+\chi)\zeta(\epsilon_t^M)^{\chi-1}}$$

Expanding the two Ω terms at the mirror allocation gives

$$\begin{aligned}
&\epsilon_2^M u'(C_2^{r,M}) + \Omega_2^M - \epsilon_1^M u'(C_1^{r,M}) - \Omega_1^M \\
&= (\epsilon_2^M - \epsilon_1^M)u'(C_1^{p,M}) + \epsilon_2^M \left[u'(C_2^{p,M}) - u'(C_1^{p,M})\right] \\
&\quad + (\epsilon_2^M - \lambda\kappa_2^M)\left[u'(C_2^{r,M}) - u'(C_2^{p,M})\right] \\
&\quad + (\epsilon_1^M - \lambda\kappa_1^M)\left[u'(C_1^{p,M}) - u'(C_1^{r,M})\right]
\end{aligned}$$

The first two terms are strictly positive. Since $NX_2^M > 0$,

$$\lambda\kappa_2^M < \epsilon_2^M$$

so the third term is strictly positive. The condition

$$-NX_1^M \leqslant \left(1 - \frac{\lambda}{1+\chi}\right) P_1^{T*}\theta\bar{N}$$

is equivalent to

$$\lambda P_1^{T*}\theta\bar{N} \leqslant (1+\chi)\zeta(\epsilon_1^M)^{\chi}$$

and hence to

$$\lambda\kappa_1^M \leqslant \epsilon_1^M$$

Therefore the fourth term is non-negative, and so

$$\epsilon_2^M u'(C_2^{r,M}) + \Omega_2^M > \epsilon_1^M u'(C_1^{r,M}) + \Omega_1^M$$

The strict-concavity argument above implies that the constrained efficient allocation lies before the mirror allocation:

$$\epsilon_1^O > \epsilon_1^M$$

Since D is strictly increasing along the feasible frontier,

$$\epsilon_1^O P_1^{T*} - \epsilon_2^O P_2^{T*} > -\left(\epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}\right)$$

Together with

$$\epsilon_1^O P_1^{T*} - \epsilon_2^O P_2^{T*} < \epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}$$

this gives

$$|\epsilon_1^O P_1^{T*} - \epsilon_2^O P_2^{T*}| < |\epsilon_1^{LF} P_1^{T*} - \epsilon_2^{LF} P_2^{T*}|$$

Multiplying by $\theta\bar{N}$ gives

$$\left|Y_2^{N,O} - Y_1^{N,O}\right| < \left|Y_2^{N,LF} - Y_1^{N,LF}\right|$$

□

A.8 Proof of Proposition 3

Proof. Since $N_t = \bar{N}$ and $A_t = A$, output is $Y_t = A\bar{N}$ at all allocations and in both periods. All share comparisons therefore reduce to comparisons of levels. Proposition 2 gives $\epsilon_1^O < \epsilon_1^{LF}$, $\epsilon_2^O > \epsilon_2^{LF}$, $NX_1^O < NX_1^{LF} < 0 < NX_2^{LF} < NX_2^O$, $Y_1^{N,O} > Y_1^{N,LF}$, and $Y_2^{N,O} < Y_2^{N,LF}$.

Export shares. Export demand satisfies $X_t = \zeta\epsilon_t^{1+\chi}$, which is strictly increasing in ϵ_t. Hence $X_1^O < X_1^{LF}$ and $X_2^O > X_2^{LF}$. The export share of output is strictly lower in period 1 and strictly higher in period 2 under the constrained efficient allocation.

Domestic consumption shares. The resource constraint gives $C_t = Y_t - X_t = A\bar{N} - X_t$. Since output is the same across allocations, the domestic share moves opposite to the export share: $C_1^O > C_1^{LF}$ and $C_2^O < C_2^{LF}$. The domestic share is strictly higher in period 1 and strictly lower in period 2 under the constrained efficient allocation.

HtM consumption shares. HtM households consume their current labor income: $C_t^p = w_t\bar{N} = Y_t^N$. Hence $C_1^{p,O} > C_1^{p,LF}$ and $C_2^{p,O} < C_2^{p,LF}$. The HtM share is strictly higher in period 1 and strictly lower in period 2 under the constrained efficient allocation.

Ricardian consumption share in period 2. From the consumption function (18), with $NX_t > 0$ in period 2, $C_2^r = Y_2^N - \epsilon_2 NX_2/(1-\lambda)$. Since $\epsilon_2^O > \epsilon_2^{LF} > 0$ and $NX_2^O > NX_2^{LF} > 0$, $\epsilon_2^O NX_2^O > \epsilon_2^{LF} NX_2^{LF}$. Therefore

$$C_2^{r,O} = Y_2^{N,O} - \frac{\epsilon_2^O NX_2^O}{1-\lambda} < Y_2^{N,LF} - \frac{\epsilon_2^{LF} NX_2^{LF}}{1-\lambda} = C_2^{r,LF}$$

The Ricardian share is strictly lower in period 2 under the constrained efficient allocation.

Ricardian consumption share in period 1. From the consumption function (18), with $NX_1 < 0$, let $B \equiv -NX_1 > 0$. Then $C_1^r = Y_1^N + \epsilon_1 B/(1-\lambda)$, so

$$C_1^{r,O} - C_1^{r,LF} = \left(Y_1^{N,O} - Y_1^{N,LF}\right) + \frac{1}{1-\lambda}\left(\epsilon_1^O B^O - \epsilon_1^{LF} B^{LF}\right)$$

The first term is strictly positive. Since $\epsilon_1^O < \epsilon_1^{LF}$ and $B^O = -NX_1^O > -NX_1^{LF} = B^{LF}$, the sign of the second term is ambiguous. Hence the sign of $C_1^{r,O} - C_1^{r,LF}$ is ambiguous. □

A.9 Proof of Proposition 4

Proof. From (77), $\hat{\tau}_t^N = 1 - v'(N_t)/(\hat{w}_t \bar{u}'_t) = \Lambda_t/(\hat{w}_t \bar{u}'_t)$. Since u is strictly increasing, $\bar{u}'_t > 0$, and by assumption $\hat{w}_t > 0$. Thus it suffices to show $\Lambda_t > 0$ whenever $NX_t \neq 0$.

From (15), $(\partial NX_t/\partial N_t)/(\partial NX_t/\partial \epsilon_t) < 0$. Moreover, (17) and (18) imply $C_t^r - C_t^p = -\epsilon_t NX_t/(1-\lambda)$. If $NX_t > 0$, then $C_t^r < C_t^p$, so strict concavity of u gives $u'(C_t^p) - u'(C_t^r) < 0$. If $NX_t < 0$, then $C_t^r > C_t^p$, so $u'(C_t^p) - u'(C_t^r) > 0$. Hence $\left[u'(C_t^p) - u'(C_t^r)\right] NX_t < 0$ whenever $NX_t \neq 0$. Since $\lambda \in (0,1)$, (76) gives $\Lambda_t > 0$, and therefore $\hat{\tau}_t^N > 0$. □

A.10 Proof of Proposition 5

Proof. Since $A_t = A$, $P_t^{T*} = P^{T*}$, and $\beta R^* = 1$, we have $\Phi_2(\cdot;\mu) = \beta\, \Phi_1(\cdot;\mu)$ for all μ. Since $\beta > 0$, the maximizer is unchanged, so by Lemma 5, $N_1^*(\mu) = N_2^*(\mu)$ and $\epsilon_1^*(\mu) = \epsilon_2^*(\mu)$ for all μ. Since the two periods share the same $(N^*, \epsilon^*, A, P^{T*})$, we have $NX_1 = NX_2$, so (78) gives $NX_1 = NX_2 = 0$. Substituting into the consumption functions yields $C_t^p = C_t^r$ for $t \in \{1,2\}$, and hence $C_1^p = C_1^r = C_2^p = C_2^r$.

Since $C_t^p = C_t^r$, we have $\Omega_t = \Lambda_t = 0$. Equation (73) then reduces to the undistorted Ricardian Euler equation, giving $\tau = 0$. Equation (77) reduces to $v'(N_t) = \bar{u}'_t \hat{w}_t$, so $1 - \tau_t^N = v'(N_t)/(\bar{u}'_t w_t) = \hat{w}_t/w_t$. Hence $\tau_t^N = \frac{1}{\chi}\epsilon_t P^{T*}\theta/w_t > 0$, since $v'(N_t) > 0$ and $\bar{u}'_t > 0$ imply $\hat{w}_t > 0$ and therefore $w_t > 0$. □

A.11 Proof of Proposition 6

Proof. For a fixed import price P, let $h(P, \mu)$ denote the trade balance chosen by the unique interior maximizer of the static problem $\Phi(N, \epsilon; \mu)$ in Lemma 5. Step 2 in the proof of that lemma implies that $h(P, \mu)$ is strictly increasing in μ.

Let $\mu^0(P)$ denote the multiplier associated with the balanced-trade static optimum, $h(P, \mu^0(P)) = 0$. At balanced trade, $C^p = C^r$, so the static problem coincides with the representative-agent static problem. The first-order condition with respect to ϵ gives $\mu^0(P) = \frac{1+\chi}{\chi}\epsilon(P)C(P)^{-\sigma}$. Along the balanced-trade optimum, the proof of Lemma 2 shows that $\epsilon(P)C(P)^{-\sigma}$ is strictly increasing in P when $0 < \chi \leqslant 1$ and $\sigma > 1/(1+\chi)$. Hence $\mu^0(P_1^{T*}) > \mu^0(P_2^{T*})$.

Let μ^* denote the multiplier for the constrained efficient allocation, and write $h_t(\mu) \equiv h(P_t^{T*}, \mu)$. Since the allocation satisfies the intertemporal budget constraint,

$$h_1(\mu^*) + \frac{1}{R^*}h_2(\mu^*) = 0$$

Moreover, $h_1(\mu^0(P_2^{T*})) < h_1(\mu^0(P_1^{T*})) = 0$ and $h_2(\mu^0(P_1^{T*})) > h_2(\mu^0(P_2^{T*})) = 0$. Since $h_1(\mu) + R^{*-1}h_2(\mu)$ is strictly increasing in μ, it follows that

$$\mu^0(P_2^{T*}) < \mu^* < \mu^0(P_1^{T*})$$

Therefore

$$NX_1 = h_1(\mu^*) < 0 < NX_2 = h_2(\mu^*)$$

Since $C_t^r - C_t^p = -\epsilon_t NX_t/(1-\lambda)$, we have $C_1^r > C_1^p$ and $C_2^r < C_2^p$. Strict concavity of u then gives $u'(C_1^p) > u'(C_1^r)$ and $u'(C_2^p) < u'(C_2^r)$. Since $-(\partial Y_t^N/\partial\epsilon_t)/(\partial NX_t/\partial\epsilon_t) > 0$, (71) gives $\Omega_1 > 0 > \Omega_2$. Using (80) and $\beta R^* = 1$, $\tau = (\Omega_1 - \Omega_2)/[\epsilon_1 u'(C_1^r)] > 0$.

Since $NX_t \neq 0$ for both periods and $\hat{w}_t > 0$, Proposition 4 implies $\hat{\tau}_t^N > 0$. □

B Quantitative Model Appendix

B.1 Unions' Optimization Problem and the Wage Phillips Curve

At time t, union k sets the nominal wage $W_{k,t}$ to maximize (52) subject to the task demand schedule

$$N_{k,t} = \left(\frac{W_{k,t}}{W_t}\right)^{-\varepsilon_w} N_t$$

Each union is infinitesimal and takes W_t and N_t as given. Household real earnings are

$$Z_t = \frac{1}{P_t}\int_0^1 W_{k,t}\left(\frac{W_{k,t}}{W_t}\right)^{-\varepsilon_w} N_t \, dk$$

By the envelope theorem, we can evaluate how each household, j, values a marginal increase in labor incomes and hours worked. Then $\frac{\partial C_t^j}{\partial W_{k,t}} = \frac{\partial Z_t}{\partial W_{k,t}}$, where

$$\frac{\partial Z_t}{\partial W_{k,t}} = \frac{1}{P_t} N_{k,t}(1-\varepsilon_w)$$

On the other hand, total hours worked by household j are $N_t^j \equiv \int_0^1 \left(\frac{W_{k,t}}{W_t}\right)^{-\varepsilon_w} N_t \, dk$ which falls when $W_{k,t}$ rises according to

$$\frac{\partial N_t^j}{\partial W_{k,t}} = -\varepsilon_w \frac{N_{k,t}}{W_{k,t}}$$

Therefore, the union's first order condition is given by:

$$\left(\frac{W_{k,t}}{W_{k,t-1}} - 1\right)\frac{W_{k,t}}{W_{k,t-1}} \quad = \quad \frac{\varepsilon_w}{\psi_{nr}}\left[N_{k,t}\nu'(N_t) - \frac{\varepsilon_w - 1}{\varepsilon_w}\frac{W_{k,t}}{P_t} N_{k,t}\bar{u}'_t\right] \; + \; \beta\left(\frac{W_{k,t+1}}{W_{k,t}} - 1\right)\frac{W_{k,t+1}}{W_{k,t}}$$

where $\bar{u}'_t \equiv \lambda u'(C_t^p) + (1-\lambda)u'(C_t^r)$. In a symmetric equilibrium, $W_{k,t} = W_t$ and $N_{k,t} = N_t$ for all k. Let $\mu_w \equiv \varepsilon_w/(\varepsilon_w - 1)$. Then

$$\left(\frac{W_t}{W_{t-1}} - 1\right)\frac{W_t}{W_{t-1}} = \frac{\varepsilon_w}{\psi_{nr}}\left[N_t\nu'(N_t) - \frac{1}{\mu_w}\frac{W_t}{P_t}N_t\bar{u}'_t\right] + \beta\left(\frac{W_{t+1}}{W_t} - 1\right)\frac{W_{t+1}}{W_t}$$

Using $w_t = W_t/P_t$ and $\Pi_t = P_t/P_{t-1}$, this can be written as

$$\left(\frac{w_t}{w_{t-1}}\Pi_t - 1\right)\frac{w_t}{w_{t-1}}\Pi_t \quad = \quad \frac{\varepsilon_w}{\psi_{nr}}\left[N_t\nu'(N_t) - \frac{1}{\mu_w}w_t N_t \bar{u}'_t\right] \; + \; \beta\left(\frac{w_{t+1}}{w_t}\Pi_{t+1} - 1\right)\frac{w_{t+1}}{w_t}\Pi_{t+1}$$

Rearranging gives $\nu'(N_t) = (1-\tau_t^N)w_t\bar{u}'_t$, where

$$1 - \tau_t^N = \frac{1}{\mu_w} + \frac{\left(\frac{w_t}{w_{t-1}}\Pi_t - 1\right)\frac{w_t}{w_{t-1}}\Pi_t - \beta\left(\frac{w_{t+1}}{w_t}\Pi_{t+1} - 1\right)\frac{w_{t+1}}{w_t}\Pi_{t+1}}{\frac{\varepsilon_w}{\psi_{nr}}w_t N_t \bar{u}'_t}$$

B.2 Balance of Payments and Intertemporal Budget Constraint

Aggregating the Ricardian and HtM household budget constraints gives the aggregate household budget constraint:

$$C_t + B_{t+1} = w_t N_t + R_t B_t + (1-\lambda)\frac{T_t}{P_t} \tag{88}$$

Adding (88) to the government budget constraint (61) eliminates transfers and gives

$$C_t + B_{t+1} + B^G_{t+1} + \epsilon_t B^{G*}_{t+1} = w_t N_t + R_t(B_t + B^G_t) + \epsilon_t R^* B^{G*}_t \tag{89}$$

Firm zero profits and goods market clearing are given by

$$w_t N_t = A_t N_t - \epsilon_t P^{T*}_t \theta N_t \qquad C_t + \zeta \epsilon_t^{1+\chi} = A_t N_t \tag{90}$$

Substituting (90) into (89) yields

$$B_{t+1} + B^G_{t+1} + \epsilon_t B^{G*}_{t+1} = \zeta \epsilon_t^{1+\chi} - \epsilon_t P^{T*}_t \theta N_t + R_t(B_t + B^G_t) + \epsilon_t R^* B^{G*}_t \tag{91}$$

Finally, using domestic bond market clearing (64), and dividing by ϵ_t, gives the period-by-period balance of payments condition (65).

To obtain the intertemporal budget constraint, use the intermediary supply rule (58) for the inherited position F^*_t when $t \geqslant 2$:

$$R_t \frac{\epsilon_{t-1}}{\epsilon_t} F^*_t = R^* F^*_t - \Gamma R^* (F^*_t)^2 \tag{92}$$

Substituting (92) into (65) gives

$$P^{T*}_t \theta N_t + \Gamma R^* (F^*_t)^2 + F^*_{t+1} + B^{G*}_{t+1} = \zeta \epsilon_t^{\chi} + R^*(F^*_t + B^{G*}_t) \tag{93}$$

for all $t \geqslant 2$. The period-1 condition stays unchanged:

$$P^{T*}_1 \theta N_1 + F^*_2 + B^{G*}_2 = \zeta \epsilon_1^{\chi} + R_1 \frac{\epsilon_0}{\epsilon_1} F^*_1 + R^* B^{G*}_1 \tag{94}$$

The no-Ponzi condition on households, (49), and government's asset domestic position, (62), together with the domestic bonds market clearing, (64), imply:

$$\lim_{T\to\infty} \frac{\epsilon_T F^*_{T+1}}{\prod_{s=2}^{T} R_s} = 0 \tag{95}$$

Since $\prod_{s=2}^{T} \tilde{R}_s = (\prod_{s=2}^{T} R_s)\epsilon_1/\epsilon_T$, (95) implies $F^*_{T+1}/\prod_{s=2}^{T} \tilde{R}_s \to 0$. The interest-rate consistency condition, (63), then gives

$$\lim_{T\to\infty} \frac{F^*_{T+1}}{R^{*T-1}} = 0 \tag{96}$$

Together with the government no-Ponzi condition on B^{G*}_{T+1}, (62), we have:

$$\lim_{T\to\infty} \frac{F^*_{T+1} + B^{G*}_{T+1}}{R^{*T-1}} = 0 \tag{97}$$

Combining (94) with (93), discounting by R^{*t-1}, summing from $t = 1$ to T, gives

$$\sum_{t=1}^{T} \frac{P^{T*}_t \theta N_t}{R^{*t-1}} + \sum_{t=1}^{T-1} \frac{\Gamma(F^*_{t+1})^2}{R^{*t-1}} + \frac{F^*_{T+1} + B^{G*}_{T+1}}{R^{*T-1}} = \sum_{t=1}^{T} \frac{\zeta \epsilon^X_t}{R^{*t-1}} + R^* B^{G*}_1 + R_1 \frac{\epsilon_0}{\epsilon_1} F^*_1 \tag{98}$$

Taking $T \to \infty$, using (97) and substituting $\Gamma(F^*_{t+1})^2 = \Gamma^{-1}(1 - \tilde{R}_{t+1}/R^*)^2$ from (58) yields the intertemporal budget constraint, (66).

B.3 Decentralized Equilibrium Definition

Given:

- Initial states: $\{R^M_1, B_1, B^{G*}_1, \epsilon_0, F^*_1, w_0\}$,
- Exogenous sequence of import prices and productivity, $\{P^{T*}_t, A_t\}^{\infty}_{t=1}$,
- Fiscal-monetary policy: $\{B^G_{t+1}, B^{G*}_{t+1}, R^M_{t+1}, \frac{T_t}{P_t}\}^{\infty}_{t=1}, R_1$,

a decentralized equilibrium consists of an allocation: $\{C^r_t, C^p_t, B_{t+1}, N_t, M_t, Y_t, F^*_{t+1}, X_t\}^{\infty}_{t=1}$ and prices $\{\epsilon_t, w_t, \Pi_t, R_{t+1}\}^{\infty}_{t=1}$ such that:

- Given prices, Ricardian households solve (47) and HtM households satisfy their period-by-period budget constraints:

$$\frac{u'(C^r_t)}{\beta u'(C^r_{t+1})} = R_{t+1}$$

$$C^r_t + \frac{B_{t+1}}{1-\lambda} = w_t N_t + R_t \frac{B_t}{1-\lambda} + \frac{T_t}{P_t}$$

$$\lim_{T\to\infty} \frac{B_{T+1}}{\prod_{s=2}^{T} R_s} = 0$$

$$C^p_t = w_t N_t$$

- Given prices, firms solve (3):

$$M_t = \theta N_t, \quad Y_t = A_t N_t$$

Zero profits imply:

$$w_t = A_t - \epsilon_t P_t^{T*} \theta$$

- Labor aggregator and unions' optimization gives the wage Phillips curve:

$$\left(\frac{w_t}{w_{t-1}}\Pi_t - 1\right)\frac{w_t}{w_{t-1}}\Pi_t = \frac{\varepsilon_w}{\psi_{nr}}\left[N_t v'(N_t) - \frac{1}{\mu_w} w_t N_t \bar{u}'_t\right] + \beta\left[\left(\frac{w_{t+1}}{w_t}\Pi_{t+1} - 1\right)\frac{w_{t+1}}{w_t}\Pi_{t+1}\right]$$

- Given prices, international financial intermediaries optimize:

$$F^*_{t+1} = \frac{1}{\Gamma}\left[1 - \frac{R_{t+1}}{R^*}\left(\frac{\epsilon_t}{\epsilon_{t+1}}\right)\right]$$

- World demand for domestic output is given by:

$$X_t = \zeta \epsilon_t^{1+\chi}$$

- Fiscal policy satisfies the period-by-period budget constraints:

$$B^G_{t+1} + \epsilon_t B^{G*}_{t+1} + (1-\lambda)\frac{T_t}{P_t} = R_t B^G_t + \epsilon_t R^* B^{G*}_t$$

where $B^G_1 = \epsilon_0 F^*_1 - B_1$; satisfies no-Ponzi conditions:

$$\lim_{T\to\infty} \frac{B^G_{T+1}}{\prod_{s=2}^T R_s} = 0, \quad \lim_{T\to\infty} \frac{B^{G*}_{T+1}}{{R^*}^{T-1}} = 0$$

together with the interest rate consistency condition:

$$0 < \inf_{T\geqslant 2} \prod_{s=2}^T \frac{\tilde{R}_s}{R^*} \leqslant \sup_{T\geqslant 2} \prod_{s=2}^T \frac{\tilde{R}_s}{R^*} < \infty$$

- Inflation is implied by monetary policy:

$$R^M_t = R_t \Pi_t, \qquad t \geqslant 1$$

- Final goods market clears:

$$\lambda C^p_t + (1-\lambda) C^r_t + X_t = A_t N_t$$

- Labor market clears.

- Domestic bonds market clears:

$$B_{t+1}+B^G_{t+1}=\epsilon_t F^*_{t+1}$$

- Balance of payments is implied by all other budget constraints and market clearing conditions holding:

$$P^{T*}_t\theta N_t+F^*_{t+1}+B^{G*}_{t+1}=\zeta\epsilon^{\chi}_t+R_t\frac{\epsilon_{t-1}}{\epsilon_t}F^*_t+R^*B^{G*}_t$$

B.4 Optimal Policy Problem

Given B^{G*}_1 and under the assumption that $F^*_1=0$, consider the constrained planning problem:

$$\max_{\{N_t,\epsilon_t,\tilde{R}_{t+1},C^r_t,C^p_t\}_{t=1}^{\infty}}\sum_{t=1}^{\infty}\beta^{t-1}\left[\lambda u\left(C^p_t\right)+(1-\lambda)u\left(C^r_t\right)-\nu(N_t)\right]\qquad \text{s.t.}$$

$$\sum_{t=1}^{\infty}\frac{\frac{1}{\Gamma}\left(1-\frac{\tilde{R}_{t+1}}{R^*}\right)^2}{{R^*}^{t-1}}+\sum_{t=1}^{\infty}\frac{P^{T*}_t\theta N_t}{{R^*}^{t-1}}=\sum_{t=1}^{\infty}\frac{\zeta\epsilon^{\chi}_t}{{R^*}^{t-1}}+R^*B^{G*}_1+R_1\frac{\epsilon_0F^*_1}{\epsilon_1}$$

$$\tilde{R}_{t+1}=\frac{u'\left(C^r_t\right)\epsilon_t}{\beta u'\left(C^r_{t+1}\right)\epsilon_{t+1}}\tag{99}$$

$$C^r_t=A_tN_t-\frac{\zeta\epsilon^{1+\chi}_t}{1-\lambda}+\frac{\lambda}{1-\lambda}\epsilon_tP^{T*}_t\theta N_t$$

$$C^p_t=(A_t-\epsilon_tP^{T*}_t\theta)N_t$$

$$N_t,\epsilon_t,\tilde{R}_{t+1},C^r_t,C^p_t\geqslant 0$$

The following Lemmas establish conditions under which a solution to (99) is implementable as a decentralized equilibrium defined in Section B.3.

Lemma 7. *Given* $F^*_1=0$*,* B^{G*}_1*,* $\beta R^*=1$*, and positive sequences* $\{A_t,P^{T*}_t\}_{t\geqslant 1}$ *satisfying* $A_t\leqslant\bar{A}<\infty$ *and* $P^{T*}_t\geqslant\underline{P}>0$ *for all* $t\geqslant 1$*, let* $\{N_t,\epsilon_t,\tilde{R}_{t+1},C^r_t,C^p_t\}_{t\geqslant 1}$ *solve* (99)*. Suppose further that the solution satisfies for all* $t\geqslant 1$*:*

$$\epsilon_t\geqslant\underline{\epsilon},\qquad \underline{C}^r\leqslant C^r_t\leqslant\bar{C}^r$$

where $\underline{\epsilon},\underline{C}^r,\bar{C}^r>0$*. In addition, assume all conditions of Lemma 8 are satisfied. Then there exists a non-unique sequence of transfers* $\{T_t/P_t\}_{t\geqslant 1}$ *such that the solution to* (99) *is implementable as a decentralized equilibrium defined in Section B.3.*

Proof. Construct w_t, M_t, Y_t, X_t, F^*_{t+1}, and R_{t+1} directly:

$$w_t = A_t - \epsilon_t P_t^{T*}\theta, \qquad M_t = \theta N_t, \qquad Y_t = A_t N_t, \qquad X_t = \zeta \epsilon_t^{1+\chi}$$

$$F^*_{t+1} = \frac{1}{\Gamma}\left(1 - \frac{\tilde{R}_{t+1}}{R^*}\right)$$

$$R_{t+1} = \tilde{R}_{t+1}\frac{\epsilon_{t+1}}{\epsilon_t}$$

Since Ricardian Euler equation and HtM budget constraint are implementability conditions of (99), they are already satisfied. Using the wage determination equation, firms' zero profit condition: $A_t N_t = w_t N_t + \epsilon_t P_t^{T*}\theta N_t$ also holds. The condition $C_t^r = A_t N_t - \frac{\zeta \epsilon_t^{1+\chi}}{1-\lambda} + \frac{\lambda}{1-\lambda}\epsilon_t P_t^{T*}\theta N_t$ then implies that the goods market clearing condition holds: $(1-\lambda)C_t^r + \lambda C_t^p + X_t = A_t N_t$.

The Ricardian Euler equation and $\beta R^* = 1$ imply

$$\prod_{s=2}^{T}\frac{\tilde{R}_s}{R^*} = \frac{\epsilon_1 u'(C_1^r)}{\epsilon_T u'(C_T^r)}$$

Since $C_t^p = (A_t - \epsilon_t P_t^{T*}\theta)N_t > 0$ and $N_t > 0$,

$$\epsilon_t < \frac{A_t}{P_t^{T*}\theta} \leqslant \frac{\bar{A}}{\underline{P}\theta}$$

The lower bound on ϵ_t and the bounds assumptions on C_t^r therefore imply that there exist constants $0 < \underline{m} < \overline{m} < \infty$ such that $\underline{m} \leqslant \epsilon_t u'(C_t^r) \leqslant \overline{m}$ for all $t \geqslant 1$. Hence the interest rate consistency condition holds:

$$0 < \inf_{T\geqslant 2}\prod_{s=2}^{T}\frac{\tilde{R}_s}{R^*} \leqslant \sup_{T\geqslant 2}\prod_{s=2}^{T}\frac{\tilde{R}_s}{R^*} < \infty$$

Choose any sequence of transfers $\{T_t/P_t\}_{t\geqslant 1}$ satisfying

$$\sum_{t=1}^{\infty}\frac{(1-\lambda)T_t/P_t}{\prod_{s=2}^{t}R_s} = \sum_{t=1}^{\infty}\frac{C_t - w_t N_t}{\prod_{s=2}^{t}R_s} - R_1 B_1 \tag{100}$$

where $C_t = (1-\lambda)C_t^r + \lambda C_t^p$. This choice is non-unique. One admissible choice sets $(1-\lambda)T_1/P_1$ equal to the right-hand side of (100) and sets $T_t/P_t = 0$ for all $t \geqslant 2$. Using goods market clearing and zero profits, we have $C_t - w_t N_t = \epsilon_t\left(P_t^{T*}\theta N_t - \zeta\epsilon_t^{\chi}\right)$. Then

$$\frac{C_t - w_t N_t}{\prod_{s=2}^{t}R_s} = \epsilon_1\frac{P_t^{T*}\theta N_t - \zeta\epsilon_t^{\chi}}{\prod_{s=2}^{t}\tilde{R}_s}$$

By the interest rate consistency condition, there exists $K < \infty$ such that $\frac{1}{\prod_{s=2}^{t}\tilde{R}_s} \leqslant \frac{K}{R^{*t-1}}$ for all $t \geqslant 1$.

The intertemporal budget constraint in (99) implies

$$\sum_{t=1}^{\infty}\left|\frac{C_t - w_t N_t}{\prod_{s=2}^{t} R_s}\right| \leqslant K\epsilon_1 \sum_{t=1}^{\infty} \frac{P_t^{T*}\theta N_t + \zeta\epsilon_t^{\chi}}{{R^*}^{t-1}} < \infty$$

Define $\{B_{t+1}\}_{t\geqslant 1}$ recursively by the consolidated household budget

$$B_{t+1} = w_t N_t + R_t B_t + (1-\lambda)\frac{T_t}{P_t} - C_t, \qquad B_1 \text{ given} \tag{101}$$

Since $C_t^p = w_t N_t$ holds, subtracting $\lambda/(1-\lambda)$ times this from (101) and dividing by $(1-\lambda)$ yields the Ricardian budget:

$$C_t^r + \frac{B_{t+1}}{1-\lambda} = w_t N_t + R_t \frac{B_t}{1-\lambda} + \frac{T_t}{P_t}$$

Iterating (101) from $t = 1$ to T gives

$$\frac{B_{T+1}}{\prod_{s=2}^{T} R_s} = R_1 B_1 + \sum_{t=1}^{T} \frac{w_t N_t + (1-\lambda)T_t/P_t - C_t}{\prod_{s=2}^{t} R_s}$$

By (100), the right-hand side converges to zero as $T \to \infty$, so $\lim_{T\to\infty} \frac{B_{T+1}}{\prod_{s=2}^{T} R_s} = 0$.

Construct B_{t+1}^G from the bonds market clearing condition:

$$B_{t+1}^G = \epsilon_t F_{t+1}^* - B_{t+1}$$

Since $F_1^* = 0$, the initial condition $B_1^G = \epsilon_0 F_1^* - B_1 = -B_1$ holds.

Construct B_{t+1}^{G*} from the period-by-period BoP conditions:

$$B_{t+1}^{G*} = \zeta\epsilon_t^{\chi} + R_t \frac{\epsilon_{t-1}}{\epsilon_t} F_t^* + R^* B_t^{G*} - P_t^{T*}\theta N_t - F_{t+1}^*$$

with $F_1^* = 0, B_1^{G*}$ given. Since the solution satisfies the intertemporal budget constraint, it satisfies the no-Ponzi condition on the net foreign asset position:

$$\lim_{T\to\infty} \frac{B_{T+1}^{G*} + F_{T+1}^*}{{R^*}^{T-1}} = 0$$

Furthermore, since the solution satisfies:

$$\sum_{t=1}^{\infty} \frac{\frac{1}{\Gamma}\left(1 - \frac{\tilde{R}_{t+1}}{R^*}\right)^2}{{R^*}^{t-1}} = \sum_{t=1}^{\infty} \frac{\Gamma\left(F_{t+1}^*\right)^2}{{R^*}^{t-1}} < \infty$$

therefore $\left(F^*_{T+1}\right)^2 / R^{*\,T-1} \to 0$. Hence, the following hold separately

$$\lim_{T\to\infty} \frac{F^*_{T+1}}{R^{*\,T-1}} = 0, \quad \lim_{T\to\infty} \frac{B^{G*}_{T+1}}{R^{*\,T-1}} = 0$$

Using the bonds market clearing condition we have:

$$\frac{B^G_{T+1}}{\prod_{s=2}^T R_s} = \frac{\epsilon_1 F^*_{T+1}}{\prod_{s=2}^T \tilde{R}_s} - \frac{B_{T+1}}{\prod_{s=2}^T R_s}$$

Taking limits: $T \to \infty$, the second term on the RHS vanishes by Household no-Ponzi condition holding. For the first: since $F^*_{T+1}/R^{*\,T-1} \to 0$ and $\sup_T \prod_{s=2}^T (R^*/\tilde{R}_s) < \infty$, then $\frac{F^*_{T+1}}{\prod_{s=2}^T \tilde{R}_s} = \frac{F^*_{T+1}}{R^{*\,T-1}} \cdot \prod_{s=2}^T \frac{R^*}{\tilde{R}_s} \longrightarrow 0$. Hence $\lim_{T\to\infty} \frac{B^G_{T+1}}{\prod_{s=2}^T R_s} = 0$.

The period-by-period government budget constraints hold by Walras' identity: multiplying period-by-period BoP conditions by ϵ_t and substituting $C_t + X_t = A_t N_t$ and $w_t N_t = A_t N_t - \epsilon_t P^{T*}_t \theta N_t$ gives

$$\epsilon_t F^*_{t+1} + \epsilon_t B^{G*}_{t+1} = w_t N_t - C_t + R_t \epsilon_{t-1} F^*_t + \epsilon_t R^* B^{G*}_t$$

Substituting $w_t N_t - C_t = B_{t+1} - R_t B_t - (1-\lambda) T_t / P_t$ and $B_{t+1} + B^G_{t+1} = \epsilon_t F^*_{t+1}$ gives

$$B^G_{t+1} + \epsilon_t B^{G*}_{t+1} + (1-\lambda)\frac{T_t}{P_t} = R_t B^G_t + \epsilon_t R^* B^{G*}_t$$

Finally, to construct the inflation path and the nominal interest rates, construct the labor wedge as:

$$\tau^N_t = 1 - \frac{v'(N_t)}{\bar{u}'_t w_t}$$

Then construct Π_t as a solution to:

$$1 - \tau^N_t = \frac{1}{\mu_w} + \left[\frac{\left(\frac{w_t}{w_{t-1}}\Pi_t - 1\right)\frac{w_t}{w_{t-1}}\Pi_t - \beta\left[\left(\frac{w_{t+1}}{w_t}\Pi_{t+1} - 1\right)\frac{w_{t+1}}{w_t}\Pi_{t+1}\right]}{\frac{\varepsilon_w}{\psi_{nr}} w_t N_t \bar{u}'_t} \right]$$

By Lemma 8, there exists $\{\Pi_t\}_{t\geqslant 1} \subset (0,\infty)$ satisfying the above equation.

Construct R_1, R^M_{t+1} using:

$$R_1 = \frac{R^M_1}{\Pi_1}, \qquad R^M_{t+1} = R_{t+1}\Pi_{t+1}, \quad t \geqslant 1$$

All decentralized equilibrium conditions in Section B.3 are therefore satisfied. Non-uniqueness follows from the non-uniqueness of any transfer sequence satisfying (100). □

Lemma 8. *Let* $\{N_t, \epsilon_t, \tilde{R}_{t+1}, C^r_t, C^p_t\}_{t\geqslant 1}$ *be a solution to* (99), *with* $w_t = A_t - \epsilon_t P^{T*}_t \theta$, *and* $w_0 > 0$ *given. Suppose that, for every* $t \geqslant 1$,

$$\mathcal{X}_t \equiv \sum_{j=t}^{\infty} \beta^{j-t} \frac{\varepsilon_w}{\psi_{nr}} \left[N_j v'(N_j) - \frac{1}{\mu_w} w_j N_j \left[\lambda u'(C^p_j) + (1-\lambda) u'(C^r_j) \right] \right] \in \left(-\frac{1}{4}, \infty \right) \tag{102}$$

Then there exists a sequence $\{\Pi_t\}_{t\geqslant 1} \subset (0,\infty)$ *such that the wage Phillips curve holds for all* $t \geqslant 1$.

Proof. Since $C^p_t = w_t N_t$, $C^p_t > 0$, and $N_t > 0$, we have $w_t > 0$ for all $t \geqslant 1$. Define

$$\Pi_t = \frac{w_{t-1}}{w_t} \frac{1 + \sqrt{1 + 4\mathcal{X}_t}}{2}$$

By (102), $\Pi_t > 0$ for all $t \geqslant 1$ and

$$\mathcal{X}_t = \left(\frac{w_t}{w_{t-1}} \Pi_t - 1 \right) \frac{w_t}{w_{t-1}} \Pi_t$$

Moreover, the definition of $\mathcal{X}_t$ implies

$$\mathcal{X}_t = \frac{\varepsilon_w}{\psi_{nr}} \left[N_t v'(N_t) - \frac{1}{\mu_w} w_t N_t \left[\lambda u'(C^p_t) + (1-\lambda) u'(C^r_t) \right] \right] + \beta \mathcal{X}_{t+1}$$

Therefore the wage Phillips curve holds for all $t \geqslant 1$. □

Lemma 9. *Given* $F^*_1 = 0$, B^{G*}_1, $\beta R^* = 1$, $w_0 > 0$, *and positive sequences* $\{A_t, P^{T*}_t\}_{t\geqslant 1}$ *satisfying* $A_t \to A_{ss} > 0$, *and* $P^{T*}_t \to P^{T*}_{ss} > 0$, *let* $\{N_t, \epsilon_t, \tilde{R}_{t+1}, C^r_t, C^p_t\}_{t\geqslant 1}$ *solve* (99) *with*

$$N_t \to N_{ss} > 0, \qquad \epsilon_t \to \epsilon_{ss} > 0, \qquad C^r_t \to C^r_{ss} > 0, \qquad C^p_t \to C^p_{ss} > 0, \qquad \tilde{R}_{t+1} \to R^* > 0$$

Suppose also that condition (102) *holds for every* $t \geqslant 1$. *Then this solution is implementable as a decentralized equilibrium. Moreover, under the larger-root convention, the inflation path* $\{\Pi_t\}_{t\geqslant 1} \subset (0,\infty)$ *satisfying the wage Phillips curve and converging to a positive finite limit is unique and converges to*

$$\Pi_{ss} = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{1-\beta} \frac{\varepsilon_w}{\psi_{nr}} \left[N_{ss} v'(N_{ss}) - \frac{1}{\mu_w} w_{ss} N_{ss} \bar{u}'_{ss} \right]}$$

where $w_{ss} = A_{ss} - \epsilon_{ss} P^{T*}_{ss} \theta$.

Proof. Since $A_t \to A_{ss} > 0$ and $P^{T*}_t \to P^{T*}_{ss} > 0$, there exist $\bar{A} < \infty$ and $\underline{P} > 0$ such that $A_t \leqslant \bar{A}$ and $P^{T*}_t \geqslant \underline{P}$ for all $t \geqslant 1$. Since $\epsilon_t \to \epsilon_{ss} > 0$ and $C^r_t \to C^r_{ss} > 0$, there exist $\underline{\epsilon}, \underline{C}^r, \bar{C}^r > 0$ such that $\epsilon_t \geqslant \underline{\epsilon}$ and $\underline{C}^r \leqslant C^r_t \leqslant \bar{C}^r$ for all $t \geqslant 1$. Hence all conditions of Lemma 7 are satisfied. Implementability follows as a corollary. Moreover by Lemma 8, there exists an inflation path

$\{\Pi_t\}_{t\geqslant 1} \subset (0,\infty)$ satisfying the wage Phillips curve. It remains to show that the inflation path is unique. Let

$$\mathcal{A}_t = \frac{\varepsilon_w}{\psi_{nr}}\left[N_t v'(N_t) - \frac{1}{\mu_w} w_t N_t \bar{u}'_t\right]$$

Since $w_t = A_t - \epsilon_t P_t^{T*}\theta$, convergence gives $w_t \to w_{ss} > 0$ then

$$\mathcal{A}_t \to \mathcal{A}_{ss} = \frac{\varepsilon_w}{\psi_{nr}}\left[N_{ss} v'(N_{ss}) - \frac{1}{\mu_w} w_{ss} N_{ss} \bar{u}'_{ss}\right]$$

For any inflation path satisfying the wage Phillips curve, the associated $\mathcal{X}_t$ satisfies

$$\mathcal{X}_t = \mathcal{A}_t + \beta \mathcal{X}_{t+1}$$

Thus

$$\mathcal{X}_t = \sum_{j=t}^{\infty} \beta^{j-t}\mathcal{A}_j + \kappa\beta^{-t}, \qquad \kappa \in \mathbb{R}$$

Since $\mathcal{A}_t \to \mathcal{A}_{ss}$,

$$\sum_{j=t}^{\infty} \beta^{j-t}\mathcal{A}_j \to \frac{\mathcal{A}_{ss}}{1-\beta}$$

If $\kappa < 0$, then $\mathcal{X}_t \to -\infty$, so $\mathcal{X}_t < -1/4$ for all sufficiently large t, which is incompatible with $\Pi_t > 0$. If $\kappa > 0$, then $\mathcal{X}_t \to +\infty$, so the larger-root inflation path cannot converge to a finite limit. Hence any positive larger-root inflation path with a finite limit must have $\kappa = 0$. In that case,

$$\mathcal{X}_t = \sum_{j=t}^{\infty} \beta^{j-t}\mathcal{A}_j \to \frac{\mathcal{A}_{ss}}{1-\beta}$$

Under the larger-root convention,

$$\Pi_t = \frac{w_{t-1}}{w_t}\frac{1+\sqrt{1+4\mathcal{X}_t}}{2}$$

Since $w_{t-1}/w_t \to 1$, it follows that

$$\Pi_t \to \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{1-\beta}\frac{\varepsilon_w}{\psi_{nr}}\left[N_{ss} v'(N_{ss}) - \frac{1}{\mu_w} w_{ss} N_{ss} \bar{u}'_{ss}\right]} \equiv \Pi_{ss} \in (0,\infty)$$

The case $\kappa = 0$ is the only case admitting a positive larger-root inflation path with a finite positive limit. Hence the convergent inflation path is unique. □

B.5 Calibration

We calibrate the quantitative model to Japan and interpret the imported input as fossil fuel imports. A period is one quarter. The calibration is designed for MIT shocks to the imported-input price, P_t^{T*}, and chooses the no-shock steady state as the initial state for all policy experiments. Table 3 reports the parameter values used in the quantitative exercises.

The normalizations are $A_{ss} = 1$, $P_{ss}^{T*} = 1$, $\theta = 1$, and $N_{ss} = 1$. The scale parameter ζ is chosen jointly with the steady real exchange rate so that the fossil-fuel import bill is 4 percent of GDP and net exports are -0.5 percent of GDP in the no-shock steady state. Since the no-shock steady state has $F^* = 0$, the public foreign asset position B^{G*} is calibrated to make the domestic value of net foreign assets equal to 60 percent of annual GDP. The world gross interest rate R^* is then chosen to satisfy the steady-state balance of payments at this net-export target.

Preferences use $u(C) = C^{1-\sigma}/(1-\sigma)$ and $v(N) = \eta N^{1+\phi}/(1+\phi)$. The household share λ is set to 0.35, in the range of estimates of hand-to-mouth households in Kaplan, Violante and Weidner (2014). The coefficient of relative risk aversion is $\sigma = 4$. In this model, χ is the Marshall-Lerner elasticity: it is the elasticity of foreign-currency export revenue, $X_t/\epsilon_t = \zeta\epsilon_t^{\chi}$, with respect to the real exchange rate. We set $\chi = 0.25$, so a one percent real depreciation raises the foreign purchasing power supplied by exports by 0.25 percent, equivalently the export-demand price elasticity is $1 + \chi = 1.25$. This imposes a positive but modest Marshall-Lerner response, consistent with empirical work that finds limited short-run trade-balance responses to exchange-rate movements; see Rose (1991) and the survey in Bahmani-Oskooee and Ratha (2004). The financial-intermediation parameter is set to $\Gamma = 9$, following Fanelli and Straub (2021). The wage elasticity is $\varepsilon_w = 6$, corresponding to a steady wage markup $\mu_w = 1.2$, as in the standard staggered-wage formulation of Erceg, Henderson and Levin (2000).

The labor-disutility curvature parameter ϕ and the nominal wage rigidity parameter ψ_{nr} discipline the response to the benchmark import-price shock. We choose $\phi = 29.5$ so that a doubling of import prices lowers gross output, $A_t N_t$, by 1 percent in the representative-agent benchmark, where no household is borrowing constrained. We choose $\psi_{nr} = 28000$ so that the same benchmark produces impact inflation of 5 percent after a doubling of import prices. The persistence of the imported-input price process is $\rho_{P^{T*}} = 0.945$, estimated from an AR(1) regression on detrended quarterly observations of the IMF's Global Price of Energy Index (International Monetary Fund

Table 3: Calibration

Parameter	Description	Value	Target or source
R^*	Gross world interest rate	1.0021	Net exports of −0.5 percent of GDP
β	Discount factor	0.9979	$1/R^*$
σ	Relative risk aversion	4.0	Standard value
η	Labor-disutility scale	0.9261	$N_{ss} = 1$ and steady labor FOC
ϕ	Inverse Frisch curvature	29.50	1 percent output decline after doubling P_t^{T*} in representative-agent benchmark
λ	HtM household share	0.35	Kaplan, Violante and Weidner (2014)
ζ	Export demand scale	1.9758	Fossil-fuel import bill of 4 percent of GDP and net exports of −0.5 percent of GDP
χ	Marshall-Lerner elasticity	0.25	Modest short-run response; Rose (1991), Bahmani-Oskooee and Ratha (2004)
$\rho_{P^{T*}}$	Import-price persistence	0.945	AR(1) on detrended quarterly IMF energy-price index from FRED
Γ	Financial-intermediation friction	9.0	Fanelli and Straub (2021)
ε_w	Wage substitution elasticity	6.0	Wage markup $\mu_w = 1.2$; Erceg, Henderson and Levin (2000)
ψ_{nr}	Nominal wage rigidity	28000.0	5 percent impact inflation after doubling P_t^{T*} in representative-agent benchmark
B_{ss}^{G*}	Public foreign asset position	60.0	Net foreign assets of 60 percent of annual GDP

2026).

B.6 Representative-Agent Benchmark

This appendix reports the responses of the representative-agent economy, in which no household is borrowing constrained and every household participates in international asset markets and remains on its Euler equation, to the same two imported-energy price paths studied in the main text. By Proposition 1, optimal policy leaves the intertemporal margin undistorted and sets the terms-of-trade-adjusted labor wedge to zero, so the paths below coincide with the RA-equivalent policy applied in this economy. All deviations are relative to the representative-agent economy's own no-shock steady state, which differs from the heterogeneous economy's steady state; comparisons with the main-text figures should therefore be made in proportionate terms only.

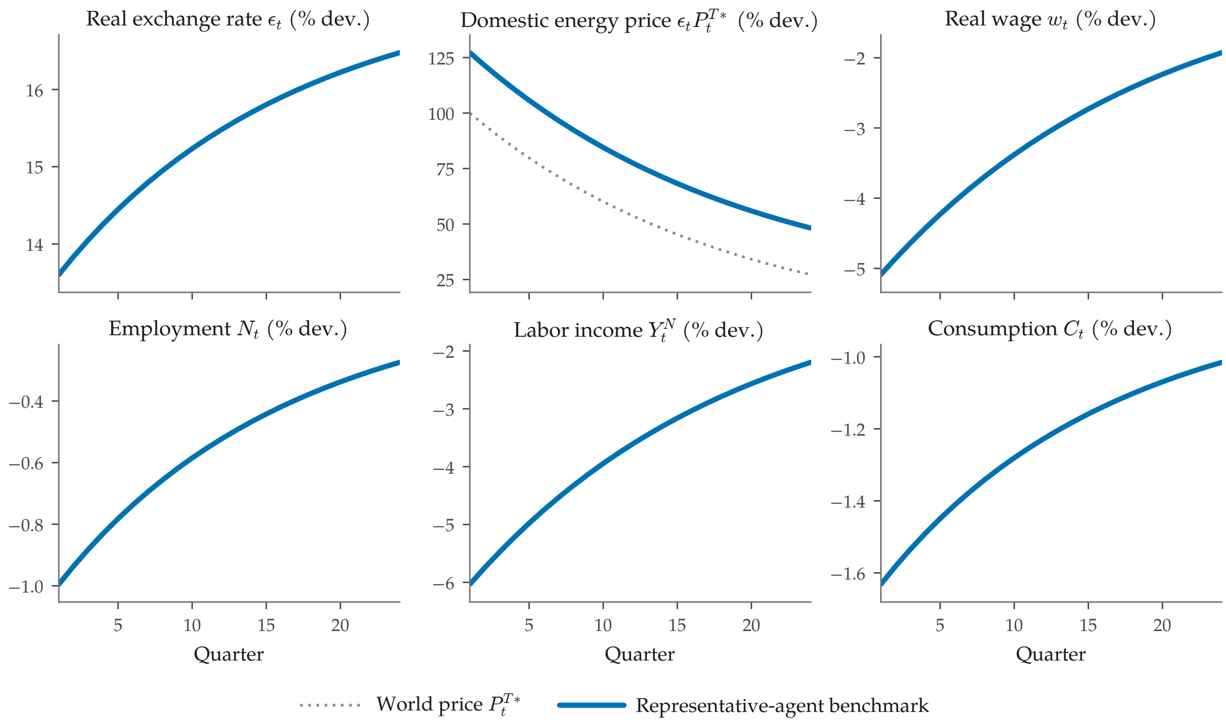

Figure 6. Representative-agent benchmark: MIT-shock allocations

Note: Percent deviations from the representative-agent economy's initial steady state. The dotted line is the world energy price.

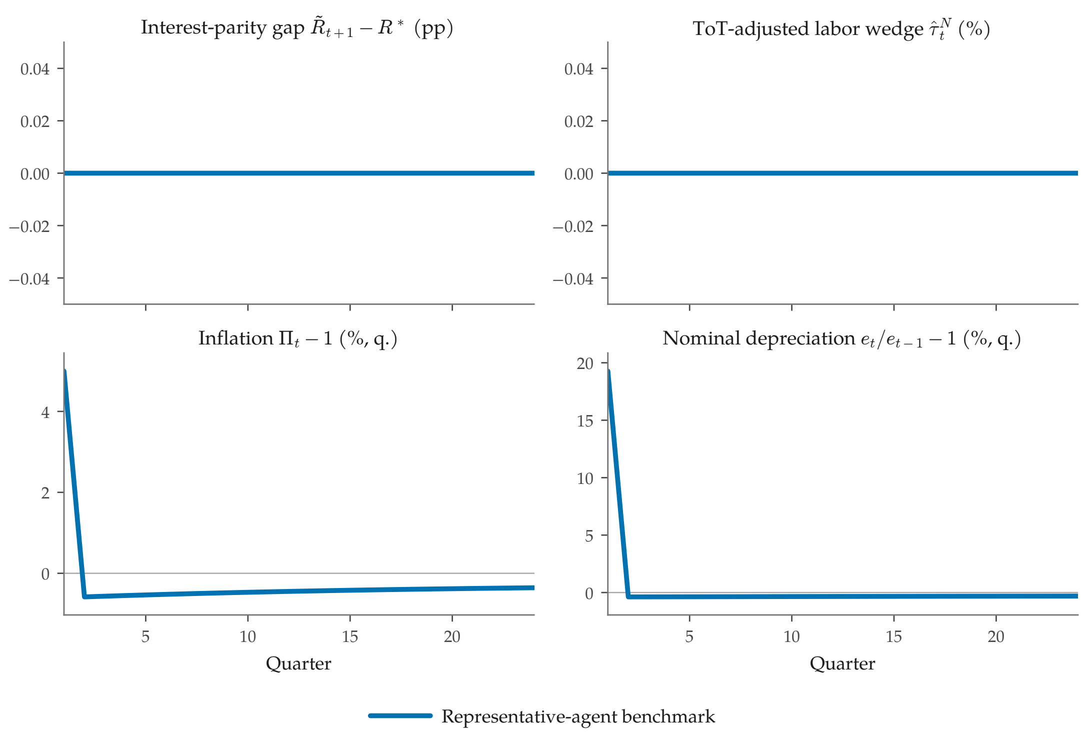

Figure 7. Representative-agent benchmark: MIT-shock policy

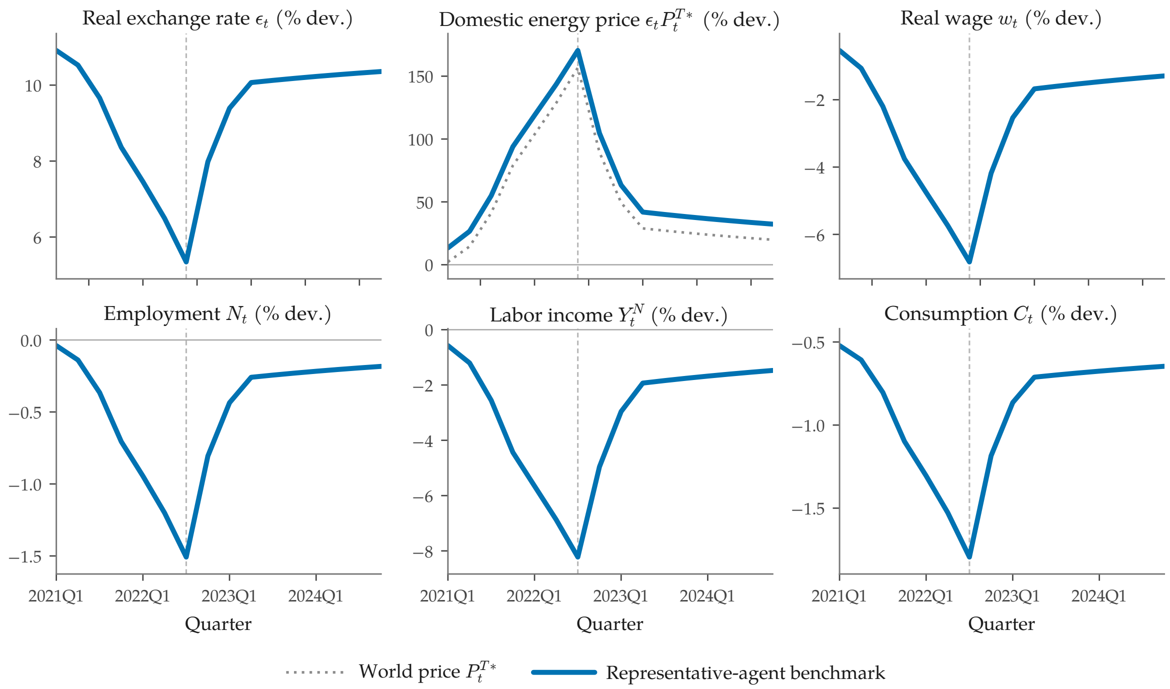

Figure 8. Representative-agent benchmark: 2021–2023 allocations

Note: Percent deviations from the representative-agent economy's initial steady state. The dotted line is the world energy price; the vertical line marks its peak.

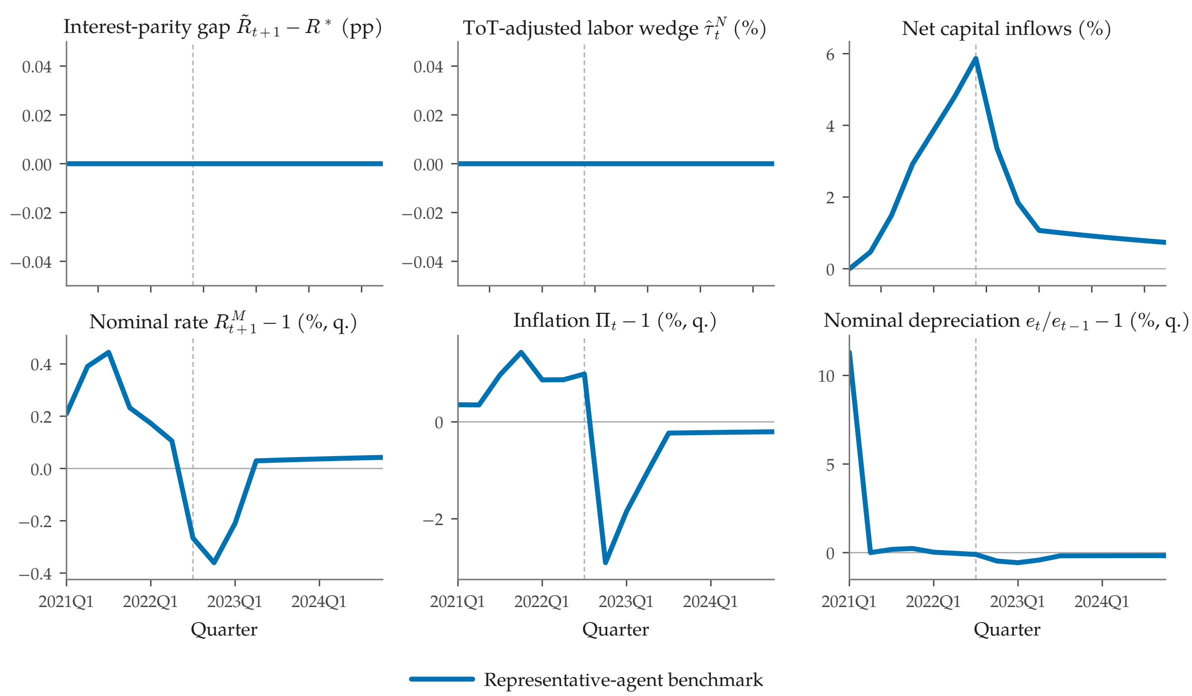

Figure 9. Representative-agent benchmark: 2021–2023 policy

Note: Net capital inflows are shown as a percent of initial steady-state gross output. The vertical line marks the energy-price peak.

B.7 Resource Shares

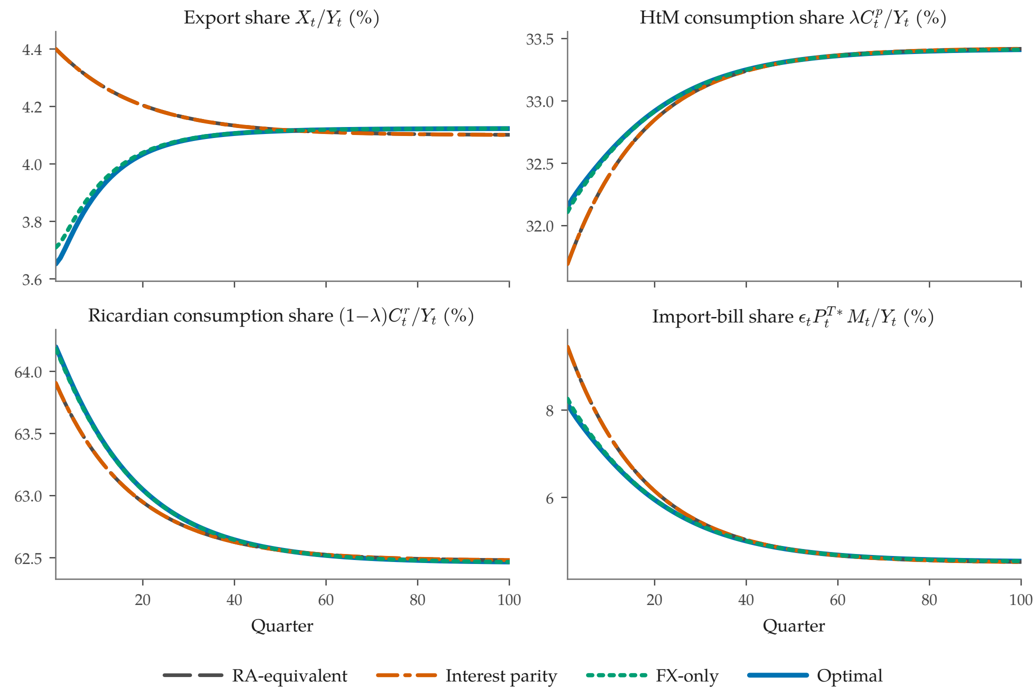

Figure 10. MIT shock: resource shares

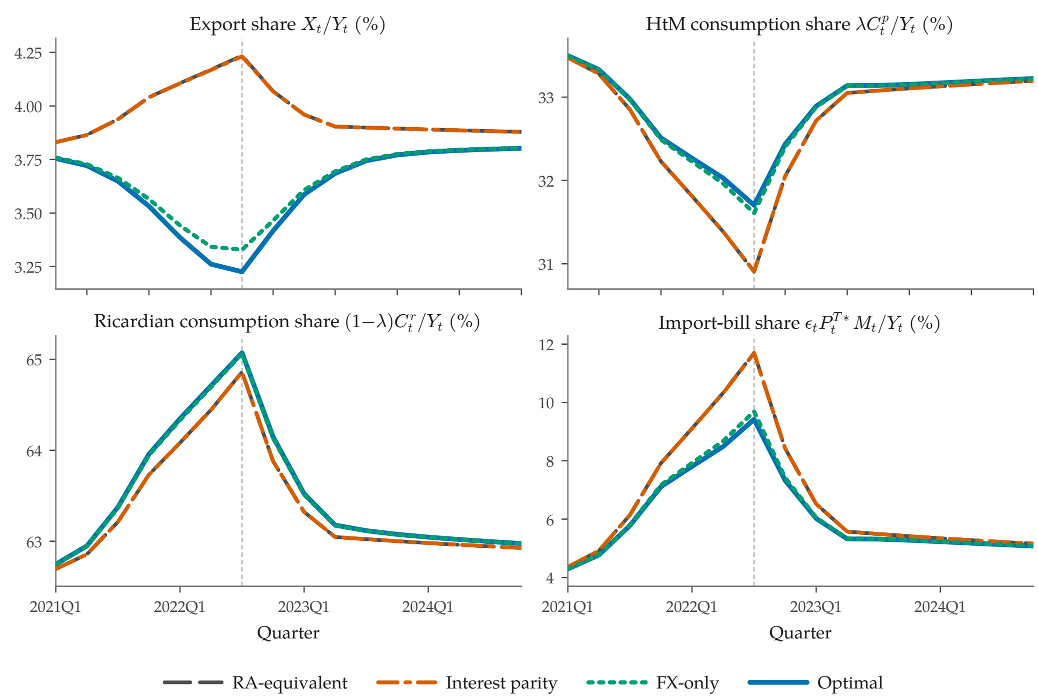

Figure 11. 2021–2023 shock: resource shares

Note: The vertical line marks the energy-price peak.